

PROTON MOTION OF THE HYDROGEN ATOM IN THE NEW DIRAC FIELD THEORY*

K. LJOLJE and S. VOBORNIK

*Department of Physics, Faculty of Sciences,
University of Sarajevo, 71000 Sarajevo*

Received 14 September 1981

UDC 530.19

Original scientific paper

Motion of the proton of the hydrogen atom in the new Dirac field theory is considered. The proton-antiproton field with respect to the electromagnetic interaction is described by the new Dirac field with the corresponding mass of the proton. The interaction between the electron-positron field and the proton-antiproton field is assumed to be realized by the electromagnetic field already incorporated in each field. The self-interaction is also considered.

1. Introduction

The new Dirac field theory¹⁾ has been recently applied to the relativistic hydrogen atom²⁾. The proton was taken to be fixed. In this article we consider the motion of the proton.

Two questions arise: (1) how to describe the proton field with respect to the electromagnetic interaction only and (2) how to describe the interaction of the electron-positron field with the proton (-antiproton) field.

Due to the one half spin of the proton we assume that the proton field with respect to the electromagnetic interaction is described analogously to the electron-positron field. As far as the second question is concerned we assume that the

* This work was supported by the SIZ of Science of SR Bosna and Hercegovina, Sarajevo.

interaction between the electron-positron and the proton-antiproton fields is given by the electromagnetic field already incorporated in each field itself. In terms of minimal action it means that the total action is the sum of the actions for each field interacting with the electromagnetic field and the free electromagnetic field. Due to the statistical interpretation of the Dirac's fields one may expect that this interaction corresponds to the Hartree-Fock approximation of the hydrogen atom in the nonrelativistic quantum mechanics. We show that it is the case. By this the general interaction among the Dirac's field is indicated. This general problem we do not consider here, we do it elsewhere.

The self-interaction, which was ignored in Ref. 2, we consider also here. We restrict ourself to show how this interaction appears in the new theory, how it has to be treated and what the consequences are in the nonrelativistic limit. The systematic analysis of this interaction we do also elsewhere.

In Section 2 we present the new Dirac field theory in a short form (free field as well as the field interacting with the electromagnetic field) and some mathematical elaborations of the procedure which leads to the definition of the normal and dual solutions. The proton-antiproton field is defined in the Section 3. The interaction of the electron-positron field with the proton-antiproton field and application to the hydrogen is given in Section 4. The self-interaction is considered in Section 5 and final results with conclusions are given in Section 6.

2. The new Dirac field theory

The new Dirac field¹⁾ is defined by the Lagrangian

$$\mathcal{L} = \kappa [(-i \partial_\alpha \bar{\Phi} \gamma^\alpha) (i \partial_\beta \gamma^\beta \Phi) - \kappa^2 \bar{\Phi} \Phi], \quad (1)$$

where Φ is a bispinor and γ^α are the Dirac's matrices. We use the coordinates $x^\alpha = (x^0, x^1, x^2, x^3)$, the metric

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1, \quad g_{\alpha\beta} = 0, \quad \alpha \neq \beta,$$

the representation of the γ^μ

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3,$$

and units $c = \hbar = 1$. κ is a constant.

The Lagrange's equations for Φ and Φ^\dagger are

$$\partial_\alpha \partial^\alpha \Phi + \kappa^2 \Phi = 0, \quad (2)$$

$$\partial_\alpha \partial^\alpha \Phi^\dagger + \kappa^2 \Phi^\dagger = 0. \quad (3)$$

The conjugate momenta to Φ and Φ^\dagger are

$$\Pi_\Phi = \kappa (-i\partial_\nu \Phi^\dagger \gamma^\nu) i, \quad (4)$$

$$\Pi_{\Phi^\dagger} = \kappa (-i) (i\partial_\mu \gamma^\mu \Phi). \quad (5)$$

The Hamiltonian density is

$$\mathcal{H} = \frac{1}{\kappa} \Pi_\Phi \gamma^0 \Pi_{\Phi^\dagger} - \Pi_\Phi \partial_t \alpha^t \Phi - \partial_t \Phi^\dagger \alpha^t \Pi_{\Phi^\dagger} + \kappa (\kappa^2 \Phi^\dagger \gamma^0 \Phi), \quad (6)$$

and the corresponding canonical equations are

$$\partial_0 \Phi = \frac{1}{\kappa} \gamma^0 \Pi_{\Phi^\dagger} - \partial_t \alpha^t \Phi, \quad (7)$$

$$\partial_0 \Phi^\dagger = \frac{1}{\kappa} \Pi_\Phi \gamma^0 - \partial_t \Phi^\dagger \alpha^t, \quad (8)$$

$$\partial_0 \Pi_\Phi = -\partial_t \Pi_\Phi \alpha^t - \kappa^3 \Phi^\dagger \gamma^0, \quad (9)$$

$$\partial_0 \Pi_{\Phi^\dagger} = -\partial_t \alpha^t \Pi_{\Phi^\dagger} - \kappa^3 \gamma^0 \Phi, \quad (10)$$

where $\alpha^t = \gamma^0 \gamma^t$.

Introducing the new quantities Ψ_I and Ψ_{II} according to

$$\Psi_I = \frac{1}{\sqrt{2}} \left(\kappa \Phi + \frac{i}{\kappa} \Pi_{\Phi^\dagger} \right), \quad (11)$$

$$\Psi_{II} = \frac{1}{\sqrt{2}} \left(\kappa \Phi - \frac{i}{\kappa} \Pi_{\Phi^\dagger} \right), \quad (12)$$

the Eqs. (7—10) become

$$(i\partial_\nu \gamma^\nu - \kappa) \Psi_I = 0, \quad (13)$$

$$(i\partial_\nu \gamma^\nu + \kappa) \Psi_{II} = 0, \quad (14)$$

$$-i\partial_\nu \bar{\Psi}_I \gamma^\nu - \kappa \bar{\Psi}_I = 0, \quad (15)$$

$$-i\partial_\nu \bar{\Psi}_{II} \gamma^\nu + \kappa \bar{\Psi}_{II} = 0 \quad (16)$$

or writing

$$\Psi = \begin{pmatrix} \Psi_I \\ \Psi_{II} \end{pmatrix}, \quad \bar{\Psi} = \Psi^\dagger \tilde{\gamma}^0, \quad \tilde{\gamma}^\alpha = \begin{pmatrix} \gamma^\alpha & 0 \\ 0 & \gamma^\alpha \end{pmatrix}, \quad (17)$$

$$\begin{pmatrix} i\partial_\nu \gamma^\nu - \kappa & 0 \\ 0 & i\partial_\nu \gamma^\nu + \kappa \end{pmatrix} \Psi = 0, \quad (18)$$

$$\begin{pmatrix} -i\partial_\nu & 0 \\ 0 & -i\partial_\nu \end{pmatrix} \bar{\Psi} \tilde{\gamma}^\nu + \begin{pmatrix} -\kappa & 0 \\ 0 & \kappa \end{pmatrix} \bar{\Psi} = 0. \quad (19)$$

As one sees Eqs. (13—16) are the Dirac's equations with positive and negative mass terms. Consequently, they define the physical object.

The solution of Eq. (18) we seek in the form

$$\Psi(\vec{r}, t) = \int \Psi(\vec{r}, \omega) e^{-i\omega t} \frac{d\omega}{\sqrt{2\pi}}. \quad (20)$$

The Fourier coefficient $\Psi(\vec{r}, \omega)$ satisfies the equation

$$\begin{pmatrix} -i\partial_t \alpha^t + \kappa \gamma^0 & 0 \\ 0 & -i\partial_t \alpha^t - \kappa \gamma^0 \end{pmatrix} \Psi(\vec{r}, \omega) = \omega \Psi(\vec{r}, \omega), \quad \alpha^t = \gamma^0 \gamma^t. \quad (21)$$

This is the eigenvalue problem of the operator

$$\begin{pmatrix} -i\partial_t \alpha^t + \kappa \gamma^0 & 0 \\ 0 & -i\partial_t \alpha^t - \kappa \gamma^0 \end{pmatrix}. \quad (22)$$

The set of its eigenfunctions with large upper two components in the nonrelativistic limit for the particle and corresponding eigenvalues are

$$\omega = \pm \sqrt{k^2 + \kappa^2} \equiv \pm k_0, \quad k = |\vec{k}|, \quad (23)$$

$$\Psi_1(\vec{r}, -k_0) = \frac{1}{\sqrt{4\kappa k_0}} \begin{bmatrix} \kappa + k_0 \\ 0 \\ k_z \\ k_x + ik_y \\ \kappa - k_0 \\ 0 \\ -k_z \\ -k_x - ik_y \end{bmatrix} \frac{e^{i\vec{k}\vec{r}}}{(2\pi)^{3/2}}, \quad (24)$$

$$\Psi_2(\vec{r}, -k_0) = \frac{1}{\sqrt{4\pi k_0}} \begin{bmatrix} 0 \\ \kappa + k_0 \\ k_x - ik_y \\ -k_z \\ 0 \\ \kappa - k_0 \\ -k_x + ik_y \\ k_z \end{bmatrix} \frac{e^{i\vec{k}\vec{r}}}{(2\pi)^{3/2}} \quad (24')$$

$$\Psi_3(\vec{r}, +k_0) = \frac{1}{\sqrt{4\pi k_0}} \begin{bmatrix} \kappa - k_0 \\ 0 \\ k_z \\ k_x + ik_y \\ \kappa + k_0 \\ 0 \\ -k_z \\ -k_x - ik_y \end{bmatrix} \frac{e^{i\vec{k}\vec{r}}}{(2\pi)^{3/2}} \quad (25)$$

$$\Psi_4(\vec{r}, +k_0) = \frac{1}{\sqrt{4\pi k_0}} \begin{bmatrix} 0 \\ \kappa - k_0 \\ k_x - ik_y \\ -k_z \\ 0 \\ \kappa + k_0 \\ -k_x + ik_y \\ k_z \end{bmatrix} \frac{e^{i\vec{k}\vec{r}}}{(2\pi)^{3/2}} \quad (25')$$

