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AN ANALYSIS OF THE OZI-ALLOWED DECAYS $\Upsilon(10.54)$ BASED ON THE PSEUDO-DIMENSION RULE

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This paper investigates the *OZI*-allowed decays of the vector bottomium $\Upsilon(10.54)$. For this resonance the *OZI*-favoured decay modes include $B\bar{B}$ and $B\bar{B}^*$ provided the $B\bar{B}^*$ mode is also energetically allowed. In this paper the pseudo-dimension rule has been employed in the analysis of the decays $\Upsilon(10.54) \to B\bar{B}, B\bar{B}^*$. This analysis reveals that the decay $\Upsilon(10.54) \to B\bar{B}$ is allowed by the pseudo-dimension rule according to which, however, the decay $\Upsilon(10.54) \to B\bar{B}^*$ is forbidden. This fact implies that the decay $\Upsilon(10.54) \to B\bar{B}^*$ should be relatively suppressed with respect to the decay $\Upsilon(10.54) \to B\bar{B}$. To be more specific, the pseudo-dimension rule claims that the former decay must be inhibited compared to the latter decay by a factor 10^{-3}. This explains the experimentally observed strong suppression of the *OZI*-allowed decay $\Upsilon(10.54) \to B\bar{B}^*$. It has also been pointed out that the phase space factor alone cannot account for the strong suppression of the $B\bar{B}^*$ mode in $\Upsilon(10.54)$-decay.

1. Introduction

Of all the upsilons so far observed the recently detected[1] $\Upsilon(10.54)$ is the only state which occurs above the $b\bar{b}$-threshold and as such it admits the interpretation of an *unbound* resonance. For reasons obvious one expects the *OZI*-allowed decay $\Upsilon(10.54) \to B\bar{B}$ which is also kinematically favoured, B being

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the pseudo-scalar bottom meson. For this vector bottomium the OZI-allowed decays $\Upsilon(10.54) \rightarrow B\bar{B}^*$, $B^*\bar{B}^*$ are also theoretically feasible. These decays, however, have not been seen[1]. This fact may be interpreted in two alternative ways. Either these decays are not energetically allowed although they are OZI-favoured or they are energetically allowed but are strongly suppressed under the influence of a selection rule. As our present status of knowledge regarding the masses of the bottom (or beauty) mesons B and B^* is not precise[1], therefore, it is not reasonable to rule out the decay $\Upsilon(10.54) \rightarrow B\bar{B}^*$ in particular on energy considerations. This is because the conservative lower limit[1] for the mass of the B-meson is 5.18 GeV. Further, $[m(B^*) - m(B)] < 0.11$ GeV. This point becomes transparent if we recall that $[m(D^*) - m(D)] = 0.142$ GeV where D and D^* are the pseudo-scalar and vector charmed meson, respectively. It is interesting to note that the mass differences between the HFS states like (D, D^*), (F, F^*) and (B, B^*) become smaller for the more massive states, F and F^* being the pseudo-scalar and vector strange-charmed meson respectively. Therefore, $[m(B^*) - m(B)] < < [m(F^*) - m(F)] = 0.11$ GeV. This in turn suggests that the mass of B^* should be around 5.29 GeV. Clearly, then, a conservative estimate indicates that $[m(B) + m(B^*)] < 10.54$ GeV. Obviously, the $B\bar{B}^*$ mode is quite likely to be energetically not disallowed. This is precisely the reason for which this mode was experimentally searched[1] but not seen[1]. This fact implies that the decay $\Upsilon(10.54) \rightarrow B\bar{B}^*$ is strongly suppressed relative to the decay $\Upsilon(10.54) \rightarrow B\bar{B}$ which has been observed[1]. Such a strong suppression of the former decay cannot be accounted for by arguing that this decay has a smaller phase space compared to that available for the latter decay. In this connection it is important to remember that the phase space factor does not play the decisive role in the matter of relative suppressions of the decay modes of particles in general and, in particular, of quarkonia. This point becomes immediately transparent if we recall that the decay $\Psi(4.03) \rightarrow D\bar{D}$ is strongly suppressed[2] with respect to the decay $\Psi(4.03) \rightarrow \rightarrow D^*\bar{D}^*$ in spite of the fact that the former decay has a larger phase space. Our intention in this paper is to show that the strong inhibition of the OZI-allowed decay $\Upsilon(10.54) \rightarrow B\bar{B}^*$ follows as a necessary consequence of the pseudo-dimension rule[3-10] which forbids the decay concerned. This selection rule has already placed a very impressive role in the decays of particles[3-10] including quarkonia.

