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COLLECTIVE MOTION IN A COUPLED ELECTRON-HOLE-PHONON SYSTEM

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A dispersion relation for a collisionless electron-hole plasma coupled to longitudinal phonons is derived in the framework of the second-order *RPA*. Assuming that both electron and hole gases are degenerate, long-wavelength frequencies of optical and acoustic modes are calculated.

1. *Introduction*

Contrary to a single-component plasma, in which frequency remains finite when the wave number goes to zero, a two-component plasma may contain another branch with a different dispersion relation. It describes long-range Coulomb forces between heavier species which are screened by the dielectric constant of lighter species. The corresponding frequency is much lower than the first one, and in the limit of long waves it becomes proportional to the wave number. By analogy to the vibrational lattice spectrum, it is called the acoustic plasma branch.

Considering a two-component plasma involving electrons and holes in semiconductors, Pines and Schrieffer[1] derived a dispersion relation for both high- and low-frequency plasma oscillations and studied the two-stream instability of acoustic waves in an applied electric field. Since than there has been increasing interest in multi-component plasmas. Frequencies of three- and n-component Maxwellian plasmas were calculated in Refs. 2 and 3. Fröhlich[4–5] investigated the properties of a two-component plasma consisting of *s*- and *d*-electrons in metals. His work was generalised by Salustri[6] to include ions into consideration. The ground-state energy of an electron-hole plasma was calculated by Combescot and Nozières[7], Brinkman and Rice[8], Vashishta et al.[9] and Bhattacharyya et al.[10] in various approximations. The generalization of these calculations was performed by Wünsche[11], who investigated an electron-hole plasma involving an arbitrary number of different particles.

The aim of the present paper is to study the collective motion of degenerate electrons and holes interacting with phonons in semimetals and heavily doped semiconductors. It is convenient to assume that in a degenerate plasma the density is high enough, so that the *random phase approximation*[12] (*RPA*) is valid. As the next step one may include exchange effects into calculation. In this approximation (often called the second-order *RPA*), we shall evaluate collisionless long-wavelength oscillation modes of electrons, holes and phonons.

2. *Dispersion relation*

The coupled system containing electrons, holes and longitudinal phonons may be represented by the following Hamiltonian:

$$H = H(e) + H(h) + H(ph) + H(e, h) + H(e, ph) + H(h, ph). \tag{1}$$

The abbreviations introduced in Eq. (1) are as follows:

$$H(i) = \sum_{s\vec{\varkappa}} E_\varkappa(i)\, N_{s\varkappa}(i) + \frac{1}{2} \sum_{\vec{k}} V_k(i, i)\, \varrho_k^+(i)\, \varrho_k(i) \tag{2}$$

$$H(ph) = \frac{1}{2} \sum_{\vec{k}} (\pi_k^+ \pi_k + \Omega_k^2 q_k^+ q_k) \tag{3}$$

$$H(e, h) = \sum_{\vec{k}} V_k(e, h)\, \varrho_k^+(e)\, \varrho_k(h) \tag{4}$$

$$H(i, ph) = \sum_{\vec{k}} v_k(i, ph)\, \varrho_k^+(i)\, q_k \qquad i = e, h. \tag{5}$$

$E_\varkappa(i)$ is the single-particle energy of electron (hole) possessing momentum $\hbar\vec{\varkappa}$, $N_{s\varkappa}(i)$ is the corresponding occupation number operator

$$N_{s\varkappa}(i) = c_{s\varkappa}^+(i)\, c_{s\varkappa}(i) \tag{6}$$

$c_{s\varkappa}^+(i)$ and $c_{s\varkappa}(i)$ being the usual creation and annihilation operators, respectively, $\varrho_k(i)$ is the density fluctuation operator

$$\varrho_k(i) = \sum_{\vec{k}} c_{s\varkappa}^+(i)\, c_{s\varkappa+k}(i) \tag{7}$$

q_k and π_k is the conjugate pair of phonon coordinate and momenta, Ω_k is the unrenormalized longitudinal phonon frequency, $V_k(e, h)$ is the Coulomb coupling constant between electrons and holes and $v_k(i, ph)$ is the bare matrix element for the interaction between i-th carriers and phonons.

