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COSMIC RAY PENUMBRAL CHARACTERISTICS AT CAIRO'S LATITUDE

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The cosmic ray penumbral characteristics at Cairo's latitude (29.6°*N*, 31.3°*E*) have been investigated. The computations have been done for vertical and 8-different directions namely 16*N*, *S*, *E*, *W* and 32*N*, *S*, *E*, *W* using the trajectory-tracing technique of McCraken et al. The results shows the penumbra width for the directions 16*E*, 32*E*, and 32*S* is equal to zero, while for the others the width varies between 1.32 GV for the direction 16*W* and 4 GV for the direction 32*N*. Also, as a result of the penumbra computations, the cutoff rigidities, main cone, störmer, and effective, have been determined. The obtained values indicates that the difference in the effective vertical cutoffs calculated by the 8th degree International Geomagnetic Reference Field (*IGRF*) model and Finch and Leaton model is approximately equal to zero, over a 20-year interval. The possibility of applying Störmer theory to estimate the non-vertical cutoffs is also investigated.

1. Introduction

It is well established that the investigation of the penumbra of the cosmic radiation is becoming important for the cosmic ray measurements, particularly those employing geomagnetic effects as part of the sensor discrimination techniques. In the past, theoretical and experimental studies have been carried out to investigate the penumbral effects in the region between the main cone cutoff rigidity and the Störmer cutoff rigidity, (e. g. Vallarte[1], Schwartz[2], Hedgecock[3]). Shea et al.[4] found that the penumbra region has an extremely complex structure of allowed and forbidden bands.

On the other hand, to determine the width of the penumbra region, for specific azimuth and zenith directions, it is necessary to compute the values of the main cone cutoff rigidity and the Störmer cutoff rigidity for this direction. In this respect, it is agreed that the most accurate method of determining cosmic ray cutoff rigidities is the trajectory-tracing method (McCracken et al[5, 6], Shea et al.[7] and Shea and Smart[8, 9]. By applying this technique and using 8th degree expansion of the geomagnetic field, Smart and Shea[10] found that, up to approximately 10 GV, the penumbral width can be approximated by a simple power low relationship with the main cone cutoff rigidity for the vertical direction.

Since the computations of the trajectory technique necessitated a large amount of computer time, cutoff rigidity calculations were made utilizing Störmer theory for the inclined directions to reduce the computer time as much as possible. Smart and Shea[10] have used this theory to determine the non-vertical cutoffs for Cape Giradeau Location (USA), where this location is characterized by low cutoff rigidity ($R_e = 2.63$ GV). Accordingly, they suggested that it is possible employ to Störmer theory to obtain the cutoff rigidities at a specific azimuth and zenith angles if the vertical cutoff is known.

In the present work, the penumbral characteristics at Cairo latitude have been investigated using the trajectory-tracing technique. As a result of the penumbra computations the cutoff rigidities (main cone cutoff, Störmer cutoff, and effective cutoff) have also been calculated for vertical and 8-different inclined directions. Moreover, the possibility of applying Störmer theory to estimate the non-vertical cutoffs at Cairo latitude which is characterized by a high cut-off rigidity, is also investigated.

2. *Model of computations*

The cutoff rigidity, at any location, for a certain direction can be determined by using the usual trajectory-tracing method (McCracken et al.[5], Shea et al.[4]). Accordingly, the orbits of the primary cosmic ray particles are traced, by numerical methods, through a model of the geomagnetic field. In this respect, the geomagnetic potential may be expressed by the expansion

$$U(r, \Theta, \varphi) = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} (g_n^m \cos m\,\varphi + h_n^m \sin m\,\varphi)\, P_n^m (\cos \Theta) \left(\frac{a}{r}\right)^{n+1} \tag{1}$$

where g_n^m and h_n^m are the Gauss coefficients, a is the mean radius of the earth, and P_n^m (cos Θ) are the normalized spherical harmonics introduced by Schmidt[11].

