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LETTER TO THE EDITOR

SOME CONSIDERATIONS ON MOMENTUM DEPENDENCE OF SUPER-CONDUCTIVITY GAP

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It is well known that in solving of superconductivity equation it was assumed that the electron spectrum gap does not depend on momentum in actual momentum region. We shall try to solve this equation without introducing of this simplifying assumption, and to find the momentum dependence of the superconductivity gap. The Cooper's pair radius will be also calculated.

The total Hamiltonian of the system, including electrons, phonons and their interactions can be written as follows

$$H = \sum_{\vec{k}} E_k a_k^+ a_k + \sum_{\vec{q}j} \hbar\omega_{qj} b_{qj}^+ b_{qj} + \sum_{\vec{k}, \vec{q}, j} F(\vec{k}, \vec{q}) a_{k-q}^+ \; a_k (b_{-qj} + b_{qj}^+) \quad (1)$$

for the matrix element characterizing the electron-phonon interaction.

In further calculation the approximate expression[1)] will be used

$$F(\vec{k}, \vec{q}) = -\frac{i}{2}\left(\frac{\hbar}{2MN\omega_{qj}}\right)^{1/2} [\vec{e}_{qj}(\vec{k} - \vec{q}_j) W_{k-q,j} - kW_k]. \quad (2)$$

This approximation means that only longitudinal interactions between electrons and phonons are taken into account.

In further analysis the well known Fröhlich's treatment[2] of the Hamiltonian (1) will be used. We go over from the Hamiltonian (1) to the equivalent one by the use of the unitary transformation

$$H_{eq} = e^{-S} He^{S} \approx H - [S,H] + \frac{1}{2} [S, [S, H]] \tag{3}$$

with

$$S = \sum_{\vec{k}, \vec{q}} \frac{F(\vec{q})}{E_k - E_{k-q} + \hbar vq} a^{+}_{k-q} a_k b_{-k} - c.c. \tag{4}$$

Averaging the result over the phonon vacuum and taking into account the interactions between electrons with opposite momenta only, we finally obtain

$$H_{eff} = \sum_{\vec{k}} E_k a^{+}_k a_k + \sum_{\vec{k}, \vec{q}} V(\vec{k} - \vec{q}) a^{+}_k a^{+}_k a_{-q} a_q \tag{5}$$

where

$$V(\vec{k} - \vec{q}) = \frac{|F(\vec{k} - \vec{q})|^2 \hbar v |\vec{k} - \vec{q}|}{|E_k - E_q|^2 - \hbar^2 v^2 |\vec{k} - \vec{q}|^2}. \tag{6}$$

After the Bogoliubov's transformation[3]

$$a_{k\uparrow} = u_k A_{k0} + v_k A^{+}_{k1}, \quad a_{-k\uparrow} = u_k A_{k1} - v_k A^{+}_{k0}, \; u^2_k + v^2_k = 1 \tag{7}$$

the Hamiltonian (5) reduces to

$$H_{eff} = -\frac{1}{2} \sum_{\vec{k}} \frac{(E_k - \varepsilon_k)^2}{\varepsilon_k} + \sum_{\vec{k}} \varepsilon_k (A^{+}_{k0} A_{k0} + A^{+}_{k1} A_{k1}) \tag{8}$$

where

$$\varepsilon_k = (E^2_k + \Delta^2_k)^{1/2}. \tag{9}$$

The superconductivity gap Δ_k is defined by the equation

$$\Delta_k + \sum_{\vec{q}} \frac{V(\vec{k} - \vec{q})}{(E^2_q + \Delta^2_q)^{1/2}} \cdot \Delta_q = 0. \tag{10}$$

In the standard approach, this equation is solved under the assumption that in the momentum region $(k_F - k_D, k_F + k_D)$, the functions $V(\vec{k} - \vec{q})$ and Δ_k are constant[4]. This approach gives the well known result of BCS theory, i. e. $\Delta_{BCS} \approx 2\hbar\omega_D \exp[-1/N(0)V]$, where $N(0)$ is the electron states density per unit Fermi sphere energy and ω_D is the Debye's frequency.

We shall solve the equation (10) substituting approximately the expression $(E_k^2 + \Delta_k^2)^{1/2}$ by the mean electron energy $\frac{3}{5}\varepsilon_F$. So we obtain the approximate equation

$$\Delta_k + \frac{10\,\overline{W}^2 m^2}{3\,\hbar^2 M \varepsilon_F} \int \frac{\Delta(\vec{q})\,\mathrm{d}\vec{q}}{(\vec{k}-\vec{q})^2 - k_0^2} = 0,\ k_0 = \frac{2mv}{\hbar} \tag{11}$$

which, after the Fourier transformations

$$\Delta_k = \int \Delta(r)\,\mathrm{e}^{ikr}\,\mathrm{d}\vec{r},\ \overline{V}(\vec{r}) = \frac{\cos k_0 r}{4\pi r},\quad \frac{1}{(\vec{k}-\vec{q})^2 - k_0^2} = \int \overline{V}(\vec{r})\,\mathrm{e}^{i(\vec{k}-\vec{q})\vec{r}}\,\mathrm{d}\vec{r} \tag{12}$$

reduces to the algebraic one

$$\left(1 + \frac{10\,\overline{W}^2 m^2}{3\,\hbar^2 M \varepsilon_F}\frac{\cos k_0 r}{4\pi r}\right)\Delta(\vec{r}) = 0. \tag{13}$$

The solution of the last equation is given by (see Ref. 5)

$$\Delta(\vec{r}) = C\,\delta(\vec{r} - \vec{r}_0),\ \Delta_k = C\frac{\sin k r_0}{k},\ r_0 \approx 10^{-7}\,\mathrm{m}. \tag{14}$$

We notice that r_0 is the minimal solution of the equation

$$\left(1 + \frac{10\,\overline{W}^2 m^2}{3\,\hbar^2 M \varepsilon_F}\frac{\cos k_0 r}{4\pi r}\right) = 0 \tag{15}$$

corresponding to the maximum of the electron attraction. So r_0 can be considered as the minimal distance for electron pairing and, consequently, identified with Cooper's pair radius.