The plan of this paper is as follows. In Sec. 2 the pseudo-dimensions of fields as well as the pseudo-dimensionality-based selection rule (abbreviated as the pseudo-dimension rule) have been discussed. This rule has been employed in Sec. 3 in $\Upsilon(10.54)$-decay. In this section it has been shown that the decays $\Upsilon(10.54) \rightarrow \rightarrow B\bar{B}$ and $\Upsilon(10.54) \rightarrow B\bar{B}^*$ enjoy the same OZI-status but they are placed on different footings from the point of view of the pseudo-dimension rule. This is because the former decay is allowed by this rule according to which the latter decay is forbidden. This point has been exploited to explain the strong suppression of the $B\bar{B}^*$ mode in $\Upsilon(10.54)$-decay.

2. *Pseudo-dimension rule*

The pseudo-dimensions of fields are defined in such a way that they can mimick some of the well known properties of their canonical dimensions[11]. Therefore, it may not be out of place if we take note of the salient features of the canonical dimensions of fields. For this purpose, we first confine our attention to the spin-half and spin-zero fields. The canonical dimensions of these fields become evident if we consider the free-particle Lagrangians indicated below.

$$L = \int (\overline{\Psi} \, i \, \gamma^{\mu} \, \partial_{\mu} \Psi - m \, \overline{\Psi} \, \Psi) \, d^4 x \tag{1}$$

and

$$L = \int (\partial^{\mu} \, \overline{\Phi} \, \partial_{\mu} \, \Phi - m^2 \, \overline{\Phi} \, \Phi) \, d^4 x \tag{2}$$

where Ψ and Φ denote, respectively, the spinor and scalar fields. For reasons obvious L should be independent of x. However, the right hand sides of relations (1) and (2) involve the 4-dimensional volume element d^4x having the dimension $+4$. Therefore, from dimensional considerations it follows that each and every term occurring in the integrands appearing in relations (1) and (2) must have the dimension -4. This is achieved by assigning the dimensions $-\frac{3}{2}$ and -1, respectively, to $\overline{\Psi}$ and $\overline{\Phi}$. A similar remark is also true for Ψ and Φ. These statements can be easily checked by noting that relations (1) and (2) have been expressed in natural units ($\hbar = c = 1$) and as such the dimension of m, appearing in these equations, turns out to be -1. At this point it is worth mentioning that the dimensions of free fields are generally termed canonical dimensions and those refering to interacting fields are called *anomalous* dimensions[11, 12].

In the above we have outlined the procedure for assigning canonical dimensions to spin-half and spin-zero fields. Following the same procedure one can easily check that canonical dimensions of *all* fermion fields are $-\frac{3}{2}$ and the same for boson fields (including the photon in the 4-vector potential description) are -1. This implies that canonical dimensions of fields are related to the statistics obeyed by them and, needless to mention, they are spin-independent quantities. On the other hand, pseudodimensions of fields are defined in such a way that they can do the book-keeping of both the statistics and the spins of the fields concerned. The pesudo-dimension, to be denoted by d, of a free field carrying a non-zero actual spin J is defined[3] by

$$d = -KJ, \qquad J \neq 0 \tag{3}$$

where K is a non-zero positive odd integer. The odd integral value of K implies that the pseudo-dimensions of the fermion and boson fields possess the odd-half integral and integral values, respectively, like their canonical dimensions. This in turn means that the pesudo-dimensions, like the canonical dimensions, are related to the statistics apart from the fact that they are spin-dependent as evident from relation (3). The negative sign in relation (3) is simply due to convention usually followed in connection with the dimensions of fields.