The functions (24–25) are orthonormal in the sense

$$\int \Psi_i^\dagger(\vec{r}, \vec{k}') \tau_+ \Psi_j(\vec{r}, \vec{k}) d^3x = \delta_{ij} \delta(\vec{k}' - \vec{k}), \quad i, j = 1, 2,$$

$$\int \Psi_l^\dagger(\vec{r}, \vec{k}') \tau_- \Psi_k(\vec{r}, \vec{k}) d^3x = \delta_{lk} \delta(\vec{k}' - \vec{k}), \quad l, k = 3, 4, \quad (26)$$

$$\int \Psi_i^\dagger \tau_- \Psi_i d^3x = \int \Psi_i^\dagger \tau_+ \Psi_i d^3x = 0,$$

where

$$\tau_+ = \begin{pmatrix} (1)_{4 \times 4} & 0 \\ 0 & (-1)_{4 \times 4} \end{pmatrix}, \quad \tau_- = \begin{pmatrix} (-1)_{4 \times 4} & 0 \\ 0 & (1)_{4 \times 4} \end{pmatrix}. \quad (27)$$

The general solution of Eq. (18) is then

$$\Psi(\vec{r}, t) = \sum_{i=1,2} \int a_i(\vec{k}) \Psi_i(\vec{r}, k_0) e^{-ik_0 t} d\vec{k} + \sum_{i=3,4} \int b_i(\vec{k}) \Psi_i(\vec{r}, -k_0) e^{+ik_0 t} d\vec{k}, \quad (28)$$

where $a_i(\vec{k})$, $b_i(\vec{k})$ are arbitrary coefficients.

The solution (28) is called *normal solution*¹⁾ and it corresponds to the solution of Eq. (2)

$$\Phi = \begin{pmatrix} \neq 0 \\ \neq 0 \\ 0 \\ 0 \end{pmatrix}. \quad (29)$$

The solution of Eq. (18) which corresponds to the solution of Eq. (2) with upper zero components

$$\Phi = \begin{pmatrix} 0 \\ 0 \\ \neq 0 \\ \neq 0 \end{pmatrix}, \quad (30)$$

is called *dual solution* due to dual relation¹⁾ between these two solutions. The dual solution one eliminates from the theory, i. e. one keeps only the normal solution (28).

Let us point out that the solution (28) one gets from Eqs. (13—14) when one takes $\Psi_{II}(\vec{r}, t; \kappa) = \Psi_I(\vec{r}, t; -\kappa)$ and Ψ_I in the form which in the nonrelativistic limit contains upper two large components.

The scalar constant of motion, which follows from

$$\partial_\mu (\bar{\Psi}_I \gamma^\mu \Psi_I - \bar{\Psi}_{II} \gamma^\mu \Psi_{II}) = 0, \quad (31)$$

for the solution (28) is

$$\underline{Q} = \text{const}_Q \left\{ \sum_{i=1,2} \int |a_i(\vec{k})|^2 d\vec{k} - \sum_{i=3,4} \int |b_i(\vec{k})|^2 d\vec{k} \right\}. \quad (32)$$

The energy-momentum constant of motion, which follows from

$$T_\alpha^\beta = \partial_\alpha \Phi^\dagger \frac{\partial \mathcal{L}}{\partial (\partial_\beta \Phi^\dagger)} + \frac{\partial \mathcal{L}}{\partial (\partial_\beta \Phi)} \partial_\alpha \Phi - \delta_\alpha^\beta \mathcal{L}, \quad \partial_\beta T_\alpha^\beta = 0, \quad (33)$$

is

$$P^0 = \text{const}_P \left\{ \sum_{i=1,2} \int k_0 |a_i(\vec{k})|^2 d\vec{k} + \sum_{i=3,4} \int k_0 |b_i(\vec{k})|^2 d\vec{k} \right\}, \quad (34)$$

$$P^l = \text{const}_P \left\{ \sum_{i=1,2} \int k^l |a_i(\vec{k})|^2 d\vec{k} + \sum_{i=3,4} \int k^l |b_i(-\vec{k})|^2 d\vec{k} \right\}. \quad (35)$$

The angular momentum constant of motion, follows from

$$M^{\alpha,\beta\gamma} = (x^\gamma T^{\beta\alpha} - x^\beta T^{\gamma\alpha}) - \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \Phi)} \left(\frac{i}{2} \sigma^{\beta\gamma} \right) \Phi - \Phi \left(-\frac{i}{2} \sigma^{\beta\gamma} \right) \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \Phi)}, \quad (36)$$

$$\partial_\alpha M^{\alpha,\beta\gamma} = 0.$$

The space components are

$$M^{ij} = L^{ij} + S^{ij},$$

$$L^{ij} = \text{const}_M \int (x^j T^{i0} - x^i T^{j0}) d^3x, \quad S^{ij} = \text{const}_M \frac{1}{2} \int \Psi^\dagger \sigma^{ij} \tau_+ \Psi d^3x. \quad (37)$$

The spin pseudovector, which comes from,

$$M_k = \frac{1}{2} \varepsilon_{ijk} M^{ij} = L_k + S_k,$$

is

$$S_k = \text{const}_M \int \left(\Psi_I \frac{1}{2} \Sigma_k \Psi_I - \Psi_{II}^\dagger \frac{1}{2} \Sigma_k \Psi_{II} \right) d^3x = \text{const}_M \int \Psi^\dagger \frac{1}{2} \Sigma_k \tau_+ \Psi d^3x,$$

$$\tau_+ = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \Sigma_k = \frac{1}{2} \varepsilon_{ijk} \sigma^{ij}, \quad (38)$$

and its z -component

$$S_z = \text{const}_M \left\{ \frac{1}{2} \int (|a_1(\vec{k})|^2 - |a_2(\vec{k})|^2 + |b_4(\vec{k})|^2 - |b_3(\vec{k})|^2) d\vec{k} \right\}. \quad (39)$$

Selecting $\text{const}_Q = q$, $\text{const}_P = \text{const}_M = 1$ and taking

$$\sum_{i=1,2} |a_i(k)|^2 = 1, \quad \sum_{i=3,4} |b_i(k)|^2 = 1, \quad (40)$$

when a_i and b_i are not identically zero, these quantities become:

(a) $b_l = 0$ for all l

$$Q = q,$$

$$P^0 = \sum_{i=1,2} \int k_0 |a_i(\vec{k})|^2 d\vec{k}, \quad (41)$$

$$P^i = \sum_{i=1,2} \int k^i |a_i(\vec{k})|^2 d\vec{k},$$

$$S_z = \frac{1}{2} \int (|a_1(\vec{k})|^2 - |a_2(\vec{k})|^2) d\vec{k},$$

(b) $a_i = 0$ for all i

$$Q = -q,$$

$$P^0 = \sum_{i=3,4} \int k_0 |b_i(\vec{k})|^2 d\vec{k}, \quad (42)$$

$$P^i = \sum_{i=3,4} \int k^i |b_i(-\vec{k})|^2 d\vec{k},$$

$$S_z = \frac{1}{2} \int (|b_4(\vec{k})|^2 - |b_3(\vec{k})|^2) d\vec{k},$$

(c) a_i and b_i are not identically zero

$$Q = 0,$$

$$P^0 = \sum_{i=1,2} \int k_0 |a_i(\vec{k})|^2 d\vec{k} + \sum_{i=3,4} \int k_0 |b_i(\vec{k})|^2 d\vec{k},$$

$$P^i = \sum_{i=1,2} \int k^i |a_i(\vec{k})|^2 d\vec{k} + \sum_{i=3,4} \int k^i |b_i(-\vec{k})|^2 d\vec{k}, \quad (43)$$

$$S_z = \frac{1}{2} \int \{|a_1(\vec{k})|^2 - |a_2(\vec{k})|^2 + |b_4(\vec{k})|^2 - |b_3(\vec{k})|^2\} d\vec{k}.$$

The particle statistical interpretation of these constants of motion is evident. The case (a) gives the average values of the physical quantities of a particle with the charge q and the mass κ (or m in the standard units). The case (b) gives the same but for a particle with opposite charge ($-q$), i. e. for the antiparticle. The case (c) gives the averages physical quantities of both present particles, the particle and its antiparticle.

Interaction with the electromagnetic field

The interaction of the new Dirac field with the electromagnetic field one introduces by the substitution²⁾

$$i\partial_\alpha \rightarrow i\partial_\alpha - e A_\alpha, \quad e = -|e|. \quad (44)$$

The new Lagrangian density is then

$$\mathcal{L} = \kappa \{ [(-i\partial_\alpha - e A_\alpha) \bar{\Phi} \gamma^\alpha] [(i\partial_\beta - e A_\beta) \gamma^\beta \Phi] - \kappa^2 \bar{\Phi} \Phi \} + \mathcal{L}_{em}. \quad (45)$$

The corresponding Lagrange's equations are

$$\partial_\alpha \partial^\alpha A^\mu = -4\pi \{ 2e^2 \kappa \bar{\Phi} \Phi A^\mu + ie\kappa [-\bar{\Phi} \gamma^\mu (\partial_\alpha \gamma^\alpha \Phi) + (\partial_\alpha \bar{\Phi} \gamma^\alpha) \gamma^\mu \Phi] \}, \quad (46)$$

$$\begin{aligned} \partial_\alpha \partial^\alpha \Phi + \kappa^2 \Phi + ie [\partial_\mu \gamma^\mu (A_\beta \gamma^\beta \Phi) + A_\alpha \gamma^\alpha \partial_\beta \gamma^\beta \Phi] - \\ - e^2 A_\alpha A^\alpha \Phi = 0 \end{aligned} \quad (47)$$

or in the form

$$\{ (i\partial_\alpha - e A_\alpha) (i\partial^\alpha - e A^\alpha) - \frac{1}{2} e \sigma^{\alpha\beta} F_{\alpha\beta} \} \Phi = \kappa^2 \Phi, \quad (48)$$

where

$$\frac{1}{2} \sigma^{\alpha\beta} F_{\alpha\beta} = \vec{\Sigma} \vec{H} - i\vec{\alpha} \vec{E}, \quad \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}. \quad (49)$$

The Lagrangian (45) describes internal electromagnetic interaction of the new Dirac field. When some other sources of the electromagnetic field are present then one has to add to the Lagrangian (45) the corresponding Lagrange's densities. The new Lagrange's equation will contain contributions of these sources.

