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Optimal Rebate Policy in a Green Product Design and Development System with Price-Sensitivity Demand

Yung-Fu Huang(1), **Ming-Wei Weng**(1*)

(1) *Department of Marketing and Logistics Management, Chaoyang University of Technology, Taichung, TAIWAN*
*e-mails: huf@cyut.edu.tw; mwweng@cyut.edu.tw**

SUMMARY

Green product design and development should be an eco-design tool for waste management throughout the design and disposal stage. Return and recycling policy plays a crucial role in a recycling system through government subsidies. To reduce raw material waste and improve energy efficiency within recycling systems, the rebate policy is proving to be a valuable tool for companies striving to achieve ambitious targets in returning used bikes on the streets. This paper contributes to investigating the role of rebate policy on a two-stage integrating optimizing model with multiple components and imperfect processes based on price-sensitivity demand. The paper aims to deepen our understanding of the role of the return policy in a profit-maximizing remanufacturing system. An algorithm analyzes optimal solutions for production and usage rebate policies to maximize the total profit per unit time. Despite inherent limitations, data from the Taiwan bike company are used to illustrate the proposed model and algorithm. Additionally, sensitivity analysis is conducted to derive valuable managerial insights.

KEYWORDS: *rebate policy; recycling system; green product design and development; bike industry; economic production quantity.*

1. INTRODUCTION

Understanding the consumption of limited resources in the Green Product Design and Development (GPDD) system leads to better determination of rebate policy and demand characteristics. The GPDD system has received increasing research interest over the past decade, with a particular focus on Corporate Social Responsibility. However, there are few studies that shed light on the interplay between corporate operations and environmental improvement. Numerous studies [1-2] emphasise that the selection process of lightweight materials has minimal environmental impact throughout its life cycle. Interest in the environment has increased due to the damage caused by the development of industry [3-4]. Early theories on the economic order quantity (EOQ) model and economic production quantity

(EPQ) model can be traced back to Taft [5] and Harris [6]. Karim and Nakade [7] offered a comprehensive evaluation of the sustainable EPQ model in terms of carbon emissions and product recycling. In the literature, Jiaqi and Meizhang [8] proposed a remanufacturing product structure information model with remanufacturing design parameters and features. Digiesi et al. [9] studied sustainability aspects in the field of spare parts logistics, considering the environmental costs associated with both the production of new parts and the disposal of used ones. Modern complex manufacturing systems involve multiple stages to produce complex products, such as aviation inertial navigation product manufacturing and automobile assembling. As a result, multistage manufacturing systems (MMSs) are becoming increasingly common in the production process. The MMSs consist of multiple stages, and the products are manufactured in the order of the stages. Wang et al. [10] proposed a production quality prediction framework for multi-task joint deep learning, which evaluates the multi-task quality of all stages. In a multi-stage manufacturing environment, all these different actors will be interacting with each other. The manufacturer is responsible for selecting the green materials with which the product can be manufactured. As recommended by Han et al. [11], when consumers have the right to return second-hand products relying on manufacturers' rebates, it affects the prices of new products. Rebate determination is a common price elasticity of demand problem in a recycling program. To correct this, the return activity is one of the most important points in the production planning system. The rebates collected through this program are transferred to the government by the manufacturers and importers. The government uses these funds to support recycling initiatives, including education campaigns and the establishment of new recycling programs. In 2010, the recycling rate in Taiwan was 94.1 tonnes but this had risen to over 100 tonnes in some areas by 2016. The local government provides funding to support bike recycling program. Figure 1 shows the recovery amount by year. Approximately 148 tonnes were accounted in 2020 (148) and 2019 (141.6), 2018 (140.2), 2017 (128.1), 2016 (112.9), 2015 (98.5), 2014 (101.3), 2013 (92.2), 2012 (91), 2011 (94.4) (EPA (2023)).

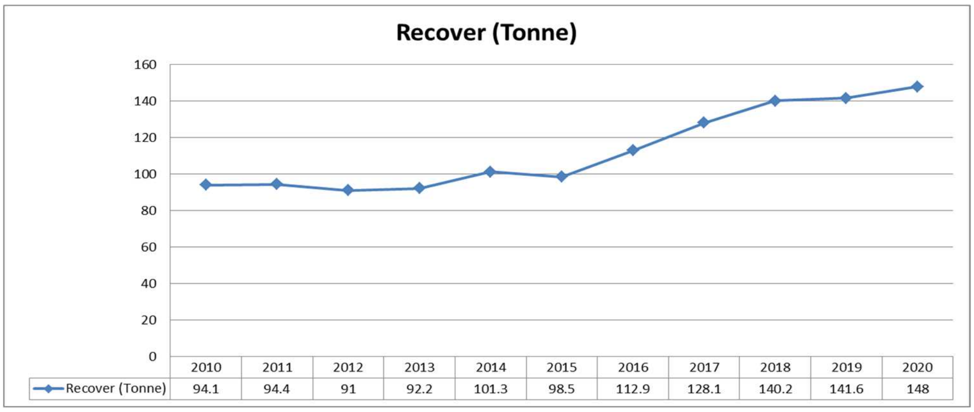

Fig. 1 *Recover amount statistics and analytics. Source: Environmental Protection Police (EPA (2023))*

Reusing/remanufacturing recovers the value of end-of-life products by reusing their durable components to manufacture a product with the original functionality. Green investments are important investments such as remanufacturing policies and mutual funds that target sustainable strategies. Here is an example from a real manufacturing company in Taiwan. At the end of a bike’s life, it can either end up in a landfill or its components can be recycled, saving large amounts of energy. The company has an advanced recycling system to reduce the waste of components and raw materials. Figure 2 summarizes the flow chart of the recycling system. The system comprises one manufacturer and one retailer. Rebate agreements play a crucial role in the supply chain as they provide companies with an effective tool to incentivize their customers. The program encourages retailers to engage in manufacturer rebates within a green supply chain and examines its impact on the green product sales.

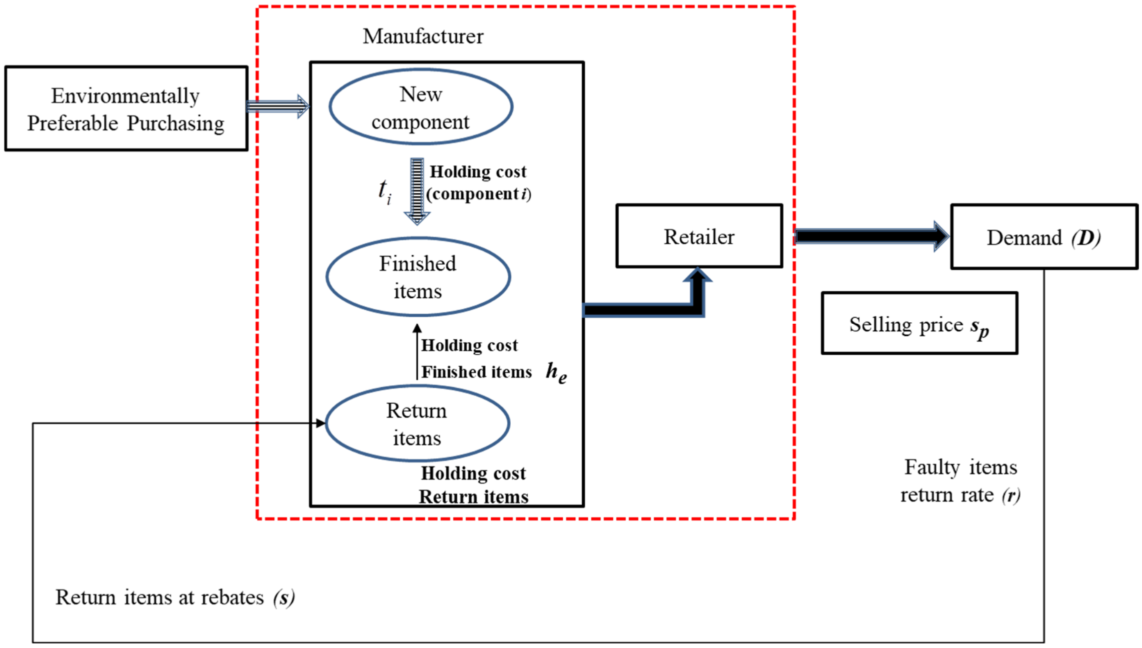

Fig. 2 *Bike recycling system*

However, there is little research that has empirically documented the relationship between inventory-production systems and rebate policies. Chen [12] studied the news-vendor problem in the case of a two-level supply chain consisting of one manufacturer and one retailer and investigated the combined effects of the cooperative advertising mechanism, the return policy, and the channel coordination. Hu et al. [13] studied how retailers under pressure to move inventory should use these two pricing tools to maximize revenue by determining the optimal use of shipping rebates together with dynamic pricing for limited inventory. The contribution of this paper is a better understanding of rebate and return policy under the green product design principle in two areas. First, new insights into the literature on demand patterns are provided, as previous studies on return policy strategy have mainly focused on a constant. In addition, this paper introduces a two-stage production system that considers the factors influencing demand, introduces ecological principles for material selection in product design, and provides guidance for designers to achieve sustainable development in their designs. The system is divided into two stages: the manufacturing of components (Stage 1) and the assembly of the end product (Stage 2). These two stages are usually studied independently of each other which does not lead to ideal results. The elements of total profit per cycle are sales revenue, set-up costs, holding costs of the end product, holding costs of all components, rework costs for all defective items, production costs and investment costs. Next, a product-inventory system was designed to investigate the identification and application of rebate policy. This is done with the motivation that it can provide an alternative solution to the rebate policy problem. Rebate programs can increase sales and demand, reduce inventory levels, and stimulate product return. Manufacturers often focus on the following three questions as key points of interest:

(1) What is the optimal rebate to increase the return rate of recycled products?

