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LETTERS TO THE EDITOR

A NOTE ON RADIATIVE DECAYS OF VECTOR MESONS*

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In this paper a phenomenological approach to the determination of effective magnetic moments of the light quarks is generalized to the SU (5) classification scheme. In the treatment of radiative decays of the η measons a mixing matrix, introduced elsewhere in connection with the broken chiral symmetry, is instrumental.

Magnetic properties of hadrons have recently been considered within a phenomenological approach[1] where estimates are given for the effective magnetic moments of quarks, due to the magnetic coupling of the photon to three or more gluons. The effect is conceivable as a quark loop implication, but in the absence of a reliable way to calculate these QCD contributions the problem is treated phenomenologically by modifying the nonrelativistic SU (6) quark model and allowing quarks to initially have arbitrary effective magnetic moments.

Our interest in the matter has been aroused by recent measurements of the hyperon magnetic moments and of magnetic dipole transitions in meson radiative decays[2], which show a remarkable reliability of the naive quark model predictions, provided one incorporates suitable anomalous magnetic moments to quarks and appropriate mixing schemes[3]. Our note, therefore, is concerned with an extension of the approach in Ref. 1 to a phenomenological consideration of the magnetic moments in an SU (5) $\times$ SU (2) nonrelativistic quark classification scheme, and

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to subsequent theoretical determination of the radiative decay widths for charmed and b-flavoured vector and pseudoscalar mesons.

A special feature in this note is an application of the η meson quark-composition matrix, which we introduced earlier[8], in the context of broken SU (5) $\times$ $\times$ SU (5) chiral symmetry. The matrix is based on the plausible assumptions that η contains only the light quarks (u, d, s) and that η' and η_c do not contain b quarks. These assumptions introduce two mixing angles φ_1 and φ_2 in the following orthogonal transformation:

$$\begin{bmatrix} \eta \\ \eta' \\ \eta_c \\ \eta_b \end{bmatrix} = \begin{bmatrix} c_1 & \frac{1}{2} s_1 & \frac{1}{2}\sqrt{\frac{3}{5}} s_1 & \sqrt{\frac{3}{5}} s_1 \\ -s_1 c_2 & \frac{1}{2} c_1 c_2 + \frac{\sqrt{3}}{2} s_2 & \frac{1}{2}\sqrt{\frac{3}{5}} c_1 c_2 - \frac{1}{2\sqrt{5}} s_2 & \sqrt{\frac{3}{5}} c_1 c_2 - \frac{1}{\sqrt{5}} s_2 \\ s_1 s_2 & -\frac{1}{2} c_1 s_2 + \frac{\sqrt{3}}{2} c_2 & -\frac{1}{2}\sqrt{\frac{3}{5}} c_1 s_2 - \frac{1}{2\sqrt{5}} c_2 & -\sqrt{\frac{3}{5}} c_1 s_2 - \frac{1}{\sqrt{5}} c_2 \\ 0 & 0 & -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} 8 \\ 15 \\ 24 \\ 0 \end{bmatrix}$$

with $s_1 = \sin\varphi_1$, $s_2 = \sin\varphi_2$, $c_1 = \cos\varphi_1$, $c_2 = \cos\varphi_2$. The symbols $|8\rangle$, $|15\rangle$, $|24\rangle$ and $|0\rangle$ stand for the regular representation zero-quantum-number states.

Starting with the general form of the decay matrix element for the radiative decay of vector mesons

$$\mathcal{M}(V \to P\gamma) \sim \varepsilon_{\mu\nu\varrho\sigma} U^\mu p^\varrho e^\nu k^\sigma, \tag{1}$$

where p and k are 4-momenta, and U and e polarization vectors, of the vector meson V and γ-quant, respectively, one routinely arrives at the decay width expression

$$\Gamma(V \to P\gamma) = \frac{1}{96\pi} a^2 (V, P, \gamma) \frac{(m_V^2 - m_P^2)^3}{m_V^3}, \tag{2}$$

and analogously

$$\Gamma(P \to V\gamma) = \frac{1}{32\pi} a^2 (V, P, \gamma) \frac{(m_P^2 - m_V^2)^3}{m_P^3}. \tag{3}$$

The SU (5) classification symmetry enables us to interrelate the amplitudes a ($VP\gamma$) for various mesons. To this end one expresses the interaction hamiltonian as a trace $Tr\, V\{P, A\}$, where P and V are the corresponding 5×5 meson matrices for 0^- and 1^-, respectively, while A is the diagonal photon matrix containing the quark magnetic moments (μ_u, μ_d, μ_s, μ_c, μ_b). This represents the most general form of the effective interaction due to the fact that photons do not carry along flavour, and consequently, A has to be diagonal and the other permissible analogous trace, but with a commutator instead of the anticommutator, must vanish.