The calculations were started at an altitude of 20 km above the sea-level of an oblate earth with average radius of 6371 km. Starting at a rigidity high above the highest possible cutoff, cosmic ray trajectories were calculated at discrete intervals decreasing in rigidity until reached cutoff is satisfactory. As the calculations progress down through the rigidity spectrum, the results change from the easily allowed orbits to a complex structure of allowed, forbidden, and quasi-trapped orbits (called penumbra) and finally to a set of rigidities where the trajectories all intersect the solid earth. As a result of the trajectory computations, three distinct cutoff rigidities are defined as follows:

i) the main cone cutoff, R_m, above which all rigidities are allowed;
ii) the Störmer cutoff, R_s, below which all rigidities are forbidden;
iii) and the effective cutoff rigidity, R_e, which is defined as

$$R_e = R_m - [\int_{R_s}^{R_m} \mathrm{d}R]_{allowed}. \tag{2}$$

3. *Results and dicsussion*

3.a) *Determination of the cutoff rigidities*

Using the trajectory-tracing method, the cutoff rigidities at Cairo's cosmic ray station (29.6°N, 31.3°E) have been calculated for zenith angles 0, 16, 32 degrees in the N-S and E-W geomagnetic azimuths. In the computation algorithm, the following considerations have been taken into account:

(i) From equation (1) it can be seen that the main contribution to the field comes from the dipole term ($n = 1$) while eccentric dipole ($n = 2$) and higher terms have smaller contributions. In the present work, the sixth degree simulation of the geomagnetic field with coefficients given by Finch and Leaton[12] was used. Shea and Smart[13] have calculated the vertical cutoff rigidities in the equatorial region utilizing the 8th degree international geomagnetic reference field (*IGRF*) model for Epochs 1965 and 1975. From the comparison between these cutoffs, with those calculated for Epoch 1955, utilizing Finch and Leaton (*FL*) model, they found that the changes (δ) in the vertical cutoff rigidity is relatively small ($\delta \leq 0.20$ GV) over the 20-years interval with the exception of the longitude range 230° to 300°E where changes up to 0.36 GV. In case of Cairo Latitude, such a comparison reveals that the value of the parameter δ is almost zero. This leads to support the fact that for low latitude cosmic ray stations, the use of six or eight degrees of expansions for geomagnetic fields is approximately the same.

(ii) All calculations through the penumbra region were made at discrete 0.01 GV intervals. The effect of the rigidity interval is not considered in these computations. The work of Shea and Smart[14] and Smart and Shea[10] have shown that the effective cutoff rigidity is not seriously affected by one or two additional allowed trajectories that may be found by searching through the penumbra at smaller rigidity intervals.

(iii) The computations of the effective cutoffs (R_e) were made according to Eq. (2), which takes into account the effect of the penumbra region.

Following equation (2) the effective cutoff rigidity, R_e, for all 9-directions have been computed. In these computations, the determination of the main cone cutoff rigidity (R_m) and the Störmer cutoff rigidity (R_s) has been made for each direction with the evaluation of the penumbral region, i. e. the difference in rigidity between R_m and R_s. Table 1 presents the results of these computations.

TABLE 1.

Direction	N_p	N_a	R_m(GV)	R_s(GV)	(R_m-R_s)	R_e(GV)
V	153	44	12.99	11.47	1.52	12.55
16E	—	—	15.07	15.07	0.00	15.07
16S	149	27	12.42	10.94	1.58	12.15
16W	133	52	11.62	10.30	1.32	11.11
16N	160	50	13.84	12.25	1.59	13.34
32E	—	—	18.16	18.16	0.00	18.16
32S	—	—	12.09	12.09	0.00	12.09
32W	151	58	10.73	9.23	1.50	10.16
32N	247	131	14.95	10.95	4.00	13.70

N_p is the total number of trajectories (allowed and forbidden), in the penumbra band.
N_a is the number of allowed trajectories in the penumbra.