(2) What is the optimal production timeframe for manufacturing a component?

(3) What is the ideal return rate per item?

To answer these questions, this paper presents a new model that examines the effects of the return policy on the production system. Overall, this paper determines the optimal production time, rebate, and return rate to maximize the total profit per unit time. This paper is organized into six key sections. Section 1 provides background information on the return policy, demand patterns, and life cycle assessment considerations. Section 2 outlines the description of green product design. Section 3 presents the industrial context and a list of notations and assumptions. Section 4 discusses the problem-solving procedure of the algorithm. Section 5 presents application examples, numerical illustrations, and sensitivity analysis. Finally, Section 6 summarizes the key findings, discussions, and recommendations for future research.

2. LITERATURE REVIEW

2.1 GREEN DESIGN

As populations and economies continue to expand, the depletion of natural resources is accelerating, and environmental pollution is becoming increasingly severe. The concept of a green supply chain aims to reduce environmental degradation and pollution by incorporating eco-friendly practices into business operations. A manufacturer rebate, in which the manufacturer refunds the consumers after the purchase, becomes an effective promotional tool. Lin [14] examined the impact of the manufacturer rebate strategy on the outcomes of channel members within a two-stage green supply chain model characterized by information asymmetry. Taleizadeh and Heydarian [15] applied a rebate strategy in a two-tier supply chain consisting of a single supplier and a manufacturer producing green and non-green products.

2.2 GREEN PRODUCT DESIGN AND DEVELOPMENT

Sustainable development represents a new approach to human survival. It not only includes the responsible use of resources and environmental protection for a sustainable ecological existence, but also serves as a basis for the development of economic and social life. Hong and Guo [16] examined various cooperation contracts within a green product supply chain and evaluated their environmental performance. Fadavi et al. [17] investigated the extent of environmental sustainability and price competition in price-sensitive markets, involving both a manufacturer and a retailer.

2.3 OPTIMAL REBATE POLICY FOR GREEN DESIGN

A durable consumer good can consist of numerous components, with each part designed by a separate team that selects type of plastic or other material best suited to its purpose. Adopting green design (GD) combined with a rebate policy (RP) can be an important solution. Manufacturers must also befully informed about demand, price, and rebate, but may face uncertain demand and selling through a competitive retail market. Giri and Chakraborty [18] investigated an imperfect production system with uncertain demand and rebate warranty. Arcelus et al. [19] studied a price-dependent stochastic demand that depends directly on the selling price of the rebate value, followed by discussions on final customer demand. Mishra et al. [20] presented a deteriorating inventory model with four-stage production rates and derived demand based on the rebate value, considering the selling price of the product at shortage. Ganguly et al. [21] presented a versatile production-inventory system designed for a single-stage assembled item where dfferent parts are both manufactured and remanufactured within the same generation cycle. Defective items occur randomly due to the imperfections inherent in the system. Different approaches for a sustainable green economy with remanufacturing are discussed in Karmakar et al. [22] Shah et al. [23] Roy and Sana [24]. Zwolinski and Brissaud [25] illustrated how the methodology is applied in the two primary design activities: redesigning products from a remanufacturing perspective and developing new products. Haziri and Sundin [26] presented a framework that supports design for remanufacturing by implementing structured feedback from remanufacturing to design. On the other hand, green technologies and green products startups offer numerous benefits beyond just being environmentally friendly. The government provides subsidies to encourage the development of green products. To deepen our understanding of subsidy and tax rebate policies for the research and development (R&D) of green enterprises in China, Chang et al. [27] called for further research on the global Malmquist–Luenberger (GML) index method. In the papers by Fadavi et al. [28], the applications of new green product structures that can help companies reduce their environmental impacts were discussed in detai. For example, marketers of environmentally friendly products can boost future purchases by promoting the company's green image. A growing number of research studies provide insights into solutions for sustainable Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ) problems by taking into account various sustainability aspects such as economic [27-28], environmental [29-34], and social issues [35-37]. There is a fairly extensive literature on the traditional EOQ/EPQ model. However, within that literature, there is a surprising lack of information on rebate policies or green supply chains. Using price or rebate to increase demand for new or existing products or services can be a good way to attract customers or boost lagging sales. Mishra et al. [20] have done extensive work with four-stage production rates and demand based on rebate value, considering the product selling price. Meng et al. [38] studied a cooperative collaborative pricing policies for products in a dual-channel green supply chain and compared the optimal solutions in two cases. However, in many real-world supply chain systems, demand patterns can be price-dependent [38-42], stock-dependent [43-44], freshness-dependent [45], age-dependent [46-47], quality-dependent [48], time-dependent [49-51], CSR impact-dependent [52-57]. Life cycle assessment (LCA) is a widely used method for measuring the environmental costs that can be attributed to a product design or service. In the following years, numerous studies have been conducted on the effectiveness of LCA for green product design [58-60]. This paper may lead to a better understanding of rebate and return policy under the principle of green product design. This paper is divided into six main sections. Section 1 provides some background information about the rebate policy, demand patterns and LCA issues. Section 2 outlines the description of green product design. Section 3 presents the industrial background and a list of notations and assumptions. Section 4 discusses the development of mathematical formulations describing some of the theoretical results encountered. Section 5 presents application examples, some numerical examples, and sensitivity analysis. Finally, Section 6 outlines some results, discussions, and conclusions for further research. Table 1 provides a brief comparison of the above demand patterns mentioned. It should be noted that demand affected by the rebate impact has been the focus of many researchers in recent years. However, it seems that various other types of product return activities can influence demand in the real world. There has been far less research on the impact of rebates and green product design development in a sustainable EPQ model. By conducting these demand classifications in Table 1, several research gaps in sustainable EOQ and EPQ models were identified. The intention is to look at the economic and social sustainability of manufacturer rebates, where the manufacturer refunds the consumers after the purchase behaviors, which is an effective promotional tool.

Table 1 *Comparison tables for different demand patterns*

References	***Demand patterns***							***Other consideration(s)***
	PD	*SD*	*FD*	*AD*	*QD*	*TD*	*CD*	
Chen and Hu [41]	V							*Adjustment cost for price changes*
Guria et al. [42]	V							*Inflation-dependent demand and immediate part payment*
Yang et al. [43]	V							*A closed-loop supply chain involving multiple retailers*
Panda et al. [44]	V							*Socially responsible manufacturer-retailer CLSC*
Saha and Goyal [46]	V	V						*Supply chain coordination contracts*
Pal et al. [47]	V	V						*An integrated supply chain inventory model for imperfect processes*
Chen et al. [48]		V	V					*Perishable items with expiration dates*
Avinadav et al. [49]	V					V		*Decaying items with the effect of fixed shelf-life*
Dobson et al. [50]				V				*Consumers' evaluation of product quality throughout its lifespan*
Glock et al. [51]	V				V			*Balancing sustainability, demand, costs, and profit trade-offs in a supply chain*
Hsieh and Dye [52]	V	V				V		*Dynamic pricing strategy for deteriorating items*
San-José et al. [53]	V					V		*Demand determined by a time-power function and a price-legit function*
Dye [54]	V	V				V		*Advertising goodwill-dependent on demand*
Raza [55]	V						V	*Supply chain coordination under a revenue-sharing contract*
Modak et al. [56]	V						V	*Dual-channel supply chain*
Seyedhosseini et al. [57]	V						V	*Coordination in a competitive supply chain*
Presented paper	V						V	*Rebate and green product design development*

Note: price-dependent demand (PD), stock-dependent demand (SD), freshness-dependent demand (FD), age-dependent demand (AD), quality-dependent demand (QD), time-dependent demand (TD), and CSR impact-dependent demand (CD).

