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BACKBENDING IN SOME LIGHT NUCLEI FROM CRANKED HFB CALCULATION

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We report a cranked *Hartree-Fock-Bogoliubov* (*CHFB*) calculation for ^{18,20,22,24}Ne with the pairing plus quadrupole-quadrupole force of Baranger and Kumar as the residual interaction. All the four isotopes show backbending at $I \sim 6^+$ to 10^+. The calculations are in fair agreement with available experimental results.

1. Introduction

The investigation of high spin states of nuclei has been of great interest in recent years. The backbending phenomenon was initially observed in the rare-earth region and consequently such nuclei were studied extensively, both theoretically and experimentally. But recently some experimental information on the high spin states of light nuclei have been obtained which aroused considerable interest in the theoretical study. Among the existing microscopic calculations *Hartree-Fock-Bogoliubov* (*HFB*) method either with angular momentum and number projection or with cranking is one of the most widely used methods for studying the high spin states[1]. Initially the cranked *HFB* (*CHFB*) calculations were

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performed for nuclei in the rare earth region[2]. Lately *YCHFC* calculations have been performed by a number of authors[3,4] for light nuclei in *s-d* shell. Experimental observations of backbending phenomenon in ^{20}Ne and ^{22}Ne by Klapdor[5] have motivated us to perform *CHFB* calculation for some neon- isotopes and compare our calculations with experiment.

2. *Theory*

The many body Hamiltonian has the form

$$\hat{H} = \sum_i \varepsilon_i a_i^\dagger a_i + \frac{1}{4} \sum_{ijkl} v_{ijkl} a_i^\dagger a_j^\dagger a_l a_k \tag{1}$$

where $(a_i^\dagger, a_l)$ are the particle creation and destruction operators. In the *Hartree--Fock-Bogoliubov* (*HFB*) method, a quasiparticle transformation is introduced[2] through

$$\beta_\mu^\dagger = \sum_n A_{n\mu} a_n^\dagger + B_{n\mu} a_n \tag{2}$$

where the variational parameters $A_{n\mu}$ and $B_{n\mu}$ are determined by minimizing the expectation value of the Hamiltonian, with the trial wave function

$$|\Psi\rangle = \prod_\mu \beta_\mu |0\rangle, \tag{3}$$

such that this state becomes the vacuum for the quasi particles ($|0\rangle$ is the particle vacuum). It is clear from Eq. (2) that this transformation mixes particles and holes. So it does not conserve particle number or angular momentum. Therefore, one minimizes the expectation value of the Hamiltonian, subject to the condition that angular momentum number operators have correct average values. These conditions are achieved through the introduction of Lagrange multipliers ω and λ. Thus one has to minimize $(\hat{H} - \omega \hat{J}_x - \lambda \hat{N})$, i.e.

$$\delta \langle \Psi_I | \hat{H} - \omega \hat{J}_x - \lambda \hat{N} | \psi_I \rangle = 0 \tag{4}$$

subject to the restrictions

$$\langle \Psi_I | \hat{J}_x | \Psi_I \rangle = \sqrt{I(I+1)} \tag{5}$$

and

$$\langle \Psi_I | \hat{N} | \Psi_I \rangle = N \tag{6}$$

where N is the correct particle number and I is the angular momentum of the state. This can be viewed as an approximate angular momentum and number projection.

HFB equations for the coefficients $A_{n\mu}$, $B_{n\mu}$ are given by[2]

$$\begin{pmatrix} h - \omega J_x & \Delta \\ -\Delta^* & -h^* + \omega J_x^* \end{pmatrix} \begin{pmatrix} A_\mu \\ B_\mu \end{pmatrix} = E_\mu \begin{pmatrix} A_\mu \\ B_\mu \end{pmatrix} \tag{7}$$

Eq. (7) is called the *CHFB* equation. The matrices h, Δ, Γ are defined as

$$h = \varepsilon - \lambda + \Gamma \tag{8}$$

$$\Gamma_{lm} = \sum_{rs} v_{lrms}\, \varrho_{sr} \tag{9}$$

$$\Delta_{lm} = \frac{1}{2} \sum_{rs} v_{lmrs}\, \varkappa_{rs} \tag{10}$$

$$\varrho_{lm} = (B^* B^T)_{lm} \tag{11}$$

$$\varkappa_{lm} = (B^* A^T)_{lm}. \tag{12}$$

It is clear that Eq. (7) is a non-linear eigenvalue problem, which has to be solved by iteration.