In order to find the eigenfrequencies of the system (1), we consider the equation of motion for the operator $\sum_{s\vec{\varkappa}} c^+_{s\varkappa}(i)\, c_{s\varkappa+k}(i) f_{\varkappa k}(i)$, where $f_{\varkappa k}(i)$ is an arbitrary function. In the second-order *RPA*, by generalizing the procedure described in Ref. 13, this yields

$$\frac{d}{dt}\sum_{s\vec{\varkappa}} c^+_{s\varkappa}(i)\, c_{s\varkappa+k}(i) f_{\varkappa k}(i) = \frac{i}{\hbar}\sum_{s\vec{\varkappa}} c^+_{s\varkappa}(i)\, c_{s\varkappa+k}(i)\, \{f_{\varkappa k}(i)\,[E_\varkappa(i) - E_{\varkappa+k}(i)] +$$

$$+ \sum_{s\vec{\varkappa}'} V_{\varkappa-\varkappa'}(i)\,[f_{\varkappa k}(i) - f_{\varkappa' k}(i)]\,[N_{s\varkappa'+k}(i) - N_{s\varkappa'}(i)]\} -$$

$$- \frac{i}{\hbar}\, Q_i(k,\omega_k)\,[v_k(i,ph) q_k + \sum_{i'=e,h} V_k(i, i')\, \varrho_k(i')] =$$

$$= -i\omega_k \sum_{s\vec{\varkappa}} c^+_{s\varkappa}(i)\, c_{s\varkappa+k}(i)\, f_{\varkappa k}(i), \tag{8}$$

where $Q_i(k, \omega_k)$ is the response function of the i-th species

$$Q_i(k, \omega_k) = \sum_{s\vec{\varkappa}} f_{\varkappa k}(i)\,[N_{s\varkappa}(i) - N_{s\varkappa+k}(i)]. \tag{9}$$

After requiring that $f_{\varkappa k}(i)$ should satisfy the integral equation

$$f_{\varkappa k}(i)\,[\hbar\,\omega_k + W_\varkappa(i) - W_{\varkappa+k}(i)] = 1 + \sum_{\vec{\varkappa}'} V_{\varkappa-\varkappa'}(i)\, f_{\varkappa' k}(i)\,[N_{s\varkappa'+k}(i) -$$

$$- N_{s\varkappa'}(i)], \tag{10}$$

$W_\varkappa(i)$ being the single-particle Hartree-Fock energy

$$W_\varkappa(i) = E_\varkappa(i) - \sum_{\vec{\varkappa}'} V_{\varkappa-\varkappa'}(i)\, N_{s\varkappa'}(i), \tag{11}$$

one arrives at

$$\sum_{i'=e,h} [V_k(i, i')\, Q_i(k, \omega_k) - \delta_{ii'}]\, \varrho_k(i) + Q_i(k, \omega_k)\, v_k(i, ph)\, q_k = 0 \quad i = e, h. \tag{12}$$

Equations (12), together with the equation of motion for the phonon coordinate

$$q_k(\Omega_k^2 - \omega_k^2) + \sum_{i=e,h} v_k^*(i, ph)\, \varrho_k(i) = 0, \tag{13}$$

represent a system of linear homogeneous algebraic equations for $\varrho_k(e)$, $\varrho_k(h)$ and q_k. The determinant of the system must be equal to zero:

$$\begin{vmatrix} V_k(e,e)\,Q_e(k,\omega_k)-1 & V_k(e,h)\,Q_e(k,\omega_k) & v_k(e,ph)\,Q_e(k,\omega_k) \\ V_k(h,e)\,Q_h(k,\omega_k) & V_k(h,h)\,Q_h(k,\omega_k)-1 & v_k(h,ph)\,Q_h(k,\omega_k) \\ v_k^*(e,ph) & v_k^*(h,ph) & \Omega_k^2-\omega_k^2 \end{vmatrix} = 0 \quad (14)$$

which is the desired dispersion relation.

In order to make the situation as simple as possible, we shell assume that both $V_k(i,i)$ and $v_k(i,ph)$ are independent of species

$$V_k(i,i) = V_k \quad (15)$$

$$V_k(i,j) = -V_k \qquad i \neq j \quad (15a)$$

$$v_k(i,ph) = v_k. \quad (16)$$

Then it can be easily shown that the dispersion relation (14) reduces to

$$1 = V_k\,[Q_e(k,\omega_k) + Q_h(k,\omega_k)] + \frac{a\,\Omega_k^2}{\omega_k^2 - \Omega_k^2(1-a)}, \quad (17)$$

where a is the carrier-phonon strength parameter

$$a = \frac{|v_k|^2}{V_k\,\Omega_k^2}. \quad (18)$$

3. *Mixing of modes*

Although the energy surfaces of carriers in solids are generaly anisotropic, it is customary to assume that they are characterized by an isotropic effective mass. This assumption makes the mathematical formalism much simpler. In this idealized model one writes

$$E_\varkappa(e) = \frac{\hbar^2\,\varkappa^2}{2m_e^*} \quad (19)$$

$$E_\varkappa(h) = \frac{\hbar^2\,\varkappa^2}{2m_h^*} \quad (20)$$

$$V_k = \frac{4\pi\,e^2}{\varepsilon\,k^2}, \quad (21)$$

where m_h^* and m_e^* are the effective mass of the conduction electron and hole, respectively, and ε is the dielectric constant of the medium.