3. NOTATIONS AND ASSUMPTIONS

In the following, an EPQ model that considers a single product, two-stage production systems, and the component replacement problem is developed. For convenience, the notations and assumptions required to develop this integrated model are given first.

3.1 NOTATIONS

The following system parameters are used to develop the model:

SYSTEM PARAMETERS

k : Set-up cost.
s_p : Selling price.
n : Number of important components for a single-end product.
p_i : Input-output ratio of the component i, where $i=1,2,...,n$ and $p_1 > p_2 > ... > p_n$.
χ_e : Rate of assembling the end product measured in units per unit time.
F_r : Fixed cost for many recycling programs per cycle.
V_r : Variable cost for many recycling programs per cycle.
θ_i : Defect rate of the component i, where $i=1,2,...,n$.
θ_e : Defect rate of the end products (per unit).
θ_r : Defect rate of the returned products (per unit).
c_w : Rework cost per item for the returned products (per unit).
h_i : Holding cost of the component i per unit time, where $i=1,2,...,n$.
h_e : End product holding cost per unit time.
h_r : Returned product holding cost per unit time.
t_{id} : Depletion period for the inventory of components i, $t_{id} \geq 0$, where $i=1,2,...,n$.
t_{ed} : Depletion period for the inventory of end products, $t_{ed} \geq 0$.
t_r : The remanufacturing time of the returned products, $t_r \geq 0$.
t_{rd} : The disassembly time of the returned products, $t_{rd} \geq 0$.
t_e : Production run time of the end products, $t_e \geq 0$.
T : Length of the cycle, $T \geq 0$.
H_i : Maximum inventory level of the component i, where $i=1,2,...,n$.
H_e : Maximum inventory level of the end products.
H_r: Maximum inventory level of the returned products.

DECISION VARIABLES

t_i : Production run time of the component i, $t_i \geq 0$, where $i=1,2,...,n$.
s : Rebate of returned products (per unit), $s \geq 0$.
r : Return rate per item.

3.2 ASSUMPTIONS

The following assumptions are made in our production model:

(1) Following Chang et al. [61], we also assume that the primary phase of this system (automated stage) is specifically designed for the production of individual components by each machine. The subsequent phase (manual stage) is focused on the assembly of the final products from the required components.

(2) In the simplest deterministic model, Modak et al. [56] proposed a linear demand function that depends on selling prices and social donation. This work extends the function established by Modak et al. [56]. The form of the demand function is: $D = a - bs_p + \delta s + w\gamma + (1-\theta_r)r$, where $a > 0$ is the market potential, b is the selling price elasticity, δ is the social donation amount elasticity, and w is the elasticity factor of investment in green design development. The total demand remains constant (composed of several different components) within a given cycle time T. Critical components is reworked and non-critical components (general components) are replaced.

(3) The investment in disposal technologies is first presented in terms of fixed and variable cost structures. Fixed costs F_r by definition, are not influenced by the volume of the waste stream (e.g., construction costs and administration). Variable costs V_r are those that are essentially directly proportional to volume (e.g., canisters and vaults built as required to contain the waste). To increase the potential benefits of compounding, the manufacturers can limit their investment to green materials or technologies. Therefore, γ is defined as a binary number of investment strategies that comprises $\gamma = 0$ and $\gamma = 1$ to evaluate the viability of investing in a recycling system to ascertain its advantages; where $\gamma = 0$ (no benefit to investing in recycling system) and $\gamma = 1$ (benefit to investing in recycling system).

(4) There is a single manufacturer and single retailer for a single product in this product system. An infinite planning horizon for the entire system is considered. An item is considered new / unused when it is reworked in the production system.

(5) The production cycle continues indefinitely.

4. MATHEMATICAL MODEL FORMULATION

This paper is primarily concerned with sustainable return on investment (S-ROI) issues through data analysis of an inventory model. Figure 3 summarizes the results of the production system. The bottom three sketches represent the inventory level of the raw material, the inventory level of the end product and the inventory level of the return product, respectively.

Assumption 1, as stated, implies that total demand remains constant within a given cycle, i.e., $p_1t_1 = p_2t_2 = ... = p_it_i = ... = p_nt_n = \chi_et_e = DT$, leading to:

$$t_i = \frac{p_nt_n}{p_i}, \tag{1}$$

and

$$T = \frac{p_nt_n}{D}, \tag{2}$$

where $i = 1,2,...,n$.

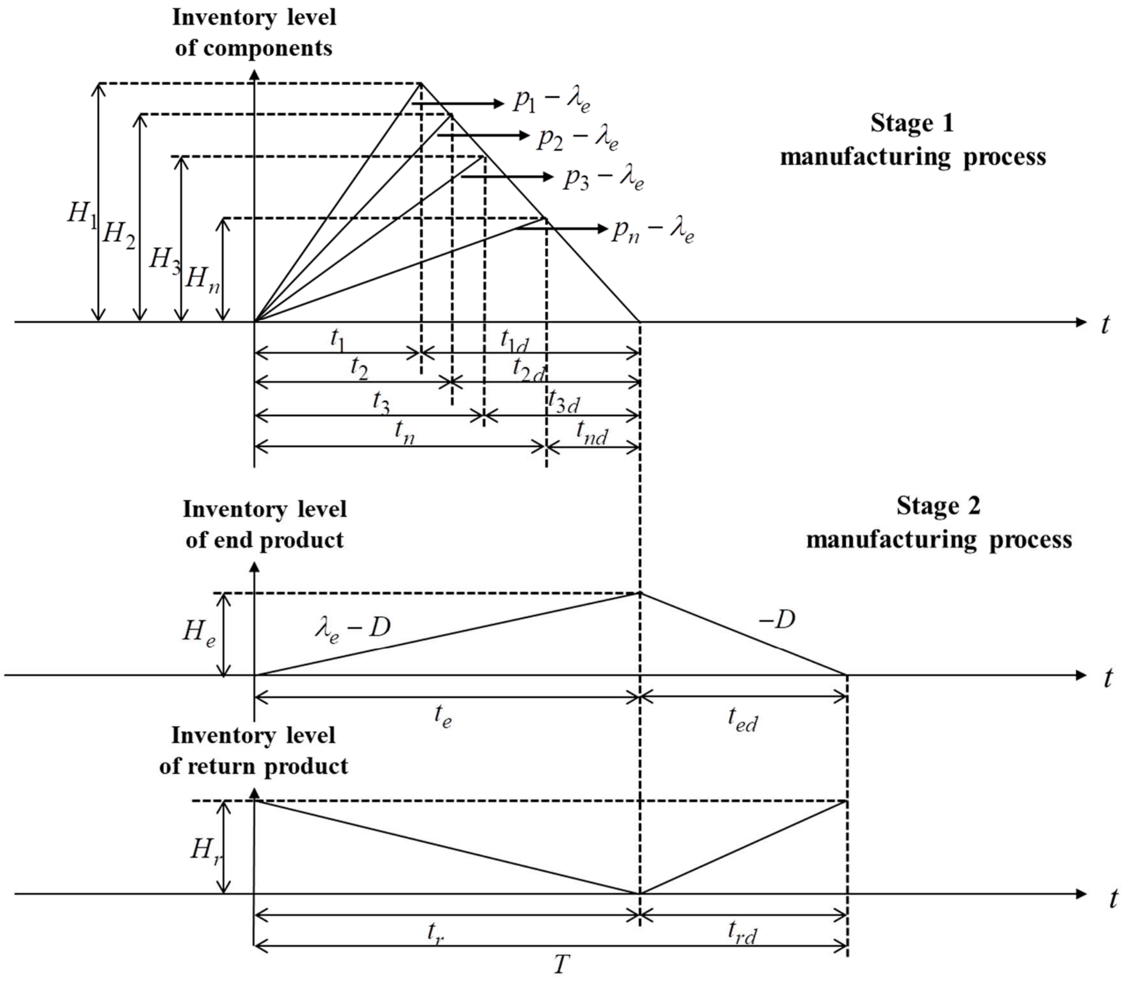

Fig. 3 *Graph illustrating inventory levels for components, end products, and returned items*

Deriving the maximum inventory level of the component i based on Eqs. (1) and (2) can be formulated as:

$$H_i = (p_i - \chi_e)t_i = \chi_e t_{id}. \tag{3}$$

At the period t_{id}, inventory depletion of the component i is:

$$t_{id} = \frac{H_e}{\chi_e} = \frac{(p_i - \chi_e)t_i}{\chi_e}, \tag{4}$$

where $i = 1,2,\ldots,n$.