The conjugate momenta to Φ and Φ^\dagger are

$$\begin{aligned} \Pi_\Phi &= \kappa [-(i\partial_\alpha + e A_\alpha) \bar{\Phi} \gamma^\alpha i\gamma^0], \\ \Pi_{\Phi^\dagger} &= \kappa [-i(i\partial_\beta - e A_\beta) \gamma^\beta \Phi]. \end{aligned} \quad (50)$$

The Hamiltonian density (corresponding to the Dirac field) is

$$\begin{aligned} \mathcal{H} &= \frac{1}{\kappa} \Pi_\Phi \gamma^0 \Pi_{\Phi^\dagger} - \Pi_\Phi (\partial_j \gamma^0 \gamma^j \Phi + ie A_\beta \gamma^0 \gamma^\beta \Phi) - (\partial_j \bar{\Phi} \gamma^j - \\ & - ie A_\alpha \bar{\Phi} \gamma^\alpha) \Pi_{\Phi^\dagger} + \kappa^3 \bar{\Phi} \Phi \end{aligned} \quad (51)$$

and the canonical equations are

$$\begin{aligned} \partial_0 \Pi_\Phi &= -\kappa^3 \bar{\Phi} + \Pi_\Phi ie A_\beta \gamma^0 \gamma^\beta + \partial_j (-\Pi_\Phi \gamma^0 \gamma^j), \\ \partial_0 \Pi_{\Phi^\dagger} &= -\kappa^3 \gamma^0 \Phi - \partial_j (\gamma^0 \gamma^j \Pi_{\Phi^\dagger}) - ie A_\alpha \gamma^0 \gamma^\alpha \Pi_{\Phi^\dagger}, \end{aligned} \quad (52)$$

$$\partial_\bullet \Phi = \frac{1}{\kappa} \gamma^0 \Pi_\Phi - (\partial_j \gamma^0 \gamma^j \Phi + ie A_\beta \gamma^0 \gamma^\beta \Phi), \quad (53)$$

$$\partial_0 \Phi^+ = \frac{1}{\kappa} \Pi_\Phi \gamma^0 - (\partial_j \bar{\Phi} \gamma^j - ie A_\alpha \bar{\Phi} \gamma^\alpha).$$

Introducing the new functions Ψ_I and Ψ_{II} according to (11—12), the canonical equations become

$$[(i\partial_\alpha - e A_\alpha) \gamma^\alpha - \kappa] \Psi_I = 0, \quad (54)$$

$$[(i\partial_\alpha - e A_\alpha) \gamma^\alpha + \kappa] \Psi_{II} = 0,$$

$$(-i\partial_\alpha - e A_\alpha) \bar{\Psi}_I \gamma^\alpha - \kappa \bar{\Psi}_I = 0, \quad (55)$$

$$(-i\partial_\alpha - e A_\alpha) \bar{\Psi}_{II} \gamma^\alpha + \kappa \bar{\Psi}_{II} = 0.$$

These are the standard Dirac field equations of the field interacting with the electromagnetic field but with the positive as well as the negative mass term.

The scalar constant of motion is²⁾

$$Q = e \int (\Psi_I^+ \Psi_I - \Psi_{II}^+ \Psi_{II}) d^3x = e \int \Psi^+ \tau_+ \Psi d^3x. \quad (56)$$

The contribution of the Dirac field to the energy-momentum constant of motion is²⁾

$$P_\alpha^D = \int \Psi^+ \tau_+ (i\partial_\alpha) \Psi d^3x \quad (57)$$

and to the angular momentum constant of motion²⁾

$$M_k^D = \int \Psi^+ \tau_+ (L_k + \frac{1}{2} \sum_k) \Psi d^3x. \quad (58)$$

It is interesting and important to notice that Eq. (46) does not contain free field solution*. Consequently, Eqs. (46—47) describe internal electromagnetic interaction including self-interactions. It can be easily seen if one writes Eq. (46) in the form

$$(\partial_\alpha \partial^\alpha + 8\pi e^2 \kappa \bar{\Phi} \Phi) A^\mu = 4\pi e \kappa [\bar{\Phi} \gamma^\mu (i\partial_\alpha \gamma^\alpha \Phi) + (-i\partial_\alpha \bar{\Phi} \gamma^\alpha) \gamma^\mu \Phi]. \quad (59)$$

The solution of equations corresponding to (46—47; 54—55) with fixed proton and without self-interactions is given in Ref. 2.

The problem of the self-interaction we consider in Section 6.

* Solution of the equation $\Delta A^\mu = 0$.

3. The proton-antiproton fields

The proton is a particle with the one half spin. With respect to the electromagnetic interaction it has a positive charge. Therefore, if the electromagnetic interaction is in question only the proton can be described by the same Dirac's equations as the electron except for the mass terms and charge. We consider this case and assume that the proton-antiproton field interacting with the electromagnetic field is given by the Lagrange's density

$$\mathcal{L} = \kappa_p \{ [(-i\partial_x + eA_x) \bar{\Psi} \gamma^\alpha] [(i\partial_\beta + eA_\beta) \Psi^\beta \Phi] - \kappa_p^2 \bar{\Psi} \Phi \} + \mathcal{L}_{em}. \quad (60)$$

The Lagrange's equations, the Hamiltonian density, the canonical equations, the constants of motion and so on are equal to those of the electron-positron field with the change $e \rightarrow -e$, $\kappa \equiv \kappa_e \rightarrow \kappa_p$. In order to distinguish the electron-positron field from the proton-antiproton field we denote quantities of the first one by the index e and of the second one by the index p . Thus, the canonical equations of the proton-antiproton field expressed by the functions Ψ (see Eqs. 11—12) are

$$[(i\partial_\alpha + eA_\alpha) \gamma^\alpha - \kappa_p] \Psi_{pI} = 0, \quad (61)$$

$$[(i\partial_\alpha + eA_\alpha) \gamma^\alpha + \kappa_p] \Psi_{pII} = 0,$$

$$(-i\partial_\alpha + eA_\alpha) \bar{\Psi}_{pI} \gamma^\alpha - \kappa_p \bar{\Psi}_{pI} = 0, \quad (62)$$

$$(-i\partial_\alpha + eA_\alpha) \bar{\Psi}_{pII} \gamma^\alpha + \kappa_p \bar{\Psi}_{pII} = 0.$$

4. Interacting fields

In the previous sections we have considered the electron-positron and the proton-antiproton fields separately. In this section we consider their mutual interaction by way of the electromagnetic field.

The first question is in what form this interaction appears. In order to make the things as simple as possible we take the simplest position in this article. In Section 2 we have presented the electron-positron field interacting with the electromagnetic field. The same we have given for the proton-antiproton field in Section 3. In both cases the electromagnetic field was incorporated in the Dirac's field by the standard procedure. The electromagnetic field potential describes the interacting electromagnetic field. The simplest extension to two Dirac's fields is just that this field potential describes interacting electromagnetic field of the both Dirac's field. In the Lagrange's language it means that the Lagrange's density of the interacting fields is

$$\mathcal{L} = \mathcal{L}_{e-p} + \mathcal{L}_{p-ap} + \mathcal{L}_{em} \quad (63)$$

where \mathcal{L}_{e-p} is given by Eq. (45), \mathcal{L}_{p-ad} by Eq. (60) without \mathcal{L}_{em} and

$$\mathcal{L}_{em} = -\frac{1}{16\pi} (\partial_\alpha A_\beta - \partial_\beta A_\alpha) (\partial^\alpha A^\beta - \partial^\beta A^\alpha) \quad (64)$$

or explicitly

$$\begin{aligned} \mathcal{L} = & \kappa_e \{ [(-i\partial_\alpha - eA_\alpha) \bar{\Phi}_e \gamma^\alpha] [(i\partial_\beta - eA_\beta) \gamma^\beta \Phi_e] - \kappa_p^2 \bar{\Phi}_e \Phi_e \} + \\ & + \kappa_p \{ [(-i\partial_\alpha + eA_\alpha) \bar{\Phi}_p \gamma^\alpha] [(i\partial_\beta + eA_\beta) \gamma^\beta \Phi_p] - \kappa_p^2 \bar{\Phi}_p \Phi_p \} + \mathcal{L}_{em}. \end{aligned} \quad (65)$$

In the next we investigate only this form of interaction between these two Dirac's fields.

We have now the Lagrange's variables: $\Phi_e, \Phi_e^\dagger, \Phi_p, \Phi_p^\dagger, A^\mu$. The electromagnetic field we describe on the standard way as in Ref. 2.