We shall further suppose that both electrons and holes form a high-density gas which may be treated in the zero-temperature approximation. Under these circumstances the integral equation (10) can be solved exactly for small wave numbers k. Confining ourselves to this case, we shall quote the final results for the response function which are valid in the high- and the low-frequency limit.

In the second-order *RPA*, the long-wavelength high-frequency response function of a degenerate electron gas was calculated by Nozières and Pines[14], Kanazawa et al.[15] and v. Roos and Zmuidzinas[16]. They obtained

$$V_k\, Q\,(k, \omega) = \left(\frac{\omega_p}{\omega}\right)^2 \left[1 + \frac{3k^2 v_F^2}{5\omega^2}\left(1 - \frac{\gamma}{3}\right)\right] \quad k \ll k_F, \quad \omega \gg k\, v_F. \tag{22}$$

An analogous calculation for the real part of the response function in the low-frequency limit was performed by v. Roos[17], and a complete expression including the imaginary part (due to Landau damping[18]) is given in Ref. 13. The result is

$$V_k\, Q\,(k, \omega) = -\left(\frac{k_s}{k}\right)^2 \frac{1}{1-\gamma}\left[1 + \frac{\mathrm{i}\,\pi\,\omega}{2k\, v_F\,(1-\gamma)}\right] \quad k \ll k_F, \ \omega \ll k\, v_F. \tag{23}$$

In Eqs. (22) and (23) ω_p is the classical plasma frequency

$$\omega_p^2 = \frac{4\pi\, N e^2}{\varepsilon\, m^*}, \tag{24}$$

N being the carrier density, k_F and v_F are the Fermi wave number and the Fermi velocity, respectively, k_s is the Thomas-Fermi screening wave number

$$k_s^2 = \frac{3\omega_p^2}{v_F^2} \tag{25}$$

and γ is the parameter measuring the relative contribution of the exchange effects. Defining the dimensionless quantity

$$r_s = \frac{1}{a_0}\sqrt[3]{\frac{3}{4\pi\, N}}, \tag{26}$$

where a_0 is the effective Bohr radius in the medium

$$a_0 = \frac{\hbar^2\, \varepsilon}{m^*\, e^2}, \tag{27}$$

γ may be expressed as

$$\gamma = \frac{r_s}{\pi}\sqrt[3]{\frac{4}{9\pi}}. \tag{28}$$

Note that the weak-coupling condition $r_s < 1$ may be satisfied in doped semiconductors and semimetals owing to small effective mass and large dielectric constant. On the other hand, this condition is not fulfilled in metals, where r_s ranges between 2 and 6.

Of course, in the limiting case $\gamma = 0$, expressions (22) and (23) may be obtained directly from Lindhard's dielectric function[19] by keeping only lowest-order terms in the wave number and considering the low- or the high-frequency limit.

So far our considerations have been valid irrespective of the type of coupling. In the following we shall confine ourselves to piezoelectric coupling and deformation potential coupling.

We shall suppose that the following inequalities hold:

$$\omega_p^2(e) \gg \omega_p^2(h) \gg a\,\Omega_k^2 \tag{29}$$

$$\frac{k^2\,\omega_p^2(h)}{\Omega_k^2\,(1-a)} \gg k_s^2(h) \gg k_s^2(e). \tag{30}$$

These conditions guarantee that collective oscillations are not strongly damped by individual particle excitations.

In solids with piezoelectric coupling (II—VI and III—V semiconductors) and deformation potential coupling (semiconductors such as Ge, Si, etc. and semimetals), Ω_k is given by

$$\Omega_k = s\,k, \tag{31}$$

where the longitudinal sound velocity s is of the order of 5×10^3 m/s and $a \approx 10^{-4} - 10^{-3}$ is a good estimate for the carrier-phonon coupling strength[20]. Making a further assumption that electron and hole gas are of equal density $N(e) = N(h) = N$, Eqs. (29) and (30) take the form

$$\frac{\hbar^2\,(3\pi^2 N)^{2/3}}{3s^2\,m_h^*} \gg m_h^* \gg m_e^*. \tag{32}$$

Let us assume that $m_h^* = 10\,m_e^* = 10\,m_e$, where m_e is the free electron mass. Then in order to satisfy Eq. (32), the carrier densities of the order $5 \times 10^{23}\ \mathrm{m}^{-3}$ are needed. As it will be shown in Table 1, this is much below the densities at which the interhole spacing $r_s(h)$ becomes less than 1. In other words, the densities required for plasma coupling to be weak are much higher than those which ensure that plasma oscillations are not strongly Landau damped.