The maximum inventory level of the end product can be expressed as follows:

$$\begin{aligned} H_e &= (\chi_e - D)t_e = (\chi_e - D)(t_n + t_{nd}) \\ &= (\chi_e - D)\frac{p_n}{\chi_e}t_n. \quad \textit{(from Eq. (4))} \end{aligned} \tag{5}$$

At the period t_{rd}, inventory depletion of the return product is:

$$H_r = p_i(1-\theta_r)t_{rd} = rt_r. \tag{6}$$

This model is composed of the following nine elements:

(1) Sales revenue (*SR*): sales revenue is the real profit, $s_p - s$, multiplied by the aggregate demand (*DT*):

$$SR=\left(s_p - s\right)DT = \left(s_p - s\right)p_n t_n .$$

(2) Setup cost (*SC*): *k*.

(3) Holding cost (HC_e) of the end product is calculated as:

$$HC_e = \frac{h_e H_e T}{2} = \frac{h_e\left(\chi_e - D\right)}{2\chi_e D} p_n{}^2 t_n{}^2 .$$

(4) Holding cost (HC_s) of all components per cycle is calculated as:

$$HC_s = \sum_{i=1}^{n} \frac{h_i H_i\left(t_i + t_{id}\right)}{2} = \frac{p_n^2 t_n^2}{2}\left[\sum_{i=1}^{n} h_i\left(\frac{1}{\chi_e} - \frac{1}{p_i}\right)\right].$$

(5) Holding cost (HC_r) of the return product is calculated as:

$$HC_r = \frac{h_r r t_r{}^2}{2}.$$

(6) Rework costs (*RC*) are calculated as:

$$RC = r\left[c_r\theta_r + \theta_e DT + \sum_{i=1}^{n}\theta_i DT\right].$$

(7) Rework costs (RC_r) of the return product are calculated as:

$$RC_r = c_w r\left(1-\theta_r\right).$$

(8) Production cost (*PC*): According to Giri et al [62], unit product cost is calculated based on the cost of end products manufactured, $p_n t_n$, that is: (a) Stage 1: raw material expenses or the cost of the components: $\beta_0 \geq 0$; (b) Stage 2: manufacturing cost (component assembly) β_1 / χ_e, where $\beta_1 \geq 0$ is unit labor cost; (c) the cost component $\beta_2\chi_e$ increases with the production rate (e.g. tool or die costs).

$$PC = \left(\beta_0 + \frac{\beta_1}{\chi_e} + \beta_2\chi_e\right)p_n t_n .$$

(9) Investment cost (*IC*): Investment cost (*IC*) of the returns program, including fixed cost and variable cost, is calculated as:

$$IC=\gamma\left(F_r + V_r H_r t_{rd}\right) = \gamma\left[F_r + V_r(rt_r)t_{rd}\right].$$

To summarize the above results, total profit per unit time (denoted by $AP\left(s,t_n,r\right)$) can be obtained as follows:

$$\begin{aligned} AP(s,t_n,r) &= \frac{1}{T}\left(SR-SC-HC_e-HC_s-HC_r-RC-RC_r-PC-IC\right) \\ &= \frac{1}{T}\Bigg\{\left(s_p-s\right)p_n t_n-k-\frac{h_e\left(\chi_e-D\right)}{2\chi_e D}p_n^2t_n^2-\frac{p_n^2t_n^2}{2}\left[\sum_{i=1}^{n}h_i\left(\frac{1}{\chi_e}-\frac{1}{p_i}\right)\right] \\ &\quad -r\left[c_r\theta_r+\theta_e DT+\sum_{i-1}^{n}\theta_i DT\right]-\left(\beta_0+\frac{\beta_1}{\chi_e}+\beta_2\chi_e\right)p_n t_n \\ &\quad -\frac{h_r r t_r{}^2}{2}-c_w r(1-\theta_r)-\gamma\left[F_r+V_r(rt_r)t_{rd}\right]\Bigg\}. \end{aligned} \tag{7}$$

Now, the first-order partial derivatives of the total profit should be calculated per unit time with respect to *s*, t_n, and *r*. Then the following is obtained:

$$\frac{\partial AP(s,t_n,r)}{\partial s}=\frac{1}{T}\left\{-p_n t_n+\left[\frac{h_e}{2D^2}\delta+\frac{h_e}{2\chi_e}\right]p_n^2t_n^2-r\left[\theta_e\delta T+\delta\sum_{i-1}^{n}\theta_i T\right]\right\}, \tag{8}$$

$$\frac{\partial AP(s,t_n,r)}{\partial t_n}=\frac{1}{T}\left\{\left(s_p-s\right)p_n-\frac{h_e\left(\chi_e-D\right)}{\chi_e D}p_n^2t_n-p_n^2t_n\left[\sum_{i=1}^{n}h_i\left(\frac{1}{\chi_e}-\frac{1}{p_i}\right)\right]-\left(\beta_0+\frac{\beta_1}{\chi_e}+\beta_2\chi_e\right)p_n\right\}, \tag{9}$$

and

$$\begin{aligned} \frac{\partial AP(s,t_n,r)}{\partial s} &= \frac{1}{T}\left\{-p_n t_n+\left[\frac{h_e}{2D^2}\delta+\frac{h_e}{2\chi_e}\right]p_n^2t_n^2-r\left[\delta\theta_e T+\delta\sum_{i-1}^{n}\theta_i T\right]\right\}, \\ \frac{\partial AP(s,t_n,r)}{\partial r} &= \frac{1}{T}\Bigg\{\left[\frac{h_e}{2D^2}\delta+\frac{h_e}{2\chi_e}\right]p_n^2t_n^2+\left[\frac{h_e}{2D^2}(1-\delta-\theta_r)-\frac{h_e}{2\chi_e}\right]p_n^2t_n^2-\left[c_r\theta_r+D\theta_e T+D\sum_{i-1}^{n}\theta_i T\right] \\ &\quad -r\left[\theta_e\delta T+\delta T\sum_{i-1}^{n}\theta_i\right]-r\left(1-\delta-\theta_r\right)\left[\theta_e T+T\sum_{i-1}^{n}\theta_i\right]-\frac{h_r t_r{}^2}{2}-c_w(1-\theta_r)-\gamma V_r t_r t_{rd}\Bigg\}. \end{aligned} \tag{10}$$

The optimal solutions for (s,t_n,r) satisfy the equations $\partial AP(s,t_n,r)/\partial s=0$, $\partial AP(s,t_n,r)/\partial t_n=0$ and $\partial AP(s,t_n,r)/\partial r=0$, simultaneously. The necessary condition for $AP(s,t_n,r)$ to be maximum is $\partial AP(s,t_n,r)/\partial t_n=0$, which gives:

$$\left(s_p-s\right)p_n-\frac{h_e\left(\chi_e-D\right)}{\chi_e D}p_n^2t_n-p_n^2t_n\left[\sum_{i=1}^{n}h_i\left(\frac{1}{\chi_e}-\frac{1}{p_i}\right)\right]-\left(\beta_0+\frac{\beta_1}{\chi_e}+\beta_2\chi_e\right)p_n=0 \tag{11}$$

It is not easy to find the closed-form solution of t_n from Eq. (11). But the value of t_n that satisfies Eq. (11) not only exists, but is also unique, as stated by the following lemma:

LEMMA 1.

The solution of t_n (say $t_n{}^*$) profit per profit per unit time $AP(s,t_n,r)$ has the global maximum value at the point $t_n=t_n{}^*$, where $t_n{}^*\in(0,\infty)$ and satisfies Eq. (11).

PROOF.

Let:

$$F\left(t_n\right)=\left(s_p-s\right)p_n-\frac{h_e\left(\chi_e-D\right)}{\chi_e D}p_n^2 t_n-p_n^2 t_n\left[\sum_{i=1}^{n}h_i\left(\frac{1}{\chi_e}-\frac{1}{p_i}\right)\right]-\left(\beta_0+\frac{\beta_1}{\chi_e}+\beta_2\chi_e\right)p_n \tag{12}$$

for $t_n \in \left(0,\infty\right)$. Taking the first derivative of $F(t_n)$ with respect to t_n, it gives:

$$\frac{\partial F\left(t_n\right)}{\partial t_n}=-p_n^2\left\{\frac{h_e\left(\chi_e-D\right)}{\chi_e D}+\left[\sum_{i=1}^{n}h_i\left(\frac{1}{\chi_e}-\frac{1}{p_i}\right)\right]\right\}<0,$$

i.e., $F(t_n)$ is a strictly decreasing function of t_n. Then, it is obtained $\lim\limits_{t_n\to\infty}F\left(t_n\right)=-\infty$ and $\lim\limits_{t_n\to 0}F\left(t_n\right)=\left[\left(s_p-s\right)-\left(\beta_0+\frac{\beta_1}{\chi_e}+\beta_2\chi_e\right)\right]p_n$.