TABLE 1.

decay	amplitude	Γ_{th} (keV)	Γ_{exp}(keV)
$\omega \to \pi\gamma$	$\left(\frac{1}{\sqrt{6}}\cos\vartheta + \frac{1}{2\sqrt{3}}\sin\vartheta\right)(\mu_u - \mu_d)$	861	889±50
$\varrho \to \pi\gamma$	$\frac{1}{2}(\mu_u + \mu_d)$	67	63±8
$\varphi \to \pi\gamma$	$\left(\frac{1}{\sqrt{6}}\sin\vartheta - \frac{1}{2\sqrt{3}}\cos\vartheta)\right)(\mu_u - \mu_d)$	5.9	5.7±2.0
$\omega \to \eta\gamma$	$\left(\frac{1}{\sqrt{2}}\cos\vartheta + \frac{1}{2}\sin\vartheta\right)\left(\frac{\sqrt{2}}{3}\sin\varphi_1 + \frac{1}{3}\cos\varphi_1\right)(\mu_u + \mu_d)$ $+ \mu_s(\sqrt{2}\sin\vartheta - \cos\vartheta)\left(\frac{\sqrt{2}}{3}\cos\varphi_1 - \frac{1}{3}\sin\varphi_1\right)$	4.4	3 +2.5 +2.5
$\varrho \to \eta\gamma$	$\left(\frac{1}{\sqrt{6}}\sin\varphi_1 + \frac{1}{2\sqrt{3}}\sqrt{3}\cos\varphi_1\right)(\mu_u - \mu_d)$	57	50±13
$\varphi \to \eta\gamma$	$\left(\frac{1}{\sqrt{2}}\sin\vartheta - \frac{1}{2}\cos\vartheta\right)\left(\frac{\sqrt{2}}{3}\sin\varphi_1 + \frac{1}{3}\cos\varphi_1\right)(\mu_u + \mu_d)$ $- \mu_s(\sqrt{2}\cos\vartheta + \sin\vartheta)\left(\frac{\sqrt{2}}{3}\cos\varphi_1 - \frac{1}{3}\sin\varphi_1\right)$	57	55±12
$\eta' \to \omega\gamma$	$\cos\varphi_2\left[\left(\frac{1}{\sqrt{2}}\cos\vartheta + \frac{1}{2}\sin\vartheta\right)\left(\frac{\sqrt{2}}{3}\cos\varphi_1 - \frac{1}{3}\sin\varphi_1\right)(\mu_u + \mu_d)\right.$ $\left.- \mu_s(\sqrt{2}\sin\vartheta - \cos\vartheta)\left(\frac{\sqrt{2}}{3}\sin\varphi_1 + \frac{1}{3}\cos\varphi_1\right)\right]$	8.6	9±3
$\eta' \to \varrho\gamma$	$\cos\varphi_2\left(\frac{1}{\sqrt{6}}\cos\varphi_1 - \frac{1}{2\sqrt{3}}\sin\varphi_1\right)(\mu_u - \mu_d)$	108	89±29
$\varphi \to \eta'\gamma$	$\cos\varphi_2\left[\left(\frac{1}{\sqrt{2}}\sin\vartheta - \frac{1}{2}\cos\vartheta\right)\left(\frac{\sqrt{2}}{3}\cos\varphi_1 - \frac{1}{3}\sin\varphi_1\right)(\mu_u + \mu_d)\right.$ $\left.+ \mu_s(\sqrt{2}\cos\vartheta + \sin\vartheta)\left(\frac{\sqrt{2}}{3}\sin\varphi_1 + \frac{1}{3}\cos\varphi_1)\right]\right.$	0.23	—
$K^{*0} \to K^0\gamma$	$\frac{1}{2}(\mu_d + \mu_s)$	139	75±35
$K^{*+} \to K^+\gamma$	$\frac{1}{2}(\mu_u + \mu_s)$	96	80
$\eta_c \to \varrho\gamma$	$\sin\varphi_2\left(\frac{1}{\sqrt{6}}\cos\varphi_1 - \frac{1}{2\sqrt{3}}\sin\varphi_1\right)(\mu_u + \mu_d)$	0.89	—