The Lagrange's equations, according to

$$\frac{\partial \mathcal{L}}{\partial \chi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \chi)} = 0 \quad (66)$$

for these variables are

$$\partial_\alpha \partial^\alpha \Phi_e + \kappa_e^2 \Phi_e + ie [\partial_\mu \gamma^\mu (A_\beta \gamma^\beta \Phi_e) + A_\alpha \gamma^\alpha \partial_\beta \gamma^\beta \Phi_e] - e^2 A_\alpha A^\alpha \Phi_e = 0, \quad (67)$$

$$\partial_\alpha \partial^\alpha \bar{\Phi}_e + \kappa_e^2 \bar{\Phi}_e - ie [\partial_\mu (\bar{\Phi}_e \gamma^\beta A_\beta) \gamma^\mu + (\partial_\beta \bar{\Phi}_e \gamma^\beta) \gamma^\alpha A_\alpha] - e^2 A_\alpha A^\alpha \bar{\Phi}_e = 0, \quad (68)$$

$$\partial_\alpha \partial^\alpha \Phi_p + \kappa_p^2 \Phi_p - ie [\partial_\mu \gamma^\mu (A_\beta \gamma^\beta \Phi_p) + A_\alpha \gamma^\alpha \partial_\beta \gamma^\beta \Phi_p] - e^2 A_\alpha A^\alpha \Phi_p = 0, \quad (69)$$

$$\partial_\alpha \partial^\alpha \bar{\Phi}_p + \kappa_p^2 \bar{\Phi}_p + ie [\partial_\mu (\bar{\Phi}_p \gamma^\beta A_\beta) \gamma^\mu + (\partial_\beta \bar{\Phi}_p \gamma^\beta) \gamma^\alpha A_\alpha] - e^2 A_\alpha A^\alpha \bar{\Phi}_p = 0, \quad (70)$$

$$\begin{aligned} \partial_\alpha \partial^\alpha A^\mu = & -4\pi \{ 2e^2 \kappa_e \bar{\Phi}_e \Phi_e A^\mu + ie \kappa_e [-\bar{\Phi}_e \gamma^\mu (\partial_\alpha \gamma^\alpha \Phi_e) + (\partial_\alpha \bar{\Phi}_e \gamma^\alpha) \gamma^\mu \Phi_e] \} - \\ & -4\pi \{ 2e^2 \kappa_p \bar{\Phi}_p \Phi_p A^\mu - ie \kappa_p [-\bar{\Phi}_p \gamma^\mu (\partial_\alpha \gamma^\alpha \Phi_p) + (\partial_\alpha \bar{\Phi}_p \gamma^\alpha) \gamma^\mu \Phi_p] \}. \end{aligned} \quad (71)$$

The conjugate momenta to $\Phi_e, \Phi_e^\dagger, \Phi_p, \Phi_p^\dagger$ are

$$\Pi_{\Phi_e} = \kappa_e [- (i\partial_\alpha + eA_\alpha) \bar{\Phi}_e \gamma^\alpha i\gamma^0], \quad (72)$$

$$\Pi_{\Phi_e^\dagger} = \kappa_e [-i (i\partial_\beta - eA_\beta) \gamma^\beta \Phi_e], \quad (73)$$

$$\Pi_{\Phi_p} = \kappa_p [- (i\partial_\alpha - eA_\alpha) \bar{\Phi}_p \gamma^\alpha i\gamma^0], \quad (74)$$

$$\Pi_{\Phi_p^\dagger} = \kappa_p [-i (i\partial_\beta + eA_\beta) \gamma^\beta \Phi_p]. \quad (75)$$

The Hamiltonian density (without the part due to A^μ) is

$$\begin{aligned} \mathcal{H} = & \frac{1}{\kappa_e} \Pi_{\Phi_e} \gamma^0 \Pi_{\Phi_e}^\dagger - \Pi_{\Phi_e} (\partial_j \gamma^0 \gamma^j \Phi_e + ie A_\beta \gamma^0 \gamma^\beta \Phi_e) - (\partial_j \bar{\Phi}_e \gamma^j - \\ & - ie A_\alpha \bar{\Phi}_e \gamma^\alpha) \Pi_{\Phi_e}^\dagger + \kappa_e^3 \bar{\Phi}_e \Phi_e + \frac{1}{\kappa_p} \Pi_{\Phi_p} \gamma^0 \Pi_{\Phi_p}^\dagger - \Pi_{\Phi_p} (\partial_j \gamma^0 \gamma^j \Phi_p - \\ & - ie A_\beta \gamma^0 \gamma^\beta \Phi_p) - (\partial_j \bar{\Phi}_p \gamma^j + ie A_\alpha \bar{\Phi}_p \gamma^\alpha) \Pi_{\Phi_p}^\dagger + \kappa_p^3 \bar{\Phi}_p \Phi_p \end{aligned} \quad (76)$$

and the corresponding canonical equations, according to

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \{F, H\}, \quad (77)$$

are

$$\partial_0 \Pi_{\Phi_e} = -\kappa_e^3 \bar{\Phi}_e + \Pi_{\Phi_e} ie A_\beta \gamma^0 \gamma^\beta + \partial_j (-\Pi_{\Phi_e} \gamma^0 \gamma^j), \quad (78)$$

$$\partial_0 \Pi_{\Phi_e}^\dagger = -\kappa_e^3 \gamma^0 \Phi_e - \partial_j (\gamma^0 \gamma^j \Pi_{\Phi_e}^\dagger) - ie A_\alpha \gamma^0 \gamma^\alpha \Pi_{\Phi_e}^\dagger, \quad (79)$$

$$\partial_0 \Phi_e = \frac{1}{\kappa_e} \gamma^0 \Pi_{\Phi_e}^\dagger - (\partial_j \gamma^0 \gamma^j \Phi_e + ie A_\beta \gamma^0 \gamma^\beta \Phi_e), \quad (80)$$

$$\partial_0 \bar{\Phi}_e = \frac{1}{\kappa_e} \Pi_{\Phi_e} \gamma^0 - (\partial_j \bar{\Phi}_e \gamma^j - ie A_\alpha \bar{\Phi}_e \gamma^\alpha), \quad (81)$$

$$\partial_0 \Pi_{\Phi_p} = -\kappa_p^3 \bar{\Phi}_p - \Pi_{\Phi_p} ie A_\beta \gamma^0 \gamma^\beta + \partial_j (-\Pi_{\Phi_p} \gamma^0 \gamma^j), \quad (82)$$

$$\partial_0 \Pi_{\Phi_p}^\dagger = -\kappa_p^3 \gamma^0 \Phi_p - \partial_j (\gamma^0 \gamma^j \Pi_{\Phi_p}^\dagger) + ie A_\alpha \gamma^0 \gamma^\alpha \Pi_{\Phi_p}^\dagger, \quad (83)$$

$$\partial_0 \Phi_p = \frac{1}{\kappa_p} \gamma^0 \Pi_{\Phi_p}^\dagger - (\partial_j \gamma^0 \gamma^j \Phi_p - ie A_\beta \gamma^0 \gamma^\beta \Phi_p), \quad (84)$$

$$\partial_0 \bar{\Phi}_p = \frac{1}{\kappa_p} \Pi_{\Phi_p} \gamma^0 - (\partial_j \bar{\Phi}_p \gamma^j + ie A_\alpha \bar{\Phi}_p \gamma^\alpha). \quad (85)$$

Again, the substitutions

$$\Psi_{e1} = \frac{1}{\sqrt{2}} \left(\kappa_e \Phi_e + \frac{i}{\kappa_e} \Pi_{\Phi_e}^\dagger \right), \quad (86)$$

$$\Psi_{eII} = \frac{1}{\sqrt{2}} \left(\kappa_e \Phi_e - \frac{i}{\kappa_e} \Pi \phi_e^\dagger \right), \quad (87)$$

$$\Psi_{pI} = \frac{1}{\sqrt{2}} \left(\kappa_p \Phi_p + \frac{i}{\kappa_p} \Pi \phi_p^\dagger \right), \quad (88)$$

$$\Psi_{pII} = \frac{1}{\sqrt{2}} \left(\kappa_p \Phi_p - \frac{i}{\kappa_p} \Pi \phi_p^\dagger \right), \quad (89)$$

transform Eqs. (78—85) into

$$[(i\partial_\alpha - eA_\alpha) \gamma^\alpha - \kappa_e] \Psi_{eI} = 0, \quad (90)$$

$$[(i\partial_\alpha - eA_\alpha) \gamma^\alpha + \kappa_e] \Psi_{eII} = 0, \quad (91)$$

$$(-i\partial_\alpha - eA_\alpha) \bar{\Psi}_{eI} \gamma^\alpha - \kappa_e \bar{\Psi}_{eI} = 0, \quad (92)$$

$$(-i\partial_\alpha - eA_\alpha) \bar{\Psi}_{eII} \gamma^\alpha + \kappa_e \bar{\Psi}_{eII} = 0, \quad (93)$$

$$[(i\partial_\alpha + eA_\alpha) \gamma^\alpha - \kappa_p] \Psi_{pI} = 0, \quad (94)$$

$$[(i\partial_\alpha + eA_\alpha) \gamma^\alpha + \kappa_p] \Psi_{pII} = 0, \quad (95)$$

$$(-i\partial_\alpha + eA_\alpha) \bar{\Psi}_{pI} \gamma^\alpha - \kappa_p \bar{\Psi}_{pI} = 0, \quad (96)$$

$$(-i\partial_\alpha + eA_\alpha) \bar{\Psi}_{pII} \gamma^\alpha + \kappa_p \bar{\Psi}_{pII} = 0. \quad (97)$$

As one sees these are the standard Dirac's equations with the electromagnetic field but with the positive and negative mass terms.

Similarly to the separated fields the constants of motion expressed by the Ψ -functions and A^μ are

$$Q = e \int \Psi_e^\dagger \tau_+ \Psi_e d^3x - e \int \Psi_p^\dagger \tau_+ \Psi_p d^3x, \quad (98)$$

$$P_\alpha^0 = \int \Psi_e^\dagger \tau_+ (i\partial_\alpha) \Psi_e d^3x + \int \Psi_p^\dagger \tau_+ (i\partial_\alpha) \Psi_p d^3x, \quad (99)$$

$$M_k^0 = \int \Psi_e^\dagger \tau_+ \left(L_k + \frac{1}{2} \Sigma_k \right) \Psi_e d^3x + \int \Psi_p^\dagger \tau_+ \left(L_k + \frac{1}{2} \Sigma_k \right) \Psi_p d^3x, \quad (100)$$

where we have written again only the parts which belong to the Dirac fields.

Hydrogen atom

We apply Eqs. (67—70) or (90—97) and (71) to the hydrogen atom in the nonrelativistic limit and for

$$A_i \doteq 0, \quad \partial_0 A_0 \doteq 0. \quad (101)$$

In these cases the solutions of Eqs. (67—70) and (90—97) can be separated on positive and negative frequencies like in the free field case. Due to presence of an electron and a proton we have to select the solutions of the same frequencies. We take the negative frequencies solutions.