Following two cases are possible:

CASE 1 *If* $\left(s_p-s\right)-\left(\beta_0+\frac{\beta_1}{\chi_e}+\beta_2\chi_e\right)>0$, *then the solution* t_n^* *which maximizes* $AP(s,t_n,r)$ *not only exists, but is also unique when applying the Intermediate Value Theorem, where* $t_n^*\in\left(0,\infty\right)$.

In this case, t_n is also greater than or equal to zero. The most important finding in this case is that the return profits for defective products are greater than the production costs. If the return is made within a limited time frame of 30 days, the manufacturer will provide a courtesy credit to the account for a cash refund for all defective products. Since the sufficient condition is met, taking the second derivative of $AP(s,t_n,r)$ with respect to t_n, and then substituting $t_n=t_n^*$ into it, the following is obtained:

$$\left.\frac{\partial^2 AP\left(s,t_n,r\right)}{\partial {t_n}^2}\right|_{t_n=t_n^*}=-\left\{\left[\sum_{i=1}^{n}h_i\left(\frac{1}{\chi_e}-\frac{1}{p_i}\right)+\frac{h_e\left(\chi_e-D\right)}{\chi_e D}\right]+\left(\beta_0+\frac{\beta_1}{\chi_e}+\beta_2\chi_e\right)p_n\right\}<0.$$

Therefore, t_n^* is the global maximum point of $AP(s,t_n,r)$. The proof is now complete.

CASE 2 *If* $\left(s_p-s\right)-\left(\beta_0+\frac{\beta_1}{\chi_e}+\beta_2\chi_e\right)<0$, *then the optimal solution is* $t_n^*=0$.

In this case, t_n is negative and against $t_n\geq 0$ because return profits for defective products are less than production costs. At this point, the manufacturer stops offering a refund policy. Therefore, the optimal length of the production run time of the component *n* should approach zero. Then substituting $t_n=t_n^*$ into Eq. (7), the total profit per unit time $AP\left(s,t_n,r\right)$ can bemodified to a new function of (*s*, *r*), given by:

$$
\begin{aligned}
AP\left(s,r\middle|t_n{=}t_n^{\;*}\right)=\frac{1}{T}\Bigg\{&\left(s_p-s\right)p_n t_n^{\;*}-k-\frac{h_e\left(\chi_e-D\right)}{2\chi_e D}p_n^2 t_n^{\;2*}-\frac{p_n^2 t_n^{\;2*}}{2}\left[\sum_{i=1}^{n}h_i\left(\frac{1}{\chi_e}-\frac{1}{p_i}\right)\right] \\
&-r\left[c_r\theta_r+\theta_e DT+\sum_{i-1}^{n}\theta_i DT\right]-\left(\beta_0+\frac{\beta_1}{\chi_e}+\beta_2\chi_e\right)p_n t_n^{\;*} \qquad (13) \\
&-\frac{h_r r t_r^{\;2}}{2}-c_w r(1-\theta_r)-\gamma\left[F_r+V_r(rt_r)t_{rd}\right]\Bigg\}.
\end{aligned}
$$

Due to the complexity of the model, it is difficult to find the close form of (s, r) and check the concavity of the manufacturer's profit function directly. Alternatively, in the following section, the concavity by numerical analysis will be verified and then a simple algorithm developed to obtain the solutions for the manufacturer.

4.1 ALGORITHM

The model proposed in Section 3 is solved using a solution procedure. First, the problem-solving procedure of the algorithm is introduced. The aim of this algorithm is to determine the optimal solution (s^*, t_n^*, r^*) for maximizing the total annual profit per unit time.

Step 1. Start with $\tau=0$ and $s_{j,\tau}=0$

Step 2. Substitute $s=s_{j,\tau}$ into Eq. (11) and solve for t_n.

Step 3. Solve Eq. (12) to find the optimal value of t_n (say $t_n(s)$ which is a function of s) and then substitute $t_n(s)$ into Eq. (13) to obtain $AP\left(s,r\middle|t_n{=}t_n^{\;*}(s)\right)$.

Step 4. Find the value of s and r by setting $\partial AP\left(s,r\middle|t_n{=}t_n^{\;*}(s)\right)/\partial r=0$ and $\partial AP\left(s,r\middle|t_n{=}t_n^{\;*}(s)\right)/\partial s=0$.

Step 5. If the difference between $s_{j,\tau}$ and $s_{j,\tau+1}$ is sufficiently small, set $\tilde{s}_j=s_{j,\tau+1}$. Otherwise, set $s_{j,\tau+1}=s_{j,\tau}+\varepsilon$, where ε is any small positive number, and set $\tau=\tau+1$, then, go back to Step 2.

Step 6. Substitute $t_n=\tilde{t}_{nj}$ and $s=\tilde{s}_j$ into Eq. (7) to calculate the value of $AP(\tilde{t}_{nj},\tilde{s}_j,\tilde{\gamma}_j)$.

Step 7. If $AP(\tilde{t}_{nj},\tilde{s}_j,\tilde{r}_j)<AP(\tilde{t}_{nj+1},\tilde{s}_{j+1},\tilde{r}_{j+1})$, then $(t_n^*,s^*,r^*)=(\tilde{t}_{nj+1},\tilde{s}_{j+1},\tilde{r}_{j+1})$ is the optimal solution. Otherwise, $(t_n^*,s^*,r^*)=(\tilde{t}_{nj},\tilde{s}_j,\tilde{r}_j)$.

Step 8. Substitute t_n^*, s^* and r^* into Eqs. (1), (2), and (13) to calculate the values of $t_1^{\;*},t_2^{\;*},\ldots,t_{n-1}^{\;*}$, and $AP(t_n^*,s^*,r^*)$.

5. APPLICATION OF THE PROPOSED ALGORITHM

Rebate can be used for many recycling purposes, such as establishing new programs or collection points and identifying markets for recovered materials. The practicality of the proposed model was evaluated through a case study with the own-brand manufacturers (OBMs) of bike companies in Taiwan. Numerical examples were solved to test the robustness and reliability of the proposed model and examine trends in the optimal policies, and managerial insights for the OBMs.

5.1 INDUSTRIAL BACKGROUND

Bike manufacturers have been trying to rethink the way bicycles are made and delivered in terms of a circular economy. The aim is to design bicycles so that they last much longer and all raw materials can be separated and reused. Ensuring a prompt return to small and medium-sized enterprise (SME) manufacturers following the consumption phase is crucial for maximizing the recovery value of durable products and, in general, minimizing negative environmental impacts [63-64]. SME manufacturers often emphasize the benefits of their products, leveraging this positive information to influence consumers in the target market during their purchasing decisions and increase their awareness of green environmental protection.

5.2 NUMERICAL EXAMPLES

Manufacturers and importers of newly regulated recyclable waste (RRW) products, as well as their packaging, containers, and specific raw materials, are obliged to take responsibility for collecting these used bicycles and old bike parts when they reach the end of their life cycle. Numerical examples were employed to validate our analytical results in the scenario and a sensitivity analysis conducted to identify trends in the optimal policies. This approach aims to provide managerial insights to the bicycle manufacturer.

Example 1. Let us consider an inventory system with the following data. A two-stage assembly system is observed. It consists of 3 component processes (*n=3*) in stage 1, and 3 components are required to assemble an end product in Stage 2.

- Demand function: $D = a - bs_p + \delta s + w\gamma + (1-\theta_r)r$, where a=500, b=0.05, δ=0.01, w=20, s_p = \$40000/per unit.
- Component 1 process: p_1=1000units/per unit time, h_1=\$0.1/per unit/per unit time, θ_1=0.01.
- Component 2 process: p_2=900units/per unit time, h_2=\$0.2/per unit/per unit time, θ_2=0.02.
- Component 3 process: p_3=800units/per unit time, h_3=\$0.3/per unit/per unit time, θ_3=0.02.
- Assembly process: χ_e=700units/per unit time, h_e=\$0.7/per unit/per unit time, θ_e=0.02.
- Other costs: k = \$5000/per cycle, β_0=\$50/per unit, β_1=\$100/per unit time, β_2=\$30/per unit time, F_r= \$500/per cycle, V_r= \$2/per unit, h_r = 0.6, θ_r = 0.03, c_r = 5, c_d = 10. The following optimal solution was calculated using the proposed algorithm: $s^* = 28.4340$, $t_n^* = 0.0127$, $r^* = 1.5150$, $\gamma^* = 1$, $ACT = 5.8569$. Note: ACT refers to the average CPU time (in seconds)

Example 2. Let us consider an inventory system with the following data. A two-stage assembly system is observed. It consists of 3 component processes (*n=3*) in Stage 1 and 3 components are required to assemble an end product in stage 2.