$\eta_c \rightarrow \omega\gamma$	$\sin\varphi_2\Big[\Big(\frac{1}{\sqrt{2}}\cos\vartheta + \frac{1}{2}\sin\vartheta\Big)\Big(\frac{\sqrt{2}}{3}\cos\varphi_1 - \frac{1}{3}\sin\varphi_1\Big)(\mu_u - \mu_d) + \mu_s(\sqrt{2}\sin\vartheta - \cos\vartheta)\Big(\frac{\sqrt{2}}{3}\sin\varphi_1 + \frac{1}{3}\cos\varphi_1\Big)\Big]$	10	—
$\eta_c \rightarrow \varphi\gamma$	$\sin\varphi_2\Big[\Big(\frac{1}{\sqrt{2}}\sin\vartheta - \frac{1}{2}\cos\vartheta\Big)\Big(\frac{\sqrt{2}}{3}\cos\varphi_1 - \frac{1}{3}\sin\varphi_1\Big)(\mu_u - \mu_d) - \mu_s(\sqrt{2}\cos\vartheta + \sin\vartheta)\Big(\frac{\sqrt{2}}{3}\sin\varphi_1 + \frac{1}{3}\cos\varphi_1\Big)\Big]$	1.7	—
$\Psi \rightarrow \eta'\gamma$	$\mu_c \sin\varphi_2$	0.16	0.16
$D^{*0} \rightarrow D^0\gamma$	$\frac{1}{2}(\mu_u + \mu_c)$	23	—
$D^{*+} \rightarrow D^+\gamma$	$\frac{1}{2}(\mu_d + \mu_c)$	3.3	—
$F^{*+} \rightarrow F^+\gamma$	$\frac{1}{2}(\mu_s + \mu_c)$	0.26	—
$\Psi \rightarrow \eta_c\gamma$	$\mu_c \cos\varphi_2$	0.7	0.7
$\Upsilon \rightarrow \eta_b\gamma$	μ_b	$5.8 \times$ $\times 10^{-3}$	
$B^{*+} \rightarrow B^+\gamma$	$\frac{1}{2}(\mu_u + \mu_b)$	0.64	—
$B^{*0} \rightarrow B^0\gamma$	$\frac{1}{2}(\mu_d + \mu_b)$	0.24	—

Predicted and experimental radiative decay widths.

Within the P-matrix we introduce an angle ϑ to represent the $\omega - \varphi$ mixing which serves to interpret some decays, e. g. $\eta' \rightarrow \omega\gamma$. We also introduce angles φ_1 and φ_2 which appear in the η meson quark-composition matrix.

In Table 1 we present, for each magnetic dipole radiative meson decay, the corresponding amplitude, predicted decay width and experimental value (where the data exist.)

Magnetic moments of the light quarks are given the usual values, $\mu_u = 0.67\,\mu_p$, $\mu_d = -0.37\,\mu_p$, $\mu_s = =0.19\,\mu_p$, with μ_p being the proton magnetic moment. Note that the data for $\psi \rightarrow \eta'\gamma$ and $\psi \rightarrow \eta_c\gamma$ contribute considerably to the value of the magnetic moment of the c quark, giving $\mu_c = \mu_{c0} - \delta_c$, with μ_{c0} being the naive quark model value[5] and δ_c amounting to $\approx 40\%$ decrement. Thus we take

$\mu_c = 0.80\ \mu_p$, while to the b quark, in the absence of experimental fits, we ascribe the naive quark model estimate $\mu_b = -0.02\ \mu_p$.

The values $\vartheta \approx 38°$ and $\varphi_1 \approx 10°$ are taken from the corresponding mixing angles in Ref. 1, while the value $\varphi_2 \approx 1°$ is a byproduct of the estimate for the c quark magnetic moment.

For processes in which only the light quarks appear the results are identical to those obtained in Ref. 1, with the exception of $\eta' \to \omega\,\gamma$, $\eta' \to \varrho\,\gamma$ and $\varphi \to \eta'\,\gamma$ which may differ on account of a correction due to the mixing angle φ_2.

In view of the obtained μ_c value we find it useful, hopefully, to use it also for the estimate of the magnetic moments of the c-flavored baryons. As definition of the baryon magnetic moment we use the expression

$$\mu_B = \langle \Phi\,\chi |\, \mu_i\, \sigma_{3i} |\, \Phi\,\chi \rangle, \tag{4}$$

where Φ and χ are flavour and spin baryon wave functions, properly symmetrized. The results are presented in Table 2.

TABLE 2.

baryon	quark content	μ/μ_p	baryon	quark content	μ/μ_p
Σ_c^{++}	uuc	0.86	Λ_c^+	udc	0.07
Σ_c^+	udc	0.17	Ξ_c^{A+}	usc	0.07
Σ_c^0	ddc	−0.52	Ξ_c^{A0}	dsc	0.07
Ξ_c^+	usc	0.29	Ξ_{cc}^{++}	ccu	−0.12
Ξ_c^0	dsc	−0.40	Ξ_{cc}^+	ccd	0.22
Ω_c^0	ssc	−0.28	Ω_{cc}^+	ccs	0.16

Ratios of the magnetic moments of charmed baryons to that of the proton.

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O RADIJACIONIM RASPADIMA VEKTORSKIH MEZONA

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Originalni znanstveni rad

U radu se generalizira jedan fenomenološki pristup određivanju efektivnih magnetskih momenata lakih kvarkova na SU (5) klasifikacionu shemu. U tretiranju radijacionih raspada η mezona primjenjuje se matrica miješanja koju su autori uveli u jednom ranijem radu u vezi razmatranja narušene kiralne simetrije.