In accordance with (14) and (9) the elementary excitations spectrum is given by

$$\varepsilon_k = \left(E_k + C^2\frac{\sin^2 k r_0}{k^2}\right)^{1/2}. \tag{16}$$

The main difficulty of the used approach is the determining of the arbitrary constant C, appearing due to the homogeneity of the equation (11). Taking the free energy minimum as the condition for determining C, we found that this leads to $C = 0$. This result denotes that the states with the energies (16) are unstable and that to the states of free electrons. So we decided to determine C by the following condition

$$\langle \Delta_k^2 \rangle = \frac{1}{2k_D}\int_{k_F-k_D}^{k_F+k_D} \Delta_k^2\,\mathrm{d}\vec{k} = \Delta_{BCS}^2. \tag{17}$$

So we find

$$\Delta_k = \frac{\sqrt{2}\, k_F \Delta_{BCS}}{\left(1 - \frac{\sin 2r_0 k_D \cos 2r_0 k_F}{2k_D r_0}\right)^{1/2}} \frac{\sin kr_0}{k} \tag{18}$$

and

$$\varepsilon_k = \left\{\left[\frac{\hbar^2}{2m}(k^2 - k_F^2)\right]^2 + \frac{2k_F^2 \Delta_{BCS}^2}{1 - \frac{\sin^2 r_0 k_D \cos 2r_0 k_F}{2k_D r_0}} \frac{\sin^2 kr_0}{k^2}\right\}^{1/2}. \tag{19}$$

It can be easily concluded that the spectrum (19) satisfies the condition of superconductive moving.

On Fig. 1. the momentum dependence of Δ_k is graphicaly expressed, together with the values of the BCS gap.

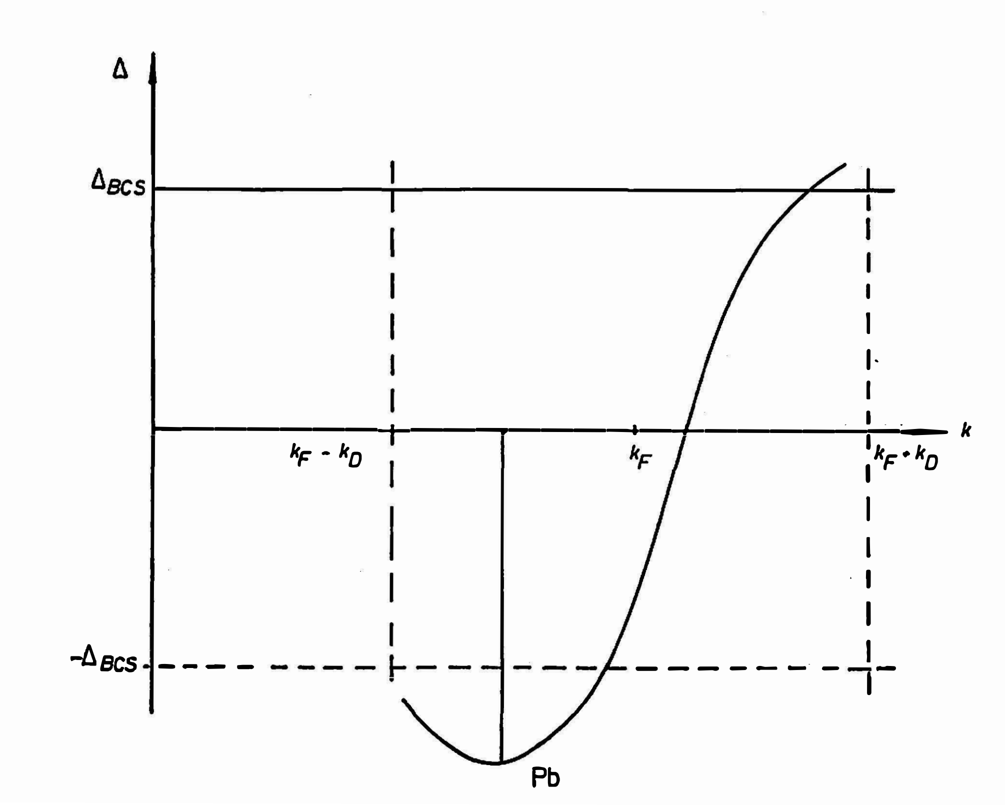

Fig. 1. Δ_k for Pb in wave vector interval $(k_F - k_D, k_F + k_D)$.

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ZAVISNOST SUPERKONDUKTIVNOG PRAGA ENERGIJE OD IMPULSA

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Ovdje je učinjen pokušaj da se jednačina za superkonduktivni prag energije riješi bez uvođenja pretpostavke da je on konstantan u aktuelnom intervalu elektronskih impulsa. Nađena je zavisnost praga od impulsa i vrijednost radijusa Cooperovog para.