Because of the known importance of n-p pairing in light nuclei, we included such terms in Eq. (1). Since *CHFB* equations are solved iteratively for each parity state of neutron and proton separately, the contributions from both parity states of neutron and proton to the pairing potential Δ is obtained from the solution of the previous iteration. This will lead to the correct treatment when self consistency is achieved. We have calculated the moment of inertia ($\mathscr{I}_I$) and the angular velocity (ω_I) following the definitions[8]

$$2\mathscr{I}_I/\hbar^2 = \left[\frac{\mathrm{d}E}{\mathrm{d}I\,(I+1)}\right]^{-1} \tag{13}$$

and

$$\hbar\omega_I = \frac{\mathrm{d}E}{\mathrm{d}\sqrt{I\,(I+1)}}. \tag{14}$$

3. *Numerical calculation*

For the two body residual interaction, v_{ljkl}, we have chosen the pairing plus quadrupole-quadrupole force of Baranger and Kumar[6]. Customarily this force is used in the medium-heavy and heavy regions. We have chosen this primarily for its simplicity. However, it turns out that this form of residual interaction is capable of reproducing experimental results with about the same degree of success as in the rare-earth region[4]. Thus the many-body Hamiltonian chosen, has the form

$$\hat{H} = \sum_{ij} \varepsilon_{ij} a_i^\dagger a_j + \frac{1}{2} \sum_{\tau_3 \tau_3'} Q_{\tau_3 \tau_3'} \sum_{\mu=-2}^{2} Y_{rs}^{2\mu}\, Y_{mn}^{2\mu}\, a_s^\dagger a_m^\dagger a_n a_r +$$

$$+ \frac{1}{4} \sum_{\tau_3 \tau_3'} G_{\tau_3 \tau_3'} \sum_{m,n>0} a_m^\dagger a_{\bar{m}}^\dagger a_{\bar{n}} a_n. \tag{15}$$

$a_i^\dagger$ and a_i are the creation and destruction operators of the chosen harmonic oscillator basis. The time reversed state of $|m\rangle$ is denoted by $|\overline{m}\rangle$. We choose the single particle Hamiltonian ε_{ij} to coincide with Nilsson energies[7] for zero deformation. To determine the parameters of the residual interaction, *CHFB* calculation has been performed for ^{20}Ne and the parameters are adjusted to get the best overall fit of the ground state rotational band. Close agreement of the calculated results for Ne isotopes (as well as previously reported calculations[4]) with the experimental results enables us to comment that the pairing plus quadrupole-quadrupole force can be safely used in the light region also.