- Demand function: $D=a-bs_p+\delta s+w\gamma+(1-\theta_r)r^2$, where $a=500$, $b=0.05$, $\delta=0.01$, $w=20$, $s_p=\$40000$/per unit
- Component 1 process: $p_1=1000$units/per unit time, $h_1=\$0.5$/per unit/per unit time, $\theta_1=0.01$.
- Component 2 process: $p_2=900$units/per unit time, $h_2=\$0.2$/per unit/per unit time, $\theta_2=0.02$.
- Component 3 process: $p_3=800$units/per unit time, $h_3=\$0.3$/per unit/per unit time, $\theta_3=0.03$.
- Assembly process: $\chi_e=700$units/per unit time, $h_e=\$0.7$/per unit/per unit time, $\theta_e=0.02$.
- Other costs: $k=\$5000$/per cycle, $\beta_0=\$50$/per unit, $\beta_1=\$100$/per unit time, $\beta_2=\$30$/per unit time, $F_r=\$500$/per cycle, $V_r=\$2$/per unit, $h_r=0.6, \theta_r=0.03$, $c_r=5$, $c_d=10$.

The algorithm presented was utilized to calculate the following optimal solution: $s^*=25.4488$, $t_n^*=1.0176$, $r^*=6.7411$, $\gamma^*=1$, $ACT=9.6519$. Note: ACT refers to the average CPU time (in seconds).

5.3 SENSITIVITY ANALYSIS

The numerical example provided in Section 5.2 was used to evaluate the impact of modifications to the system parameters (β_0, β_1, β_2, a, b, δ, w, F_r, V_r, h_1, h_2, h_3, h_e, h_r, θ_1, θ_2, θ_e, θ_r) on the values s^*, t_n^*, r^*, and $AP(s^*,t_n^*,r^*)$. Each parameter was individually adjusted (keeping other parameters constant) by +50%, +25%, −25%, or −50%. Analytical results analytical results for examples 1 and 2 are shown in Tables 2 and 3, respectively. These results provider noteworthy observations and managerial insights that can guide decision-making. Specifically, rebate programs are found to boost revenue, incentivize customers to choose one manufacturer over the competition, and foster long-term buyer-seller relationships due to their higher return profit i.e., $(s_p-s)>\left(\beta_0+\frac{\beta_1}{\chi_e}+\beta_2\chi_e\right)$. This would help quickly set up rebate programs based on product lines, customer segments, or other criteria. Additional detailed results are tabulated in Table 4 to provide some managerial insights for bike firms.

Table 2 *Effect of changes in various parameters of the model for Example 1*

		$\gamma=0$					$\gamma=1$				
Parameter		s	t_n	r	AP	ACT	s	t_n	r	AP	ACT
β_0	+50%	26.9411	0.0123	1.5186	4335	6.551	258.855	0.0125	1.51764	4413	7.192
	+25%	28.2105	0.0124	1.5173	4363	5.978	271.678	0.0126	1.51632	4442	6.214
	-25%	30.7022	0.0125	1.5147	4420	6.911	296.844	0.0127	1.51373	4499	4.761
	-50%	31.9253	0.0126	1.5134	4448	4.491	309.193	0.0128	1.51245	4528	5.911
β_1	+50%	29.4569	0.0124	1.5160	4391	6.991	284.268	0.0127	1.51502	4470	4.791
	+25%	29.4605	0.0124	1.5160	4391	4.178	284.304	0.0127	1.51502	4470	6.314
	-25%	29.4676	0.0124	1.5160	4392	6.162	284.376	0.0127	1.51501	4471	5.971
	-50%	29.4712	0.0124	1.5160	4392	4.226	284.412	0.0127	1.51501	4471	6.552
β_2	+50%	71.5485	0.0164	1.4726	5678	4.613	219.134	0.0174	1.47020	8477	5.134
	+25%	127.928	0.0237	1.4145	5188	6.412	285.650	0.0264	1.41180	7503	4.134
	-25%	189.138	0.0352	1.3512	2412	3.182	291.340	0.0360	1.34935	2694	7.190
	-50%	259.110	0.0509	1.2793	1901	4.553	397.830	0.0533	1.27651	1745	5.131
a	+50%	29.7874	0.0108	1.2580	3834	6.013	375.155	0.0110	1.25823	3890	8.312
	+25%	32.9683	0.0119	1.3835	4194	5.067	313.894	0.0121	1.38310	4262	7.145
	-25%	45.6460	0.0129	1.6488	4573	6.145	251.237	0.0132	1.64730	4662	6.109
	-50%	47.5701	0.0130	1.7819	4739	5.235	215.239	0.0137	1.77987	4839	5.191
b	+50%	11.1318	0.0161	2.5775	5669	5.718	56.8325	0.0168	2.56940	5931	7.213
	+25%	12.8404	0.0143	2.0486	5034	5.013	147.571	0.0147	2.04458	5183	6.134
	-25%	33.5536	0.0094	0.9963	3341	7.121	297.395	0.0095	0.99821	3375	5.617
	-50%	34.0801	0.005	0.490	208	5.019	341.1	0.005	0.498	208	6.011
δ	+50%	523.88	0.0142	1.4654	5003	5.178	521.70	0.0145	1.4636	5119	8.191
	+25%	419.31	0.0133	1.4924	4709	5.245	408.72	0.0136	1.4916	4793	5.135
	-25%	123.72	0.0113	1.5368	4006	6.198	103.30	0.0115	1.5363	4057	6.167
	-50%	66.226	0.0098	1.5550	3470	5.178	89.283	0.0099	1.5540	3514	8.129
w	+50%	29.464	0.0124	1.5160	6392	5.124	28.459	0.0127	1.5139	8469	9.123
	+25%	29.464	0.0124	1.5160	5392	6.081	28.445	0.0127	1.5144	6470	8.145
	-25%	29.464	0.0124	1.5160	4392	5.345	28.421	0.0127	1.5155	4471	7.135
	-50%	29.464	0.0124	1.5160	3392	6.178	28.408	0.0127	1.5160	3472	6.145
F_r	+50%	29.464	0.0124	1.5160	4392	5.013	27.776	0.0128	1.5156	4508	7.091
	+25%	29.464	0.0124	1.5160	4392	6.154	28.114	0.0127	1.5153	4490	6.981
	-25%	29.464	0.0124	1.5160	4392	5.197	28.735	0.0126	1.5147	4451	5.617
	-50%	29.464	0.0124	1.5160	4392	6.071	29.017	0.0125	1.5144	4431	5.789
V_r	+50%	29.464	0.0124	1.5160	4392	5.134	28.433	0.0127	1.5150	4470	8.134
	+25%	29.464	0.0124	1.5160	4392	5.456	28.433	0.0127	1.5150	4471	9.234
	-25%	29.464	0.0124	1.5160	4392	6.171	28.434	0.0127	1.5150	4471	7.123
	-50%	29.464	0.0124	1.5160	4392	6.571	28.434	0.0127	1.5150	4471	6.123
h_1	+50%	29.463	0.0124	1.5160	4392	6.123	28.432	0.0127	1.5150	4470	6.891
	+25%	29.463	0.0124	1.5160	4392	6.231	28.433	0.0127	1.5150	4470	7.134