From this table, it is easy to observe that, the penumbra width $(R_m - R_s)$ for the directions 16E, 32E, and 32S is equal to zero, while for the others the width varies between 1.32 GV for the direction 16W and 4 GV for the direction 32N. Accordingly, it can be stated that, the penumbra width has a directional dependence, i. e. the width depends on the azimuth and zenith angles.

3.b) *Determination of the penumbral characteristics*

The results of the trajectory calculations are presented in Figs. 1 and 2 for the nine concerned directions. In these figures the dotted areas denote the penumbra

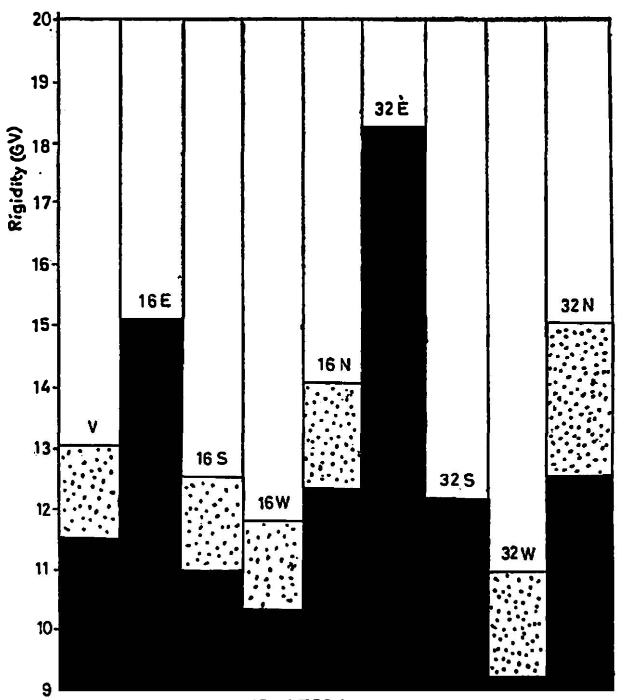

FIG.(1)

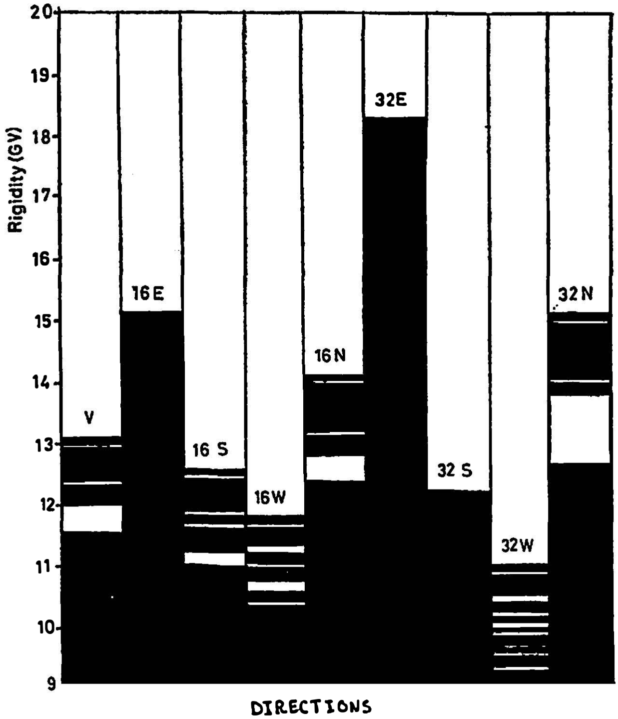

FIG.(2)

region, the dark areas denote forbidden trajectories, and white areas denote allowed trajectories. The figures show that:

(a) Except the directions $16E$, $32E$, $32S$ there is a persistant pattern in the penumbra region for all other directions. The pattern consisting of a very well defined forbidden band below the main cone cutoff, followed by a set of allowed trajectories below which there is a complex pattern of allowed and forbidden trajectories until the Störmer cutoff is reached, where below which all the trajectories are forbidden.