The three modes of the dispersion relation (17) are obtained in the following way: The electron-plasma frequency $\omega_k(e)$ is associated with the high-frequency expansion for both $Q_e(k, \omega)$ and $Q_h(k, \omega)$, the hole-plasma frequency $\omega_k(h)$ corresponds to the low-frequency expansion for $Q_e(k, \omega)$ and the high-frequency expansion for $Q_h(k, \omega)$, while in the calculation of the phonon frequency $\omega_k(ph)$ both $Q_e(k, \omega)$ and $Q_h(k, \omega)$ are represented by the low-frequency approximation. After utilizing Eqs. (29) and (30), this yields:

$$\omega_k(e) = \sqrt{\omega_p^2(e) + \frac{3}{5}\,k^2\,v_F^2(e)\left[1 - \frac{\gamma(e)}{3}\right]} \tag{33}$$

$$\omega_k(h) = k\,\frac{\omega_p(h)}{k_s(e)}\left[\sqrt{1-\gamma(e)} - \mathrm{i}\,\frac{\pi\,\omega_p(h)}{4\sqrt{3}\,\omega_p(e)}\right] \tag{34}$$

$$\omega_k(ph) = \Omega_k\sqrt{1 - a\left[1 - k^2\,\frac{1-\gamma(h)}{k_s^2(h)}\right]} - \mathrm{i}\,\frac{\pi\,v_F(h)\,k\,a\,\Omega_k^2}{12\omega_p^2(h)}. \tag{35}$$

Contrary to the high-frequency optical plasma mode $\omega_k(e)$, acoustic plasma modes $\omega_k(h)$ and $\omega_k(ph)$ are Landau damped. Formally, the acoustic damping rates are constructed in a similar way, but owing to the proportionality between Ω_k and k, Im $\omega_k(ph)$ is proportional to k^3, while Im $\omega_k(h)$ increases linearly with wave number k.

As we stated before, the damping is relatively weak by virtue of Eqs. (29) and (30). It is also worth pointing out that neither the phonon nor the hole-plasma damping rate is affected by exchange effects. Hence, the criteria for the two-stream instability of acoustic waves in the second -order *RPA* are the same as in the simple *RPA*[1,20].

Comparing the real parts of acoustic frequencies, one easily observes that in degenerate plasmas Re $\omega_k(h)/k$ is much larger than the longitudinal sound velocity s, whose typical values do not exceed 10^4 m/s. By assuming that electron and hole gases have equal densities, for a special choice of the electron-hole mass ratio $m_h^*/m_e^* = 10$ and the static dielectric constant $\varepsilon = 15$, in Table 1 we tabulate numerical results for the real part of the plasma-acoustic frequency as a function of carrier density and effective mass.

TABLE 1.

	$\frac{m_h^*}{m_e}$	$r_s(h)$	Re $\omega_k(h)/k$
$N = 10^{24}$ m^{-3}	0.1	0.782	6.49×10^5 m/s
	0.2	1.56	3.23
	0.3	2.35	2.13
	0.4	3.13	1.59
	0.5	3.91	1.26
$N = 5 \times 10^{24}$ m^{-3}	0.1	0.457	11.13
	0.2	0.915	5.54
	0.4	1.37	3.69
	0.5	2.29	2.19
	0.6	2.74	1.82
	0.8	3.66	1.35
$N = 10^{25}$ m^{-3}	0.1	0.363	14.03
	0.2	0.726	6.99
	0.4	1.45	3.48
	0.5	1.81	2.78
	0.6	2.18	2.30
	0.8	2.90	1.72
	1	3.63	1.36

The real part of $\omega_k(h)/k$ calculated for different values of carrier effective masses and particle densities. In all cases we have chosen $\varepsilon = 15$, $N(e) = N(h) = N$ and $m_h^* = 10\,m_e^*$. Hence it immediately follovs that $r_s(e) = r_s(h)/10$.

4. Conclusion

We have investigated the collective motion in semiconductors and semimetals which are considered as systems consisting of electrons, holes and longitudinal phonons. Neglecting collisional damping and taking into account the lowest-order corrections to the *RPA*, we derived the dispersion relation of the system.

Explicit calculation is performed for degenerate plasma. We adopted an idealized model in which electrons and holes are replaced by free charged particles with isotropic effective masses and assumed that both types of carriers are coupled to phonons with equal strenghts. Confining the consideration to two coupling types (piezoelectric and deformation potential coupling), the expressions for optical and two acoustic frequencies in the long-wavelength approximation are derived. Acoustic waves are Landau damped, but this damping is not influenced by the second-order *RPA* contributions. As a consequence, we may conclude that the condition for the two-stream instability in an external field[1,20] is not modified by the exchange effects.

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KOLEKTIVNO GIBANJE ELEKTRONA, ŠUPLJINA I FONONA

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Primjenom aproksimacije slučajnih faza drugog reda izvedena je disperziona relacija za plazmu elektrona i šupljina vezanu na longitudinalne fonone. Uz pretpostavku da nosioci naboja tvore degeneriranu plazmu određene su dugovalne frekvencije sistema.