Writing

$$\Phi_e = e^{-i\kappa_e t} \varphi_e, \quad (102)$$

$$\Phi_p = e^{-i\kappa_p t} \varphi_p, \quad (103)$$

and assuming

$$\begin{aligned} |\kappa_e \varphi_e| &\gg |\partial_t \varphi_e|, & |\kappa_p \varphi_p| &\gg |\partial_t \varphi_p|, \\ |A^i \partial_i \varphi_e| &\ll |\kappa_e A^0 \varphi_e|, & |A^i \partial_i \varphi_p| &\ll |\kappa_p A^0 \varphi_p|, \end{aligned} \quad (104)$$

the nonrelativistic approximation of Eqs. (67) and (69) is

$$[-2i \kappa_e \partial_t - \Delta - ie(2i \kappa_e A^0 + \gamma^\mu \gamma^0 (\partial_\mu A^0)) - e^2 A_\alpha A^\alpha] \varphi_e = 0, \quad (105)$$

$$[-2i \kappa_p \partial_t - \Delta + ie(2i \kappa_p A^0 + \gamma^\mu \gamma^0 (\partial_\mu A^0)) - e^2 A_\alpha A^\alpha] \varphi_p = 0. \quad (106)$$

For the cases (101) and neglecting the terms $e^2 A_\alpha A^\alpha$ they become

$$(-2i \kappa_e \partial_t - \Delta + 2e \kappa_e A^0) \varphi_e = 0, \quad (107)$$

$$(-2i \kappa_p \partial_t - \Delta - 2e \kappa_p A^0) \varphi_p = 0 \quad (108)$$

or

$$i\partial_t \varphi_e = \left(-\frac{1}{2\kappa_e} \Delta + eA^0 \right) \varphi_e, \quad (109)$$

$$i\partial_t \varphi_p = \left(-\frac{1}{2\kappa_p} \Delta - eA^0 \right) \varphi_p, \quad (110)$$

where we have assumed also the solutions with small down two components.

The corresponding nonrelativistic approximation of Eq. (71) is

$$-\Delta A^0 = (-8\pi e^2 \kappa_e \bar{\varphi}_e \varphi_e A^0 + 8\pi e \kappa_e^2 \bar{\varphi}_e \varphi_e) + (-8\pi e^2 \kappa_p \bar{\varphi}_p \varphi_p A^0 - 8\pi e \kappa_p^2 \bar{\varphi}_p \varphi_p). \quad (111)$$

The right-hand side contains both fields (electron's and proton's). Consequently, in Eqs. (109—110) there will be also the self-interaction terms. We might ignore them, as it was done in Ref. 2, but we investigate the self-interaction separately in the next Section and then come back again to the hydrogen atom.

5. The self-interaction in the new Dirac field

The self-interaction is important in the classical as well as quantum electrodynamics. It leads to known infinities and the renormalization procedure.

In this section we investigate the basic effects of the self-interaction in the new Dirac field theory in the nonrelativistic limit. The systematic analysis of this interaction we perform in one of subsequent papers.

One electron problem

We first investigate the self-interaction of one electron. The equations of the electron interacting with its own electromagnetic field, according to (67) and (71), are

$$\partial_\alpha \partial^\alpha \bar{\Phi} + \kappa^2 \bar{\Phi} + ie [\partial_\mu \gamma^\mu (A_\beta \gamma^\beta \Phi) + A_\alpha \gamma^\alpha \partial_\beta \gamma^\beta \Phi] - e^2 A_\alpha A^\alpha \bar{\Phi} = 0, \quad (112)$$

$$\partial_\alpha \partial^\alpha A^\mu = -4\pi \{2e^2 \kappa \bar{\Phi} \Phi A^\mu + ie \kappa [-\bar{\Phi} \gamma^\mu (\partial_\alpha \gamma^\alpha \Phi) + (\partial_\alpha \bar{\Phi} \gamma^\alpha) \gamma^\mu \Phi]\}. \quad (113)$$

Writing Φ in the form (102) and using the conditions (104) they become

$$[-2i\kappa \partial_t - \Delta - ie(2i\kappa A^0 + \gamma^\mu \gamma^0 (\partial_\mu A^0)) - e^2 A_\alpha A^\alpha] = 0, \quad (114)$$

$$\partial_\alpha \partial^\alpha A^0 = -8\pi e^2 \kappa \bar{\varphi} \varphi A^0 + 8\pi e \kappa^2 \bar{\varphi} \varphi, \quad (115)$$

$$\partial_\alpha \partial^\alpha A^i = -8\pi e^2 \kappa \bar{\varphi} \varphi A^i + 8\pi e \kappa^2 \bar{\varphi} \gamma^i \gamma^0 \varphi. \quad (116)$$

In the nonrelativistic limit, writing

$$\varphi = \begin{pmatrix} \varphi_a \\ \varphi_b \end{pmatrix}, \quad (117)$$

we select the solutions

$$|\varphi_a| \gg |\varphi_b|. \quad (118)$$

The second term on the right-hand side of Eq. (116) is small ($\rightarrow 0$) and there is a solution

$$A^i \doteq 0, \quad (\text{and } \partial_t A^0 \doteq 0, \text{ due to } L. \text{ condition}). \quad (119)$$

In the next we consider only this case.

Taking (119) in Eqs. (114—115) and neglecting also the square potential term we find

$$i\partial_t \varphi = \left(-\frac{1}{2\kappa} \Delta + eA^0 \right) \varphi, \quad (120)$$

$$\Delta A^0 - 8\pi e^2 \kappa \bar{\varphi} \varphi A^0 = -8\pi e \kappa^2 \bar{\varphi} \varphi. \quad (121)$$

Simultaneous solutions of these equations we need now. The general solution of Eq. (121) is

$$A^0 = A_h^0 + A_p^0, \quad (122)$$

where A_h^0 is the general solution of the homogeneous equation,

$$\Delta A_h^0 - 8\pi e^2 \kappa \bar{\varphi} \varphi A_h^0 = 0, \quad (123)$$

and A_p^0 is a particular solution. One easily sees that

$$A_p^0 = \frac{\kappa}{e}. \quad (124)$$

The solution of Eq. (120) we seek further in the form

$$\varphi(\vec{r}, t) = e^{-i\epsilon t} \varphi(\vec{r}). \quad (125)$$

After substitution in (120) follows

$$\epsilon \varphi(\vec{r}) = \left(-\frac{1}{2\kappa} \Delta + eA^0 \right) \varphi(\vec{r}). \quad (126)$$

One may expect a small contribution of eA^0 . Therefore, we apply the perturbation method:

$$\varphi(\vec{r}) = \varphi^{(0)}(\vec{r}) + \varphi^{(1)}(\vec{r}) + \dots, \quad (127)$$

$$\epsilon = \epsilon^{(0)} + \epsilon^{(1)} + \dots$$

and

$$\epsilon^{(0)} \varphi^{(0)}(\vec{r}) = -\frac{1}{2\kappa} \Delta \varphi^{(0)}(\vec{r}), \quad (128)$$

$$\epsilon^{(1)} \varphi^{(0)}(\vec{r}) + \epsilon^{(0)} \varphi^{(1)}(\vec{r}) = -\frac{1}{2\kappa} \Delta \varphi^{(1)}(\vec{r}) + eA^0 \varphi^{(0)}(\vec{r}), \quad (129)$$

The box normalized solutions of Eq. (128) are

$$\varphi_{ik}^{(0)}(\vec{r}) = \frac{1}{L^{3/2}} e^{i\vec{k}\vec{r}}, \quad \varepsilon^{(0)} = \frac{\hbar^2}{2\kappa}, \quad i = 1, 2. \quad (130)$$

The down two components of φ we take to be zero (normal solution).

The substitution of (127) into (123) gives

$$\Delta A_h^0 - 8\pi e^2 \kappa \bar{\varphi}_{ik}^{(0)} \varphi_{ik}^{(0)} A_h^0 - 8\pi e^2 \kappa (\bar{\varphi}_{ik}^{(0)} \varphi_{ik}^{(1)} + \bar{\varphi}_{ik}^{(1)} \varphi_{ik}^{(0)}) A_h^0 + \dots = 0. \quad (131)$$

The third term is second order in A_h^0 , the fourth term is third order in A_h^0 and so on. Since, we expect small A_h^0 we write

$$A_h^0 = A_h^{0(0)} + A_h^{0(1)} + \dots \quad (132)$$

and

$$\Delta A_h^{0(0)} - \delta^2 A_h^{0(0)} = 0, \quad \delta^2 = 8\pi e^2 \kappa \bar{\varphi}_{ik}^{(0)} \varphi_{ik}^{(0)} = 4\pi e^2 \kappa \frac{1}{L^3}, \quad (133)$$

$$\Delta A_h^{0(1)} - \delta^2 A_h^{0(1)} - 8\pi e^2 \kappa [\bar{\varphi}_{ik}^{(0)} \varphi_{ik}^{(1)} (A_h^{0(0)}) + \bar{\varphi}_{ik}^{(1)} (A_h^{0(0)}) \varphi_{ik}^{(0)}] A_h^{0(0)} = 0, \quad (134)$$

$$i = 1, 2.$$

To simplify the analysis we substitute the box L^3 with the sphere $4\pi R^3/3$. The spherically symmetrical solution of Eq. (133) is then

$$A_h^{0(0)} = \frac{1}{r} (C e^{-\delta r} + D e^{\delta r}), \quad (135)$$

where C and D are arbitrary constants.

The condition

$$A^0(0) = \text{const} \equiv \zeta \quad (136)$$

leads to

$$C + D = 0 \quad (137)$$

and

$$A^{0(0)} = 2D \frac{\text{sh } \delta r}{r}. \quad (138)$$

The solution (122) in zero order approximation of A^0 is then

$$A^{0(0)} = 2D \frac{\text{sh } \delta r}{r} + \frac{\kappa}{e}. \quad (139)$$

Application of the condition (136) gives

$$D = -\frac{\kappa}{2\delta e} + \frac{\zeta}{2\delta} \quad (140)$$

and

$$A^{0(0)} = \frac{\kappa}{e} \left(1 - \frac{\text{sh } \delta r}{\delta r} \right) + \frac{\text{sh } \delta r}{\delta r}. \quad (141)$$

Because of $r < R$, for $R \rightarrow \infty$

$$\delta r \rightarrow 0 \quad \text{and} \quad A^{0(0)} \rightarrow \zeta. \quad (142)$$

We may select the constant ζ to be zero. Therefore, the self-interaction can be ignored.