	-25%	*29.464*	*0.0124*	*1.5160*	*4392*	*7.678*	*28.434*	*0.0127*	*1.5150*	*4471*	*7.456*
	-50%	*29.465*	*0.0124*	*1.5160*	*4392*	*6.124*	*28.435*	*0.0127*	*1.5150*	*4471*	*8.112*
h_2	*+50%*	*29.463*	*0.0124*	*1.5160*	*4392*	*6.134*	*28.433*	*0.0127*	*1.5150*	*4470*	*7.134*
	+25%	*29.463*	*0.0124*	*1.5160*	*4392*	*7.081*	*28.433*	*0.0127*	*1.5150*	*4471*	*8.135*
	-25%	*29.464*	*0.0124*	*1.5160*	*4392*	*6.213*	*28.434*	*0.0127*	*1.5150*	*4471*	*11.34*
	-50%	*29.464*	*0.0124*	*1.5160*	*4392*	*6.145*	*28.434*	*0.0127*	*1.5150*	*4471*	*9.034*
h_3	*+50%*	*29.4638*	*0.0124*	*1.5160*	*4392*	*7.190*	*28.4337*	*0.0127*	*1.5150*	*4471*	*10.11*
	+25%	*29.4639*	*0.0124*	*1.5160*	*4392*	*6.179*	*28.4338*	*0.0127*	*1.5150*	*4471*	*8.121*
	-25%	*29.4642*	*0.0124*	*1.5160*	*4392*	*5.181*	*28.4341*	*0.0127*	*1.5150*	*4471*	*9.101*
	-50%	*29.4643*	*0.0124*	*1.5160*	*4392*	*6.341*	*28.4342*	*0.0127*	*1.5150*	*4471*	*7.134*
h_r	*+50%*	*12.1632*	*0.0024*	*1.5410*	*5192*	*7.019*	*18.2357*	*0.0107*	*1.5560*	*5871*	*6.189*
	+25%	*29.2631*	*0.0114*	*1.5330*	*4192*	*5.178*	*16.1328*	*0.0116*	*1.5229*	*4261*	*9.112*
	-25%	*35.1842*	*0.0124*	*1.5220*	*3292*	*6.891*	*14.0331*	*0.0125*	*1.5011*	*3221*	*8.924*
	-50%	*39.2633*	*0.0164*	*1.5160*	*2192*	*6.171*	*12.1312*	*0.0127*	*1.4921*	*2411*	*8.191*
h_e	*+50%*	*14.7322*	*0.0099*	*1.5616*	*5270*	*6.134*	*16.46.67*	*0.0101*	*1.5613*	*5353*	*7.891*
	+25%	*25.7158*	*0.0109*	*1.5405*	*4834*	*4.146*	*14.3392*	*0.0111*	*1.5398*	*4915*	*11.11*
	-25%	*55.3295*	*0.0147*	*1.4894*	*3882*	*6.179*	*14.2680*	*0.0140*	*1.4882*	*3953*	*9.134*
	-50%	*83.8754*	*0.0184*	*1.4599*	*3246*	*7.134*	*12.9197*	*0.0180*	*1.4589*	*3298*	*9.245*
θ_1	*+50%*	*28.7813*	*0.0124*	*1.5167*	*4376*	*7.189*	*27.7456*	*0.0126*	*1.5157*	*4455*	*10.12*
	+25%	*29.1233*	*0.0124*	*1.5164*	*4384*	*6.531*	*28.0904*	*0.0126*	*1.5154*	*4463*	*9.869*
	-25%	*29.8037*	*0.0125*	*1.5157*	*4399*	*5.718*	*28.7764*	*0.0127*	*1.5147*	*4478*	*7.123*
	-50%	*30.1421*	*0.0125*	*1.5153*	*4407*	*6.741*	*29.1176*	*0.0127*	*1.5143*	*4486*	*6.345*
θ_2	*+50%*	*28.0939*	*0.0123*	*1.5174*	*4361*	*7.543*	*27.0524*	*0.0126*	*1.5164*	*4439*	*9.112*
	+25%	*28.7813*	*0.0124*	*1.5167*	*4376*	*6.989*	*27.7456*	*0.0126*	*1.5157*	*4455*	*8.981*
	-25%	*30.1421*	*0.0125*	*1.5153*	*4407*	*7.012*	*29.1176*	*0.0127*	*1.5143*	*4486*	*8.451*
	-50%	*30.8156*	*0.0125*	*1514.62*	*4422*	*6.813*	*29.7966*	*0.0127*	*1.5136*	*4502*	*9.145*
θ_e	*+50%*	*28.0939*	*0.0123*	*1517.43*	*4361*	*6.918*	*27.0524*	*0.0126*	*1.5164*	*4439*	*11.34*
	+25%	*28.7813*	*0.0124*	*1516.72*	*4376*	*6.201*	*27.7456*	*0.0126*	*1.5157*	*4455*	*10.12*
	-25%	*30.1421*	*0.0125*	*1515.32*	*4407*	*7.189*	*29.1176*	*0.0127*	*1.5143*	*4486*	*9.121*
	-50%	*30.8156*	*0.0125*	*1514.62*	*4422*	*5.091*	*29.7966*	*0.0127*	*1.5136*	*4502*	*8.911*
θ_r	*+50%*	*29.2355*	*0.0125*	*1540.07*	*4412*	*6.011*	*28.2475*	*0.0127*	*1.5390*	*4492*	*9.451*
	+25%	*29.3510*	*0.0125*	*1527.95*	*4402*	*5.981*	*28.3418*	*0.0127*	*1.5269*	*4481*	*8.192*
	-25%	*29.5748*	*0.0124*	*1504.27*	*4381*	*7.011*	*28.5240*	*0.0126*	*1.5033*	*4460*	*9.456*
	-50%	*29.6834*	*0.0124*	*1492.71*	*4371*	*6.911*	*28.6121*	*0.0126*	*1.4918*	*4449*	*10.23*

Note: ACT refers to the average CPU time (in seconds)

Table 3 *Effect of changes in various parameters of the model for Example 2*

	$\gamma=0$					$\gamma=1$				
Parameter	s	t_n	r	AP	ACT	s	t_n	r	AP	ACT
β_0 +50%	254.688	1.01833	6.7411	13156	5.156	254.488	1.0177	6.7461	13137	9.814
+25%	254.688	1.01829	6.7411	13155	4.198	254.488	1.0176	6.7461	13136	8.891
-25%	254.688	1.01821	6.7411	13153	6.189	254.488	1.0176	6.7461	13133	8.123
-50%	254.688	1.01818	6.7411	13152	6.179	254.488	1.0175	6.7461	13132	8.045
β_1 +50%	254.688	1.01825	6.7411	13154	4.123	254.488	1.0177	6.7461	13135	9.456
+25%	254.688	1.01825	6.7411	13154	5.123	254.488	1.0177	6.7461	13135	9.671
-25%	254.688	1.01825	6.7411	13154	6.164	254.488	1.0177	6.7461	13135	9.014
-50%	254.688	1.01825	6.7411	13154	6.011	254.488	1.0177	6.7461	13135	8.901
β_2 +50%	254.688	1.04966	6.8222	14164	5.891	254.488	1.0491	6.8271	14144	8.101
+25%	254.688	1.03395	6.7821	13654	6.180	254.488	1.0334	6.7871	13635	7.987
-25%	254.688	1.00255	6.7012	12663	6.253	254.488	1.0020	6.7061	12644	6.991
-50%	254.688	0.98684	6.6613	12180	6.190	254.488	0.9862	6.6601	12162	7.212
a +50%	229.688	0.94347	6.5514	10895	6.891	254.488	1.0491	6.8271	10878	9.101
+25%	242.188	0.98086	6.6515	11999	5.719	254.488	1.0334	6.7871	11981	9.341
-25%	267.188	1.05564	6.8418	14360	6.451	254.488	1.0020	6.7061	14340	8.914
-50%	279.688	1.09303	6.9419	15617	6.911	254.488	0.9862	6.6601	15597	9.014
b +50%	404.688	2.32111	8.2129	59062	6.019	410.001	2.2121	8.1031	-58952	10.11
+25%	304.688	1.16781	7.0121	18286	5.910	304.488	1.1672	7.1001	18263	11.34
-25%	204.688	0.86870	6.3621	88417	5.991	204.488	0.8681	6.3611	88261	9.011
-50%	154.68	0.71911	5.9711	5349	6.105	154.411	0.7124	5.9741	9336	9.321
δ +50%	169.792	0.65994	8.7311	54550	4.017	169.659	0.6595	8.7312	54469	9.451
+25%	203.751	0.80326	7.7412	81345	6.910	203.591	0.8028	7.7412	81226	9.761
-25%	339.585	1.37656	5.7521	24187	6.819	339.318	1.3758	5.7511	24152	8.191
-50%	432.134	1.69811	3.6221	21260	7.191	411.216	1.6111	4.1211	21212	7.919
w +50%	254.688	1.01825	6.7411	52154	6.091	254.388	1.0174	6.7411	63126	9.567
+25%	254.688	1.01825	6.7411	42154	6.478	254.438	1.0175	6.7412	53130	9.675
-25%	254.688	1.01825	6.7411	31154	6.981	254.538	1.0178	6.7412	43140	9.891
-50%	254.688	1.01825	6.7411	20154	8.431	254.588	1.0179	6.7411	33144	10.06
F_r +50%	254.688	1.01825	6.7411	13154	5.198	254.488	1.0177	6.7411	23135	9.111
+25%	254.688	1.01825	6.7411	13154	6.571	254.488	1.0177	6.7411	13135	9.321
-25%	254.688	1.01825	6.7411	13154	5.178	254.488	1.0177	6.7411	13135	9.678
-50%	254.688	1.01825	6.7411	13154	8.001	254.488	1.0177	6.7411	13135	9.981
V_r +50%	254.688	1.01825	6.7411	13154	6.178	254.488	1.0177	6.7411	13135	9.991
+25%	254.688	1.01825	6.7411	13154	6.481	254.488	1.0177	6.7411	13135	10.54
-25%	254.688	1.01825	6.7411	13154	6.661	254.488	1.0177	6.7411	13135	8.671
-50%	254.688	1.01825	6.7411	13154	6.891	254.488	1.0177	6.7411	13135	9.771
h_1 +50%	274.664	1.35485	7.6121	26922	6.015	274.464	1.3541	7.6111	26886	9.991
+25%	263.806	1.16623	7.1221	18493	6.789	263.606	1.1656	7.1211	18467	9.561
-25%	246.923	0.89971	6.4412	16341	7.141	246.723	0.8992	6.4411	6197	9.456
-50%	240.230	0.80308	6.1912	12339	7.456	240.030	0.8026	6.1911	2227	10.66