(b) No penumbra is observed in the directions $16E$, $32E$ and $32S$, where the width of the penumbra for these directions is almost equal to zero.

3.c) *Determination of the non-vertical cutoffs by using Störmer theory*

Störmer theory can be used to calculate the cutoff at a specific azimuth and zenith angles if the vertical cutoff is known. At a location of geomagnetic latitude λ, the Störmer equation for determining the cutoff is given by:

$$R(A, Z) = \frac{4 R_V}{[1 + (1 - \sin Z \cdot \cos A \cdot \cos^3 \lambda)^{1/2}]^2} \tag{3}$$

where $R(A, Z)$ is the cutoff rigidity at a specific azimuth, A (measured clockwise from the north) and zenith angle, Z (measured from the vertical), R_V is the cutoff rigidity in the vertical direction.

Following Störmer equation and using the obtained effective cutoff rigidity in the vertical direction, derived from the trajectory-tracing method, the cutoff at the concerned non-vertical directions (16*N*, *S*, *E*, *W* and 32*N*, *S*, *E*, *W*) have been calculated for Cairo latitude. The results of such computations, R_e, are listed in Table 2.

TABLE 2.

Direction	R_e (GV)	R_e^* (GV)	Difference ($R_e - R_e^*$)
16*E*	15.07	12.48	+2.59
16*S*	12.15	11.45	+0.70
16*W*	11.11	12.62	−1.51
16*N*	13.34	13.97	−0.63
32*E*	18.16	12.42	+5.74
32*S*	12.09	10.62	+1.47
32*W*	10.16	12.69	−2.53
32*N*	13.70	15.70	−2.00

From Table 2, it is clear that the difference between R_e calculated by using trajectory tracing method (*TTM*) and the corresponding ones R_e^*, calculated by using Störmer method (*SM*) is fluctuating from −2.53 to +5.74 GV for the directions 32*W* and 32*E*, respectively. These differences can be ascribed to the fact *SM* (Eq. (3)) does not consider the penumbra width in calculating the R_e. On the other hand, Smart and Shea[10] have concluded that *SM* may be used to replace the *TTM* for computing R_e at non-vertical directions at a location characterized by low cutoff rigidity. In fact, their conclusion is in contradiction with the present one. This is mainly due to the effect of the penumbra width which appears only with low latitude stations which have high cutoff rigidities (e. g., Cairo's latitude).

Accordingly, one may modify Smart and Shea conclusion as follows: the *SM* can be applied only for low latitude stations where the penumbra width has negligible effect. Once the penumbra width is expected to play a role in the calculations of R_e at non-vertical directions, the *TTM* is recommended for computations, even if it needs longer computational time. Moreover, one may suggest another approach, i. e. to modify the *SM* (Eq. (3)) to deal with the effect of the penumbra width. This indeed is considered to be beyond the scope of the present work.

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PENUMBRALNA KARAKTERISTIKA KOZMIČKIH ZRAKA NA ŠIRINI KAIRA

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Istražuju se penumbralne karakteristike kozmičkih zraka na geografskoj širini Kaira (29.6°*N*, 31.3°*E*). Račun je proveden za vertikalni i 8 drugih raznih smjerova, tj. 16*N*, *S*, *E*, *W* i 32*N*, *S*, *E*, *W* upotrebljavajući tehniku McCrakena i suradnika za utvrđivanje trajektorije. Rezultati pokazuju da je penumbralna širina za smjerove 16*E*, 32*E* i 32*S* jednaka nuli, dok za ostale varira od 1.32 GV za smjer 16*W* do 4 GV za smjer 32*N*. Također su izračunate druge veličine kao posljedice penumbralnih računa. Dobivene vrijednosti ukazuju da je razlika u efektivnom vertikalnom cutoff-u i *IGRF* modelu te u modelu Fincha i Leatona aproksimativno jednaka nuli u intervalu od 20 godina. Istražena je također mogućnost primjene störmer teorije za ocjenu nevertikalnih cutoff-a.