A correlation between the pseudo-dimensions and canonical dimensions is established by requiring that the former reduce to the latter for the fields having the lowest possible spins $\left(J = \frac{1}{2} \text{ for fermion and } J = 0 \text{ for boson fields}\right)$. This requirement implies that

$$d = -1 \text{ for fields having } J = 0 \tag{4}$$

as the canonical dimensions of boson fields (irrespective of their spins) are -1. Also, the above mentioned requirement applied to fermion fields having $J = \frac{1}{2}$ suggests that

$$d = -\frac{3}{2} \quad \text{for} \quad J = \frac{1}{2} \tag{5a}$$

since the canonical dimensions of fermion fields (irrespective of their spins) are $-\frac{3}{2}$. Further, it follows from relation (3) that

$$d = -\frac{K}{2} \quad \text{for} \quad J = \frac{1}{2}. \tag{5b}$$

A comparison between relation (5a) and relation (5b) indicates that $K = 3$ which enables us to rewrite relation (3) in the following form

$$d = -3J \quad \text{for fields having } J \neq 0. \tag{6}$$

Obviously, relation (6) assigns the value $d = -3$ to all vector bosons including the photon. Stated differently, relation (6) places the photon on the same footing with the other massive vector bosons. However, the photon is a class by itself and its special status should be recognized. This can be done so by assigning a value different from -3 to the pseudo-dimension appropriate for it. In order to do this we can exploit the accidental coincidence between the *numerical* value (which is 3) of the pseudo-dimensions for the vector fields and the same for the number of states of polarization (which is also 3) for the *massive* vector fields. If we demand that for the photon also the equality between the numerical value of its pseudo-dimension and its number of states of polarization must hold true, then, we can write

$$d = -2 \quad \text{for the photon} \tag{7}$$

as the photon has two states of polarization. It may be emphasized that the arguments for arriving at relation (7) cannot be extended to non-vector fields. Hereafter we will be concerned with the magnitudes of the pseudo-dimensions of fields. Therefore, we give below the magnitudes of the pseudo-dimensions of fields for our convenience of further discussions.

$$d\,(\text{magnitude}) = 3J \text{ for fields having } J \neq 0 \tag{8a}$$

$$d\,(\text{magnitude}) = 1 \quad \text{for fields having } J = 0 \tag{8b}$$

$$d\,(\text{magnitude}) = 2 \quad \text{for the photon.} \tag{8c}$$

In passing we may also note that relations (8a)—(8c) are strictly valid for free fields as we have demanded that the pseudo-dimensions for the fields having the lowest possible spins must reduce to their respective canonical dimensions. We have noted earlier that canonical dimensions refer to free field only.

In order to formulate the pseudo-dimensionality-based selection rule (to be abbreviated as the pseudo-dimension rule) for particle decays, we consider the decay $A \to B + C + \ldots$. We denote by d_u the magnitude of the pseudo-dimension of the unstable (i. e. the decaying) field A. Further, we denote by D the sum of the magnitudes of the pseudo-dimensions of the fields $B, C, \ldots$ constituting a decay mode. By definition, therefore, $D = d_B + d_C + \ldots$. Obviously, for a particular decaying field d_u is fixed whereas D can take, in general, a finite spectrum of discrete values corresponding to the finite number of theoretically allowed decay modes of the decaying field concerned. To proceed further we may note that in any decay process the spin in the initial state is not, in general, equal to the same in the final state (as the total angular momentum is conserved always and not the spin). This, along with the fact that the quantities d_u and D (defined above) are spin-dependent scalar quantities, implies that both of the following relations

$$d_u > D \tag{9a}$$

and

$$d_u < D \tag{9b}$$

can hold true, in principle at least, if there exists a perfect democracy amongs the theoretically allowed decay channels for a given unstable particle. The pseudo-dimension rule for particle decays is finally obtained by sacrifying a perfect democracy and, instead, introducing a favoured class amongst the theoretically allowed decay channels of a decaying particle. This is reflected in the statement of this rule which reads: All the allowed decays of an unstable particle must be governed by *one and only one of* the two constraints

$$d_u > D \tag{9a}$$

and

$$d_u < D. \tag{9b}$$

At this point we may note that the pseudo-dimension rule does not specify which one of the two constraints, given by relations (9a) and (9b), will be operative in the allowed decays of a given particle. The application of this rule, however, in the decays of a particle necessitates the knowledge of the constraint appropriate for the decaying particle concerned. The specification of the constraints effective in the decays of vector quarkonia (with which we will be concerned in this note) is particularly simple and can be easily done with the help of the following considerations.

(i) The allowed decays of *all* the vector quarkonia, which admit the interpretation of *bound* resonances, are described by the constraint $d_u < D$.

(ii) The allowed decays of vector quarkonia, which admit the interpretation of *unbound* resonances, are described by the constraint as specified below

$$d_u > D \quad \text{for } n = 1, 3, 5, \ldots$$

$$d_u < D \quad \text{for } n = 2, 4, 6, \ldots$$

where $n = 1$ represents the first *unbound* resonance, $n = 2$ the second *unbound* resonance, and so on. The validity of the above statements can be easily checked by considering the observed dominant decays of the vector quarkonia. This point has been exhaustively discussed in our previous papers[4-10] on the decays of vector quarkonia.