It is important to notice that the self-interaction in the new Dirac field theory does not lead to serious difficulties.

Electron in the field of fixed proton

At the presence of a fixed proton there appears the term $-4\pi e \delta(\vec{r} - \vec{r}_0)$ on the right-hand side of Eq. (113), so that this equation now reads

$$\begin{aligned} \mathbf{I}_\alpha \partial^\alpha A^\mu = & -4\pi \{ 2e^2 \kappa \bar{\Phi} \Phi A^\mu + ie \kappa [-\bar{\Phi} \Phi \gamma^\mu (\partial_\alpha \gamma^\alpha \Phi) + (\partial_\alpha \Phi \gamma^\alpha) \gamma^\mu \Phi] \} - \\ & - 4\pi e \delta(\vec{r} - \vec{r}_0). \end{aligned} \quad (143)$$

on the next we take $\vec{r}_0 = 0$.

The nonrelativistic approximation of this equation, according to (121), is

$$\Delta A^0 - 8\pi e^2 \kappa \bar{\varphi} \varphi A^0 = -8\pi e \kappa^2 \bar{\varphi} \varphi - 4\pi e \delta(r). \quad (144)$$

The equation for φ remains the same, i. e.

$$i\partial_t \varphi = \left(-\frac{1}{2\kappa} \Delta + eA^0 \right) \varphi. \quad (145)$$

Outside of the proton Eq. (144) is

$$\Delta A^0 - 8\pi e^2 \kappa \bar{\varphi} \varphi A^0 = -8\pi e \kappa^2 \bar{\varphi} \varphi. \quad (146)$$

The general solution of this equation is

$$A^0 = A_n^0 + A_p^0, \quad (147)$$

where A_h^0 is the general solution of the equation

$$\Delta A_h^0 - 8\pi e^2 \kappa \bar{\varphi} \varphi A_h^0 = 0 \quad (148)$$

and A_p^0 is the particular solution

$$A_p^0 = \frac{\kappa}{e}. \quad (149)$$

Solution of Eq. (148) one can find applying the same procedure as in the free electron case.

First, we take

$$\varphi = e^{-i\epsilon t} \varphi(\vec{r}). \quad (150)$$

The equation for $\varphi(\vec{r})$ is

$$\epsilon \varphi(\vec{r}) = \left(-\frac{1}{2\kappa} \Delta + eA^0 \right) \varphi(\vec{r}). \quad (151)$$

Writing

$$A^0 = A_{pr}^0 + A_s^0, \quad (152)$$

$$\varphi(\vec{r}) = \varphi^{(0)}(\vec{r}) + \varphi^{(1)}(\vec{r}) + \dots, \quad \epsilon = \epsilon^{(0)} + \epsilon^{(1)} + \dots, \quad (153)$$

$$A^0 = A^{(0)} + A^{(1)} + \dots, \quad (154)$$

where A_{pr}^0 is the potential due to the proton and A_s^0 the potential due to the self-interaction, and connecting $\varphi^{(0)}$ with A_{pr}^0 the zero order equations for φ and A^0 are

$$\epsilon^{(0)} \varphi^{(0)} = \left(-\frac{1}{2\kappa} \Delta + eA_{pr}^0 \right) \varphi^{(0)}, \quad (155)$$

$$\Delta A_h^{(0)} - 8\pi e^2 \kappa \bar{\varphi}^{(0)} \varphi^{(0)} A_h^{(0)} = 0. \quad (156)$$

Eq. (155) is the nonrelativistic stationary state equation of the hydrogen atom for upper components of $\varphi^{(0)}$, since we have selected down two components to be zero in the zero order approximation (elimination of the dual solution), and separates the rest into

$$\varphi^{(0)} = a_1 \begin{pmatrix} \varphi_1^{(0)} \\ 0 \\ 0 \\ 0 \end{pmatrix} + a_2 \begin{pmatrix} 0 \\ \varphi_2^{(0)} \\ 0 \\ 0 \end{pmatrix}, \quad (157)$$

where a_1, a_2 are constants. Solutions of Eq. (155) are the hydrogen stationary state wave functions

$$\varphi_i^{(0)} = \varphi_{intm}(\vec{r}), \quad \varepsilon^{(0)} = \varepsilon_n. \quad (158)$$

After substitution of the solutions (157) into (156) one gets

$$\Delta A_h^{0(0)} - 8\pi e^2 \kappa |\varphi_{intm}|^2 A_h^{0(0)} = 0. \quad (159)$$

The functions φ_{intm} are localized around the proton. Due to this reason we approximate the expression $|\varphi_{intm}|^2$ by the step function:

$$|\varphi_{intm}|^2 = \begin{cases} \frac{1}{2\kappa^2 \Omega_a}, & r \leq a, \quad i = 1, 2. \\ 0 & r > a \end{cases} \quad (160)$$

$$\Omega_a = \frac{4\pi a^3}{3}.$$

(Let us remember that the Ψ -functions are normalized to one, according to (26)). Eq. (159) then becomes

$$\Delta A_h^{0(0)} - 4\pi e^2 \frac{1}{\kappa \Omega_a} A_h^{0(0)} = 0, \quad r \leq a \quad (161)$$

$$A_h^{0(0)} = 0, \quad r > a.$$

We are interested for solutions in the region $r \leq a$. These solutions are

$$A_h^{0(0)} = \frac{1}{r} (C e^{-\delta r} + D e^{\delta r}), \quad (162)$$

where

$$\delta^2 = \frac{4\pi e^2}{\kappa \Omega_a} \quad (163)$$

and C, D are arbitrary constants.

The solution (162) has to be now extended to the proton position. In order to do it we integrate Eq. (144) over a sphere with the proton in the center:

$$\int_{\Omega} \Delta A^0 d^3x - 8\pi e^2 \kappa \int_{\Omega} \bar{\varphi} \varphi A^0 d^3x = 8\pi e \kappa^2 \int_{\Omega} \bar{\varphi} \varphi d^3x - 4\pi e,$$

$$\Omega = \frac{4\pi R^3}{3}, \quad R < a.$$

Using the approximation (160), substituting A^0 from (162), (149), (147) and evaluating integrals we find

$$-(Ce^{-\delta r} + De^{\delta r}) + R \delta (-C e^{-\delta r} + D e^{\delta r}) - e^2 \frac{1}{\kappa \Omega_a} \left\{ \frac{\kappa}{e} + 4\pi C \left[\left(-\frac{R}{\delta} - \frac{1}{\delta^2} \right) e^{\delta r} + \frac{1}{\delta^2} \right] + 4\pi D \left[\left(\frac{R}{\delta} + \frac{1}{\delta^2} \right) e^{\delta r} - \frac{1}{\delta^2} \right] \right\} = 4\pi e \frac{1}{\kappa \Omega_a} - 4\pi e.$$

In the limit $R \rightarrow 0$ it becomes

$$C + D = -e \quad (164)$$

or

$$C = -D - e. \quad (165)$$

The substitution C from (165) into (162) gives

$$A_h^0 = 2D \frac{\text{sh } \delta r}{r} - \frac{e}{r} e^{-\delta r} \quad (166)$$

and

$$A^{(0)} = \frac{\kappa}{e} + 2D \frac{\text{sh } \delta r}{r} - \frac{e}{r} e^{-\delta r}. \quad (167)$$

The constant D we determine from the condition (like in the free electron case $\zeta = 0$, for the sake of simplicity)

$$\left(\frac{\kappa}{e} + 2D \frac{\text{sh } \delta r}{r} \right)_{r \rightarrow 0} \rightarrow 0. \quad (168)$$

From here we find

$$2D = -\frac{\kappa}{e \delta} \quad (169)$$

and finally

$$A^{(0)} = \frac{\kappa}{e} \left(1 - \frac{\text{sh } \delta r}{\delta r} \right) - \frac{e}{r} e^{-\delta r}, \quad r < a. \quad (170)$$

The parameter δ is less than 10^6 cm^{-1} for $a = 10^{-8} \text{ cm}$. For the largest value of $r = a$, we have

$$\delta a < 10^{-2}. \quad (171)$$

Therefore, we may expand $A^{(0)}$ in powers of δr and keep the first order terms:

$$A^{(0)} = -\frac{e}{r} + e\delta - \frac{e\delta^2}{2}r + \dots, \quad r \leq a. \quad (172)$$

The absolute value of the ratio of the first subsequent terms is

$$\left| \frac{e\delta}{\frac{e}{r}} \right| = \delta r \leq \delta a < 10^{-2}. \quad (173)$$

Similarly, the absolute value of the ratio of the first and third terms is

$$\left| \frac{\frac{e\delta^2}{2}r}{\frac{e}{r}} \right| = \frac{\delta^2}{2}r^2 \leq \frac{\delta^2 a^2}{2} < 10^{-4}. \quad (174)$$

The second term in (172) causes small shifts in the energy spectrum. Consequently, we may start keeping only the first term in (172). With this approximation we have

$$A^{(0)} = -\frac{e}{r} \equiv A_{pr}^0. \quad (175)$$

From here the zero order approximation is

$$\begin{aligned} \varphi^{(0)} &= \varphi_{nlm}, & \varepsilon^{(0)} &= \varepsilon_n, \\ A^{(0)} &= A_{pr}^0 \end{aligned} \quad (176)$$

and it is a good starting approximation. This explains the role of the self-interaction in the nonrelativistic limit. However, as one sees, this interaction can not be ignored. But, as in the free electron case in the correct (new) Dirac field theory no infinities appear.