h_2	+50%	259.907	1.10178	6.9613	16034	6.451	259.707	1.1012	6.9611	16012	9.578
	+25%	257.234	1.05861	6.8514	14503	6.891	257.034	1.0580	6.8511	14483	9.678
	-25%	252.261	0.98045	6.6513	11961	6.981	252.061	0.9800	6.6411	11943	9.981
	-50%	249.943	0.94499	6.5513	10902	7.123	249.743	0.9444	6.5511	10886	10.01
h_3	+50%	259.057	1.0879	6.92	15535	4.515	258.857	1.0874	6.9211	15513	9.997
	+25%	256.828	1.0521	6.83	14281	5.671	256.628	1.0515	6.8311	14261	8.673
	-25%	252.633	0.9862	6.66	12138	7.819	252.433	0.9856	6.6611	12120	8.145
	-50%	250.656	0.9558	6.58	11220	7.011	250.456	0.9553	6.5811	11203	9.104
h_r	+50%	229.157	0.8179	5.52	45221	6.178	258.857	0.5874	5.9211	45513	10.01
	+25%	256.428	1.0521	5.83	64111	6.289	356.128	0.6515	5.8311	64261	10.19
	-25%	282.233	1.9862	9.66	72031	7.014	358.133	1.9856	8.1611	10111	10.15
	-50%	350.156	5.9558	10.58	81510	7.321	451.256	5.9553	19.1811	111213	10.09
h_e	+50%	239.847	0.5318	5.49	47415	8.015	239.647	0.5315	5.4911	47342	9.116
	+25%	245.248	0.7000	5.92	71861	7.914	245.048	0.6996	5.9211	71752	9.567
	-25%	275.403	1.8242	8.82	76785	7.781	275.203	1.8232	8.8212	96737	9.912
	-50%	357.529	6.4762	20.80	97218	7.671	357.329	6.4738	20.811	107173	10.67
θ_1	+50%	254.688	1.0182	7.00	13154	6.981	254.488	1.0177	7.0012	13135	10.11
	+25%	254.688	1.0182	6.87	13154	7.019	254.488	1.0177	6.8712	13135	9.911
	-25%	254.688	1.0182	6.61	13154	7.134	254.488	1.0177	6.6114	13135	9.011
	-50%	254.688	1.0182	6.49	13154	7.891	254.488	1.0177	6.4821	13135	8.911
θ_2	+50%	254.688	1.0182	7.26	13154	6.891	254.488	1.0177	7.2641	13135	9.114
	+25%	254.688	1.0182	7.00	13154	6.991	254.488	1.0177	7.0041	13135	9.321
	-25%	254.688	1.0182	6.49	13154	6.981	254.488	1.0177	6.4815	13135	9.421
	-50%	254.688	1.01825	6.23	13154	7.014	254.488	1.0177	6.2361	13135	9.514
θ_e	+50%	254.688	1.01825	7.26	13154	6.123	254.488	1.0177	7.2613	13135	9.567
	+25%	254.688	1.01825	7.00	13154	6.245	254.488	1.0177	7.0051	13135	9.876
	-25%	254.688	1.01825	6.49	13154	6.431	254.488	1.0177	6.4861	13135	9.881
	-50%	254.688	1.01825	6.23	13154	7.194	254.488	1.0177	6.2315	13135	9.911
θ_r	+50%	254.688	1.01825	6.85	13154	6.001	254.488	1.0177	6.8516	13135	9.145
	+25%	254.688	1.01825	6.80	13154	6.012	254.488	1.0177	6.7914	13135	8.561
	-25%	254.688	1.01825	6.70	13154	6.103	254.488	1.0177	6.6916	13135	8.981
	-50%	254.688	1.01825	6.64	13154	6.121	254.488	1.0177	6.6417	13135	9.014

Note: ACT refers to the average CPU time (in seconds)

Table 4 *Some detail results of sensitivity analysis*

	Parameter(s)	*Example 1*	*Example 2*
Increasing	$\beta_0, \beta_1, a, \theta_1, \theta_2, \theta_e$	*Reducing costs would enable the company to invest in quality controls, gaining a competitive advantage under a linear demand pattern.*	*Reducing costs would enable the company to invest in quality controls, gaining a competitive advantage. On the other hand, gaining more rebates under the power demand pattern.*
Decreasing	$\beta_2, \delta, b, h_1, h_2, h_r$	*This indicates that if there were an increase in the holding cost, and tool (die) cost, then the company should reduce the procurement and handling fee of raw materials.*	*This indicates that if there were an increase in the holding cost, and tool (die) cost, then the company should reduce the procurement and handling fee of raw materials.*
Other(s)	1. *The effect of decreasing change to* F_r *and* θ_r *on the value of* $AP(s^*, t_n^*, r^*)$ *is maximum, with decreases of 1.7% and 0.95%, respectively. This suggests that efforts to enhance total profits per unit time should concentrate on reducing fixed costs, particularly in the context of recycling programs, to lower the defect rate of returned products.* 2. *In the realm of green design investment, augmenting the values of parameters resulted in a proportional increase in* $AP(s^*, t_n^*, r^*)$ *. This indicates that greening strategies in the manufacturing process increase profits by producing products or providing services in an environmentally friendly manner.*		

6. CONCLUSION AND DISCUSSION

6.1 RESULTS

In this section, the findings indicate that a rebate/recycling program positively affects the production-inventory system. With regard to these concerns, the case study has led to the following observations:

(1) The grants provide funding to stimulate companies to establish convenient bike recycling programs;

(2) The overall recycling rate of used components increased to 57%, largely due to the rebate policy encouraging companies to promote recyclable and reusable components. Some of these findings are worth summarizing: (a) An eligible individual who purchases a green-component bicycle can assign a rebate to a bike manufacturer at the time of purchase. (b) The amount of the rebate is reduced based on the bike manufacturer's gross income. Therefore, one will have to apply for a rebate program through the government rebate support. The results of this paper could have a significant impact on the recycling system estimates.

6.2 DISCUSSION

There are several possible discussions of this paper.

(1) Throught the proposed rebate policy, the bike manufacturer inventory management can benefit from optimized costs, improved decision-making accuracy, and adaptability to a dynamic decision environment. These benefits, in turn, contribute to the operational efficiency and effectiveness of bike manufacturer inventory management by promoting the rebate policy and improving the product run time, respectively.

(2) Under the proposed rebate policy, bike manufacturers are given the opportunity to achieve an optimal inventory policies through a recycling system. That is, the bike manufacturers improve the production run time and rebate rate.

(3) In cases when cost parameters remain stable over a long period of time, the bike manufacturer's inventory system is still able to utilize the values of the table simulated under different scenarios. Consequently, the proposed rebate policy provides valuable decision support for inventory managers, and the resulting model, once trained, can be used in real-time.

6.3 CONCLUSIONS

This paper primarily focuses on the optimal rebate policy of the bike manufacturer to stimulate the recycling of old products under business-to-customer platforms. Several implications can be drawn from this study. First, it explores the potential applications of computer simulation models in analyzing selected production decisions in the bicycle industry. Second, the main benefit of implementing a rebate policy is to increase sales. Third, companies that offer rebate incentives can provide the technology that allows tracking customers through a digital rebate system. In this paper, the condition that the unit profit exceeds the production cost must be fulfilled and the effect of rebate policy on the recycling system are presented. Our results suggest that it is far more likely to increase sales when money is refunded to a credit card or checking account. Further research can be conducted on the following aspects. First, since this research focuses on the manufacutrer 's product design decisions, the exogenous production capacities of the companies have been assumed. Another limitation is the industry concentration calculated with simulation modeling, and compustat data, covering only the private companies in the bike industry. An important focus of future research in the coming years will be to investigate to what extent and for which types of products, materials, and personalization profitability can be expected.

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