We now want to obtain an expression for the suppression factor appropriate for a decay forbidden by the pseudo-dimension rule. Obviously, a forbidden mode will be relatively suppressed with respect to an allowed one. The relative suppression factor Λ for a forbidden mode may be formally expressed as

$$\Lambda = f(d_u, D_a \sim D_f) \tag{10}$$

where D_a and D_f refer to an allowed and a forbidden mode, respectively, of a given decaying particle described by d_u. At this point it is worth noting that expression for Λ, given by relation (10), involves $(D_a \sim D_f)$ instead of D_f. This is because, as mentioned above, Λ describes the relative suppression of a forbidden mode compared to an allowed mode. To proceed further we may recall that our previous investigations[3-10] reveal that the pseudo-dimension rule inhibits the forbidden decays very severely. This fact is ensured by the following form of the function $f(d_u, D_a \sim D_f)$.

$$f \sim \exp\left[-\lambda\, d_u \{|D_a - D_f)|\}\right] \tag{11}$$

where λ is an adjustable parameter. With the help of relation (11), relation (10) can be recast as

$$\Lambda = N \exp\left[-\lambda\, d_u \{|(D_a - D_f)|\}\right] \tag{12}$$

where the constant N takes the interpretation of the normalization factor as relation (12) describes the relative suppression of a forbidden decay with respect to an allowed one of a given decaying particle. If, further, the two decay channels concerned behave identically under the action of all other selection rules (except the pseudo-dimension rule) appropriate for the decaying particle, then, the normalization factor N can be set equal to unity. For such decays, relation (12) reduces to

$$\Lambda = \exp\left[-\lambda\, d_u \{|(D_a - D_f)|\}\right]. \tag{13}$$

Finally, the numerical value of the parameter λ is found to be 0.89 with the help of the existing data[2,13] on particle decays. Therefore, relation (13) takes the following form

$$\Lambda = \exp\left[-0.89 d_u \{|(D_a - D_f)|\}\right]. \tag{14}$$

We repeat to emphasize that relation (14) can only be used if the two decay channels, one of them allowed and the other forbidden by the pseudo-dimension rule, must be identically influenced by other selection rule(s). In clearer words, either both of the decay channels concerned are allowed or both of them forbidden by other selection rule(s). This point is important and it has been used for setting N, occurring in relation (12), equal to unity.

3. Pseudo-dimension rule and Υ *(10.54)-decay*

In this section our intention is to examine the bearing of the pseudo-dimension rule on the *OZI*-allowed decays of Υ (10.54). In Sec. 1 we have argued that for this vector resonance the *OZI*-allowed as well as kinematically favoured decay modes may include BB^* apart from $B\bar{B}$. We now want to see whether both these *OZI*-allowed modes are also allowed by the pseudo-dimension rule. With this end in view we proceed by recalling that Υ (10.54) is the first state which occurs above the $b\bar{b}$-threshold. This in turn means that the vector bottomium concerned takes the interpretation of the first *unbound* vector resonance occurring above the $b\bar{b}$-threshold. This interpretation of Υ (10.54) may be utilized in the specification of the constraint which describes its decays allowed by the pseudo-dimension rule. This is because, as discussed in Sec. 2, the allowed decays of vector quarkonia, which admit the interpretation of *unbound* resonances, are described by the constraints as specified below