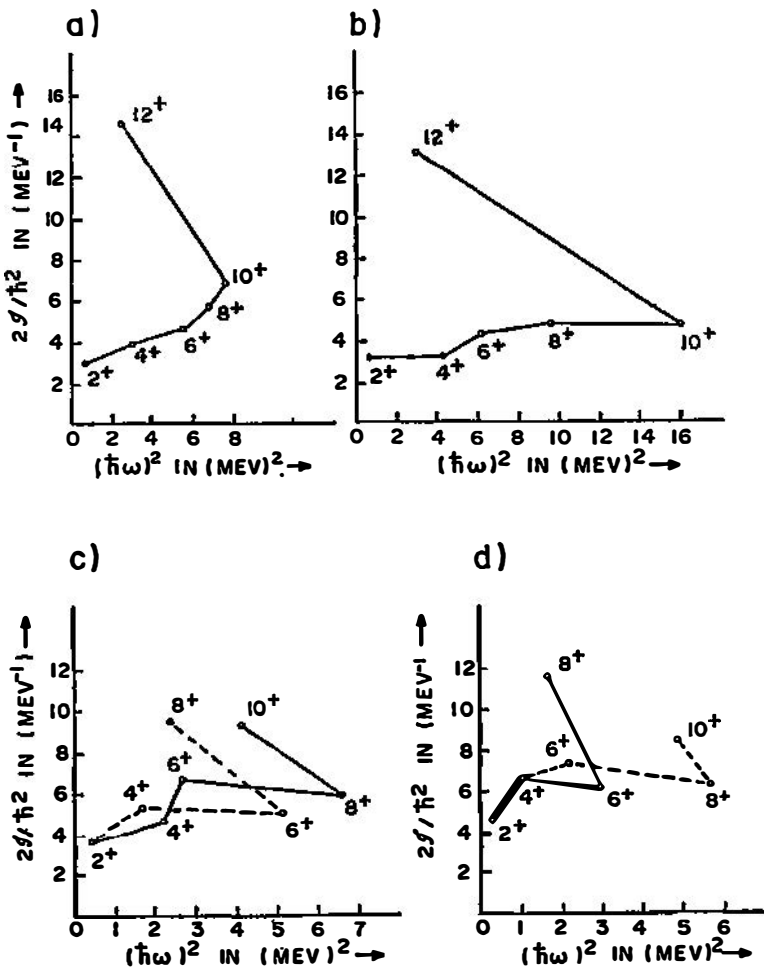

Fig. 1. a) ^{24}Ne, b) ^{18}Ne, c) ^{20}Ne, d) ^{22}Ne.

Our configuration space includes the first three major shells open to all nucleons. The only free parameter, namely, the energy scale is obtained by fitting the first 2^+ state of the ground state rotational band with experiment.

We plot the calculated moment of inertia ($\mathcal{J}_I$) against $(\hbar\omega_I)^2$ in Fig. 1 (a—. One notices that all the four isotopes of neon (^{18,20,22,24}Ne) show backbending at $I = 6^+$ to $I = 10^+$. For ^{20}Ne and ^{22}Ne, comparison with experiment[5] is possible, for which the experimental result is shown by dotted curve. It is seen from Figs. 1 (c) and 1 (d), that the qualitative feature is well reproduced in our calculation. However the exact position of the backbend differs slightly. In ^{20}Ne the experimental backbend occurs at 6^+, whereas our calculation shows this at 8^+. The situation is reversed for ^{22}Ne. In the case of ^{18}Ne, we see that a sharp backbending occurs at 10^+ whereas ^{24}Ne shows a gradual backbending at 10^+. A look at Fig. 1 (a)—(d) reveals that the sharpness of the backbend decreases as the neutron number increases. At present, experimental informations are insufficient for ^{18}Ne and ^{24}Ne, to compare with our calculations.

Summarising the results, we can remark that cranked *HFB* calculation with pairing plus quadrupole-quadrupole interaction can well represent the high spin states of the ground state rotational band in light *s-d* shell nuclei.

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REDUCIRANI HFB RAČUN MOMENATA USTRAJNOSTI STANJA VISOKOG SPINA U NEKIM LAKIM JEZGRAMA

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U radu su računatimomenti ustrajnosti za stanja visokog spina izotopa ^{18,20,22,24}Ne. Korištena je metoda *Hartree-Fock-Bogoljubova* uz uvjet na moment količine gibanja i broj čestica. U ostatno međudjelovanje uključena je sila sparivanja i kvadrupol-kvadrupol sila. Kod sva četiri izotopa u intervalu spinova od 6^+ do 10^+ pojavljuje se smanjivanje iznosa momenta ustrajnosti, tzv. backbending. Izračunati iznosi su u približnoj suglasnosti s eksperimentalnom vrijednošću.