Electron-proton system

Now, we consider the self-interaction of the electron-proton system interacting with the electromagnetic field. The nonrelativistic field equations are

$$i\partial_t \varphi_e = \left(-\frac{1}{2\kappa_e} \Delta + eA^0 \right) \varphi_e, \quad (177)$$

$$i\partial_t \varphi_p = \left(-\frac{1}{2\kappa_p} \Delta - eA^0 \right) \varphi_p, \quad (178)$$

$$\Delta A^0 - 8\pi e^2 (\kappa_e \bar{\varphi}_e \varphi_e + \kappa_p \bar{\varphi}_p \varphi_p) A^0 = 8\pi e (-\kappa_e^2 \bar{\varphi}_e \varphi_e + \kappa_p^2 \bar{\varphi}_p \varphi_p). \quad (179)$$

Taking

$$\varphi_e = e^{-ie_0 t} \varphi_e(\vec{r}), \quad (180)$$

$$\varphi_p = e^{-ie_0 t} \varphi_p(\vec{r}),$$

selecting down two components of φ_e and φ_p to be zero and separating the rest according to

$$\varphi = a_1 \begin{pmatrix} \varphi_1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + a_2 \begin{pmatrix} 0 \\ \varphi_2 \\ 0 \\ 0 \end{pmatrix} \equiv a_1 \varphi_1 + a_2 \varphi_2,$$

these equations for a given solution of φ_e and φ_p become

$$\varepsilon_e \varphi_{ei}(\vec{r}) = \left[-\frac{1}{2\kappa_e} \Delta + eA_e^0 + (eA^0 - eA_e^0) \right] \varphi_{ei}(\vec{r}), \quad (181)$$

$$\varepsilon_p \varphi_{pj}(\vec{r}) = \left[-\frac{1}{2\kappa_p} \Delta - eA_p^0 - (eA^0 - eA_p^0) \right] \varphi_{pj}(\vec{r}), \quad i, j = 1, 2 \quad (182)$$

$$\Delta A^0 - 8\pi e^2 [\kappa_e \bar{\varphi}_{ei}(\vec{r}) \varphi_{ei}(\vec{r}) + \kappa_p \bar{\varphi}_{pj}(\vec{r}) \varphi_{pj}(\vec{r})] A^0 = 8\pi e [-\kappa_e^2 \bar{\varphi}_{ei}(\vec{r}) \varphi_{ei}(\vec{r}) + \kappa_p^2 \bar{\varphi}_{pj}(\vec{r}) \varphi_{pj}(\vec{r})], \quad i, j = 1, 2. \quad (183)$$

In order to find solutions of these equations we apply again the perturbation method:

$$\bar{\varphi}_{ei}(\vec{r}) = \varphi_{ei}^{(0)}(\vec{r}) + \varphi_{ei}^{(1)}(\vec{r}) + \dots, \quad \varepsilon_e = \varepsilon_e^{(0)} + \varepsilon_e^{(1)} + \dots, \quad (184)$$

$$\varphi_{pj}(\vec{r}) = \varphi_{pj}^{(0)}(\vec{r}) + \varphi_{pj}^{(1)}(\vec{r}) + \dots, \quad \varepsilon_p = \varepsilon_p^{(0)} + \varepsilon_p^{(1)} + \dots,$$

with the selections

$$\varepsilon_e^{(0)} \varphi_{ei}^{(0)}(\vec{r}) = \left(-\frac{1}{2\kappa_e} \Delta + eA_e^0 \right) \varphi_{ei}^{(0)}(\vec{r}), \quad (185)$$

$$\begin{aligned} \varepsilon_e^{(1)} \varphi_{ei}^{(0)}(\vec{r}) + \varepsilon_{ei}^{(0)} \varphi_{ei}^{(1)}(\vec{r}) - &= \frac{1}{2\kappa_e} \Delta \varphi_{ei}^{(1)}(\vec{r}) + (eA^0 - eA_e^0) \varphi_{ei}^{(0)}(\vec{r}), \\ \vdots \\ \varepsilon_p^{(0)} \varphi_{pj}^{(0)}(\vec{r}) &= \left(-\frac{1}{2\kappa_p} \Delta + eA_p^0 \right) \varphi_{pj}^{(0)}(\vec{r}), \end{aligned} \tag{186}$$

$$\varepsilon_p^{(1)} \varphi_{pj}^{(0)}(\vec{r}) + \varepsilon_{pj}^{(0)} \varphi_{pj}^{(1)}(\vec{r}) - = \frac{1}{2\kappa_p} \Delta \varphi_{pj}^{(1)}(\vec{r}) + (eA_e^0 - eA_p^0) \varphi_{pj}^{(0)}(\vec{r}),$$

and \vdots

$$eA_e^0 = -2e^2 \kappa_e^2 \int \frac{|\varphi_{pj}^{(0)}(\vec{r}')|}{|\vec{r} - \vec{r}'|} d^3x', \tag{187}$$

$$eA_p^0 = -2e^2 \kappa_p^2 \int \frac{|\varphi_{ei}^{(0)}(\vec{r}')|}{|\vec{r} - \vec{r}'|} d^3x'. \tag{188}$$

After substitution of (184—188) into (183) one gets the equation for A^0 .

The functions $\varphi_{ei}(\vec{r})$ and $\varphi_{pj}(\vec{r})$ are space localized. Let us denote the effective volume of the functions $\varphi_{ei}^{(0)}(\vec{r})$ by $\Omega_e = 4\pi R_e^3/3$ and of the functions $\varphi_{pj}^{(0)}(\vec{r})$ by $\Omega_p = 4\pi R_p^3/3$ with mutual center. Due to $\kappa_p \gg \kappa_e$, we have

$$\Omega_e \gg \Omega_p. \tag{189}$$

Further analysis we simplify as in the previous case:

$$|\varphi_{ei}^{(0)}(\vec{r})|^2 = \begin{cases} \frac{1}{2\kappa_e^2} \frac{1}{\Omega_e}, & r \leq R_e, \\ 0, & r > R_e, \end{cases} \tag{190}$$

$$|\varphi_{pj}^{(0)}(\vec{r})|^2 = \begin{cases} \frac{1}{2\kappa_p^2} \frac{1}{\Omega_p}, & r \leq R_p, \\ 0, & r > R_p. \end{cases} \tag{191}$$

a) *The potential of the electron motion* ($R_p \leq r < R_e$)

In the region of the effective electron motion the equation for $A^{0(0)}$ reads

$$\Delta A^{0(0)} - 4\pi e^2 \frac{1}{\kappa_e \Omega_e} A^{0(0)} = 4\pi e \frac{1}{\Omega_e}. \tag{192}$$

The general solution of this equation is

$$A^{0(0)} = A_h^{0(0)} + A_p^{0(0)}, \tag{193}$$

where $A_h^{0(0)}$ satisfies equation

$$\Delta A_h^{0(0)} - 4\pi e^2 \frac{1}{\kappa_e \Omega_e} A_h^{0(0)} = 0 \tag{194}$$

and

$$A_p^{0(0)} = \frac{\kappa_e}{e}. \tag{195}$$

From here, we have

$$A^{0(0)} = \frac{1}{r} (C e^{-\delta r} + D e^{\delta r}) + \frac{\kappa_e}{e}, \tag{196}$$

$$\delta^2 = 4\pi e^2 \frac{1}{\kappa_e \Omega_e}. \tag{197}$$

C and D are constants and they are determined by the extension of the solution (196) into the region $0 < r < R_p$.

Performing the integration of Eq. (179) over the volume Ω_p one gets

$$\begin{aligned} & 4\pi [-(C e^{-\delta R_p} + D e^{\delta R_p}) + R_p \delta (-C e^{-\delta R_p} + D e^{\delta R_p})] - 4\pi e^2 \left(\frac{\Omega_p}{\Omega_e} + \frac{\kappa_p}{\kappa_e} \right) - \\ & - 4\pi \left(1 + \frac{\kappa_e}{\kappa_p} \frac{\Omega_e}{\Omega_p} \right) \{ [(-R_p \delta - 1) e^{-\delta R_p} + 1] C + [(R_p \delta + 1) e^{\delta R_p} - 1] D \} = \\ & = 4\pi e \left(\frac{\Omega_p}{\Omega_e} - 1 \right). \end{aligned} \tag{198}$$

Due to $\delta R_p \ll 1$, $\Omega_e \gg \Omega_p$, $\kappa_e \gg \kappa_p$ this equation can be approximated by

$$-(C + D) = -e. \tag{199}$$

From here we have

$$C = -D + e. \tag{200}$$

After substitution C from (200) into (196) follows

$$A^{(0)} = 2D \frac{\text{sh } \delta r}{r} + \frac{\kappa_e}{e} - \frac{e}{r} e^{-\delta r}. \tag{201}$$

The condition

$$\left(2D \frac{\text{sh } \delta r}{r} + \frac{\kappa_e}{e}\right)_{r \rightarrow 0} = 0 \quad (202)$$

gives

$$2D = -\frac{\kappa_e}{\delta e} \quad (203)$$

and

$$A^{0(0)} = \frac{\kappa_e}{e} \left(1 - \frac{\text{sh } \delta r}{\delta r}\right) - \frac{e}{r} e^{-\delta r}, \quad R_p < r < R_e. \quad (204)$$

This expression is analogous to (170). Consequently, expanding $A^{0(0)}$ in powers of δr we may keep only the first term:

$$A^{0(0)} = -\frac{e}{r}. \quad (205)$$

The potential (187) in the region $R_p < r$ with the approximation (191) is

$$A^{0(0)} = -\frac{e}{\Omega_p \Omega_p} \int \frac{1}{|\vec{r}' - \vec{r}|} d^3 x' = -\frac{e}{r}. \quad (206)$$

From here we have the same conclusion as in the previous case.

b) *The potential of the proton motion* ($0 < r < R_p$)

In the region of the effective proton motion the equation for $A^{0(0)}$ reads

$$\Delta A^{0(0)} - 4\pi e^2 \left(\frac{1}{\kappa_e \Omega_e} + \frac{1}{\kappa_p \Omega_p}\right) A^{0(0)} = 4\pi e \left(-\frac{1}{\Omega_e} + \frac{1}{\Omega_p}\right). \quad (207)$$

The electron motion in this region is of no interest. Thus, we look for the potential of the proton motion only. Due to this reason we solve first the equation of the proton self-interaction:

$$\Delta A_s^{0(0)} - 4\pi e^2 \frac{1}{\kappa_p \Omega_p} A_s^{0(0)} = 4\pi e \frac{1}{\Omega_p}. \quad (208)$$

In accordance to (121—135) we have

$$A_s^{0(0)} = \frac{1}{r} (C e^{-\delta r} + D e^{\delta r}) - \frac{\kappa_p}{e}, \quad (209)$$

$$\delta^2 = \frac{4\pi e^2}{\kappa_p \Omega_p}$$

and from the condition

$$A_s^{0(0)}(0) = 0 \quad (210)$$

$$A_s^{0(0)} = \frac{\kappa_p}{e} \left(\frac{\text{sh } \delta r}{\delta r} - 1 \right). \quad (211)$$

Now, we look for a particular solution of Eq. (207) with the electron source term $(-4\pi e/\Omega_e)$. Writing

$$A^{0(0)} = A_s^{0(0)} + A_{pr}^{0(0)} \quad (212)$$

the equation for $A^{0(0)}$ is

$$\Delta A_{pr}^{0(0)} - 4\pi e^2 \frac{1}{\kappa_p \Omega_p} A_{pr}^{0(0)} = -4\pi e \frac{1}{\Omega_e} \quad (213)$$

where we have neglected $1/\kappa_e \Omega_e$ in comparison to $1/\kappa_p \Omega_p$.