$$d_u \geqslant D \text{ for } v = 1, 3, 5, \ldots$$

and

$$d_u \leqslant D \text{ for } n = 2, 4, 6, \ldots$$

where $n = 1$ represents the first *unbound* vector resonance, $n = 2$ the second *unbound* vector resonance, and so on. For the reasons indicated above Υ (10.54) turns out to be the $n = 1$ state. This implies that its decays, allowed by the pseudo-dimension rule, are governed by the constraint $d_u \geqslant D$. This rule, it may be emphasized, demands that the allowed decays of the vector bottomium under considerations is controlled by one and the same constraint, namely, $d_u \geqslant D$. Those decays of this meson which fail to be consistent with this constraint must be treated as the forbidden modes of the rule concerned. In order that both the *OZI*-allowed decays $\Upsilon (10.54) \to B\bar{B}$ and $\Upsilon (10.54) \to B\bar{B}^*$ may also be regarded as the allowed ones from the point of view of the pseudo-dimension rule, they have to be consistent with the constraint $d_u \geqslant D$. To check whether the decays just mentioned are in conformity with this constraint we may proceed as follows. For the decaying particle Υ (10.54), which is a vector meson, we have $d_u = 3$ which follows from relation (8a). Further, it is evident from relations (8b) and (8a) that $d_B = d_{\bar{B}} = 1$ and $d_B{}^* = d_{\bar{B}}{}^* = 3$ since B and B^* are $J = 0$ and $J = 1$ particles, respectively. Therefore, for the $B\bar{B}$ mode we get $D = d_B + d_{\bar{B}} = 1 + 1 = 2$ and for the $B\bar{B}^*$ mode we have $D = d_B + d_{\bar{B}}{}^* = 1 + 3 = 4$. Clearly, then, the decay $\Upsilon (10.54, d_u = 3) \to B\bar{B}$ $(D = 2)$ is consistent with the constraint $d_u \geqslant D$ which, however, is not satisfied for the decay $\Upsilon (10.54, d_u = 3) \to B\bar{B}^*$ $(D = 4)$. This means that the former decay is allowed whereas the latter decay is forbidden by the pseudo-dimension rule. This rule, therefore, claims that the decay $\Upsilon (10.54) \to$ $\to B\bar{B}^*$ must be suppressed relative to the decay $\Upsilon (10.54) \to B\bar{B}$. To be more specific, the forbidden mode $B\bar{B}^*$ must be inhibited by a factor of 5×10^{-3} with respect to the allowed mode $B\bar{B}$. This follows from relation (14) since for the decaying particle Υ (10.54) we have $d_u = 3$ and for the allowed mode $B\bar{B}$, $D_a = 2$ and for the forbidden mode $B\bar{B}^*$, $D_f = 4$. From our above discussions it is clear that the pseudo-dimension rule demands a strong suppression of the $B\bar{B}^*$ mode relative to the $B\bar{B}$ mode of Υ (10.54)-decay. This is excellently corroborated by the experimentally observed facts[1] regarding the modes concerned.

In this section we have tried to emphasize that the strong suppression of the decay $\Upsilon(10.54) \rightarrow B\bar{B}^*$ is mainly caused by the pseudo-dimension rule. However, the phase space factor also contributes to the overall suppression of the decay under considerations. In other words, the phase space factor enhances the suppression of the $B\bar{B}^*$ mode but it is alone not sufficient to account for the observed strong suppression of the $\bar{B}B^*$ mode.

4. *Concluding remarks*

In this paper we have argued that it is not logical to rule out the *OZI*-allowed decay $\Upsilon(10.54) \rightarrow B\bar{B}^*$ on energy considerations particularly because the mass of the B^*-meson has not yet been determined. On the other hand, if we assume that the above mentioned decay is energetically allowed, then, its strong suppression becomes difficult to explain in terms of the phase space factor alone. This suppression, however, receives an elegant explanation in the light of the pseudo-dimension rule which has already played an impressive role in the suppression problems of quarkonium decays.

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ANALIZA OZI-DOZVOLJENIH RASPADA $\Upsilon(10.54)$ NA OSNOVI PSEUDO-DIMENZIONALNOG PRAVILA

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U ovom radu istražuju se *OZI*-dozvoljeni raspadi $\Upsilon(10.54)$. Za tu rezonancu *OZI*-dozvoljeni raspadi su $B\bar{B}$ i $B\bar{B}^*$. U ovom radu korišteno je pseudo-dimenzionalno pravilo za analizu $\Upsilon(10.54) \rightarrow B\bar{B}, B\bar{B}^*$. Pokazuje se da je $\Upsilon(10.54) \rightarrow B\bar{B}^*$ zabranjen. Ta činjenica ukazuje da bi raspad $\Upsilon(10.54) \rightarrow B\bar{B}^*$ trebao biti potisnut obzirom na raspad $\Upsilon(10.54) \rightarrow B\bar{B}$ i to za faktor 10^{-3}. To objašnjava eksperimentalne opaženo potisnuće *OZI*-dozvoljenog raspada $\Upsilon(10.54) \rightarrow B\bar{B}^*$. Također je ukazano da sam fazni faktor ne može objasniti jako potisnuće $B\bar{B}^*$-moda u raspadu $\Upsilon(10.54)$.