Comparing this equation with the following equation from the classical electrodynamics

$$\Delta \Phi(\vec{r}) + \frac{\omega^2}{c^2} \Phi(\vec{r}) = -4\pi \rho(\vec{r}) \quad (214)$$

and its solution

$$\Phi(\vec{r}) = \int \frac{\rho(\vec{r}') e^{i\frac{\omega}{c}|\vec{r}'-\vec{r}|}}{|\vec{r}'-\vec{r}|} d^3x' \quad (215)$$

we find the particular solution of Eq. (213) in the form

$$A_{pr}^{0(0)} = - \int \frac{4\pi e}{\Omega_e} \frac{e^{-\delta|\vec{r}'-\vec{r}|}}{|\vec{r}'-\vec{r}|} d^3x'. \quad (216)$$

(The procedure which is applied here may be also applied in the previous cases.)

The parameter δ is less of the order 10^8 cm^{-1} . Thus, δr is less of 10^{-3} . Consequently we may expand $A^{0(0)}$ in powers of δr and start only with the first term. Then we get

$$A^{0(0)} = - \int \frac{4\pi e}{\Omega_e} \frac{1}{|\vec{r}'-\vec{r}|} d^3x'. \quad (217)$$

Again we get the same result concerning the self-interaction.

Finally, we come to the conclusion that the self-interaction does not lead to infinities and that it can be neglected in a good starting point but not ignored.

In this analysis of the self-interaction we have used the Lagrange's equations for the fields. Naturally, we can use also the canonical equations. The advantage of the Lagrange's equations is in the separation of the field variables, what is evident from Eq. (67—71). (The Ψ -functions contain both the Dirac and the electromagnetic field variables.)

6. Conclusions

Neglected the self-interaction Eqs. (109—110) become

$$i\partial_t \varphi_e = \left(-\frac{1}{2\kappa_e} \Delta - 2e^2 \kappa_p \int \frac{|\varphi_p(\vec{r}')|^2}{|\vec{r}' - \vec{r}|} d^3x' \right) \varphi_e, \quad (218)$$

$$i\partial_t \varphi_p = \left(-\frac{1}{2\kappa_p} \Delta - 2e^2 \kappa_e \int \frac{|\varphi_e(\vec{r}')|^2}{|\vec{r}' - \vec{r}|} d^3x' \right) \varphi_p. \quad (219)$$

The stationary solutions of these equations are

$$\varphi_e(\vec{r}, t) = e^{-i\epsilon_e t} \varphi_e(\vec{r}), \quad (220)$$

$$\varphi_p(\vec{r}, t) = e^{-i\epsilon_p t} \varphi_p(\vec{r}),$$

where $\varphi_e(\vec{r})$ and $\varphi_p(\vec{r})$ satisfy the equations

$$\epsilon_e \varphi_e(\vec{r}) = \left(-\frac{1}{2\kappa_e} \Delta - 2e^2 \kappa_p^2 \int \frac{|\varphi_p(\vec{r}')|^2}{|\vec{r}' - \vec{r}|} d^3x' \right) \varphi_e(\vec{r}), \quad (221)$$

$$\epsilon_p \varphi_p(\vec{r}) = \left(-\frac{1}{2\kappa_p} \Delta - 2e^2 \kappa_e^2 \int \frac{|\varphi_e(\vec{r}')|^2}{|\vec{r}' - \vec{r}|} d^3x' \right) \varphi_p(\vec{r}) \quad (222)$$

or in terms of Ψ -functions

$$\epsilon_e \Psi_e(\vec{r}) = \left(-\frac{1}{2\kappa_e} \Delta - e^2 \int \frac{|\Psi_p(\vec{r}')|^2}{|\vec{r}' - \vec{r}|} d^3x' \right) \Psi_e(\vec{r}), \quad (223)$$

$$\epsilon_p \Psi_p(\vec{r}) = \left(-\frac{1}{2\kappa_p} \Delta - e^2 \int \frac{|\Psi_e(\vec{r}')|^2}{|\vec{r}' - \vec{r}|} d^3x' \right) \Psi_p(\vec{r}). \quad (224)$$

Writing these equations in the form

$$\varepsilon_e \Psi_e(\vec{r}_e) = \left(-\frac{1}{2\mathcal{N}_e} \Delta_e - e^2 \int \frac{|\Psi_p(\vec{r}_p)|^2}{|\vec{r}_p - \vec{r}_e|} d^3x_p \right) \Psi_e(\vec{r}_e), \quad (225)$$

$$\varepsilon_p \Psi_p(\vec{r}_p) = \left(-\frac{1}{2\mathcal{N}_p} \Delta_p - e^2 \int \frac{|\Psi_e(\vec{r}_e)|^2}{|\vec{r}_e - \vec{r}_p|} d^3x_e \right) \Psi_p(\vec{r}_p), \quad (226)$$

we see that they are the Hartree-Fock approximation of the nonrelativistic quantum mechanical equation for the stationary states of the hydrogen atom. The total energy is also of the Hartree-Fock type, according to

$$P_0 = P_e^0 + P_p^0 + P_0^{em} \quad (227)$$

and (57) for Dirac's fields, within given approximations.

Therefore, we conclude that the superposition of the actions of the electron-positron and the proton-antiproton fields interacting with the electromagnetic field leads in the nonrelativistic limit to the Hartree-Fock approximation of the hydrogen atom of the conventional nonrelativistic quantum mechanics.

The self-interaction does not contain infinities and in a good approximation can be neglected, but not ignored. Some profound questions are still present in the self-interaction and they require further analysis.

In the conventional theory the proton motion of the hydrogen atom was considered on the basis of semi-relativistic two-body quantum equation^{3,4,5}) and fully relativistic two-body quantum equation^{6,7,8}). The later one was based on the Feynman⁹) formalism in the quantum field theory. In both cases the equations are constructed for the relative motion. The proton was assumed to be a point particle of Fermi-Dirac type without internal structure (the internal structure was neglected; anomalous magnetic moment, mesonic charge cloud, etc.). This assumption we have also taken in our work. The substantial difference between the conventional and our theory one may state in two points: (1) the procedure of the conventional theory is not canonically correct and (2) the interaction in the conventional theory is introduced according to the quantum electrodynamics what involves the Dirac's field operators for particle-antiparticle pairs and it is in contrast to our statistical interpretation of the Dirac's field. This difference, as well as concentration on the relative electron-proton motion, makes unnecessary further comparison of the conventional and our theory.

References

- 1) J. Brana and K. Ljolje, *Fizika* **12** (1980) 287;
- 2) J. Brana and K. Ljolje, *Fizika* **13** (1981) 265;
- 3) G. Breit, *Phys. Rev.* **34** (1929) 553;
- 4) G. Breit and G. E. Brown, *Phys. Rev.* **74** (1948) 1278;
- 5) G. Breit, G. E. Brown and Arfken, *Phys. Rev.* **76** (1949) 1299;
- 6) E. E. Salpeter and H. A. Bethe, *Phys. Rev.* **84** (1951) 1232;
- 7) M. Gell-Mann and F. Low, *Phys. Rev.* **84** (1951) 350;
- 8) E. E. Salpeter, *Phys. Rev.* **87** (1952) 328;
- 9) R. P. Feynman, *Phys. Rev.* **76** (1949) 749;
- 10) R. P. Feynman and M. Gell-Mann, *Phys. Rev.* **109** (1958) 193.

KRETANJE PROTONA U VODIKOVOM ATOMU U NOVOJ TEORIJI DIRACOVOG POLJA

K. LJOLJE and S. VOBORNIK

*Prirodno-matematički fakultet, Odsjek za fiziku,
Univerzitet u Sarajevu, 71000 Sarajevo*

UDK 530.19

Originalni znanstveni rad

Razmatrano je kretanje protona u vodikovom atomu u novoj teoriji Diracovog polja. Proton-antiproton polje u međudjelovanju sa elektromagnetskim poljem pretpostavljeno je da je opisano analogno elektronsko-pozitronskom polju. Elektromagnetsko međudjelovanje ova dva polja uzeto je da se ostvaruje preko elektromagnetskog polja koje je ugrađeno u svako od ovih polja. Nađeno je da ovakav oblik međudjelovanja vodi na Hartree-Fockovu aproksimaciju vodikovog atoma nerelativističke kvantne mehanike. Razmatrana su i osnovna svojstva samomeđudjelovanja u novoj teoriji. Utvrđeno je da ovo međudjelovanje ne dovodi do beskonačnosti i da se može u polaznoj aproksimaciji zanemariti, ali ne i ignorirati. Neka suštinska pitanja su prisutna i ona zahtijevaju dalju analizu. Provedeno razmatranje ukazuje i na put uvođenja općenitijeg međudjelovanja Diracovih polja, kao i na porijeklo nekih fundamentalnih pojmova konvencionalne teorije.