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ZEEMAN EFFECT AT THE HYDROGEN ATOM IN THE NEW DIRAC FIELD THEORY*

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The weak constant magnetic field effect at the hydrogen atom is considered in the new Dirac field theory. The difference from the conventional theory is found beginning with the $Z^2\alpha^2$ term.

1. Introduction

The new Dirac field theory[1] applied to the relativistic hydrogen atom[2] gives difference from the standard theory in the electron space probability distribution. What are the experimental manifestations of this difference? In this article we consider the Zeeman effect for weak fields.

Section 2 contains a short presentation of the new relativistic hydrogen atom theory. The interaction of the hydrogen atom with a constant magnetic field is considered in Section 3. In the same Section the corresponding approximative procedure is developed. The energy splitting of the ground state and the first higher states ($2S_{1/2}$, $2P_{1/2}$) is evaluated in Section 4. Conclusions are given in Section 5.

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2. The new relativistic hydrogen atom theory

The new Dirac field is defined by the Lagrangian density[1]

$$\mathscr{L} = \varkappa\,[(-\mathrm{i}\,\partial_\alpha \bar{\Phi}\,\gamma^\alpha)(\mathrm{i}\,\partial_\beta\,\gamma^\beta\,\Phi) - \varkappa^2\,\bar{\Phi}\,\Phi], \tag{1}$$

where Φ is a bispinor field and $\varkappa$ is a constant. We use the coordinates $x^\alpha = (x^0, x^1, x^2, x^3)$, the metric

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1,\ g_{\alpha\beta} = 0,\ \alpha \neq \beta,$$

the representation of the γ matrices

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix},\quad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\quad \gamma^5 = -\mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3,$$

and units $c = \hbar = 1$.

The canonical equations of this field can be written in the form

$$\begin{aligned}
(\mathrm{i}\partial_\nu\,\gamma^\nu - \varkappa)\,\Psi_I &= 0,\\
(\mathrm{i}\partial_\nu\,\gamma^\nu + \varkappa)\,\Psi_{II} &= 0,\\
-\mathrm{i}\partial_\nu\,\bar{\Psi}_I\,\gamma^\nu - \varkappa\bar{\Psi}_I &= 0,\\
-\mathrm{i}\partial_\nu\,\bar{\Psi}_{II}\,\gamma^\nu + \varkappa\bar{\Psi}_{II} &= 0.
\end{aligned} \tag{2}$$

These are Dirac equations with positive and negative mass terms.

The interaction of this field with the electromagnetic field one introduces on the usual way, i. e. by the substitution

$$\mathrm{i}\partial_\alpha \to \mathrm{i}\partial_\alpha - eA_\alpha,\ e = -|e|. \tag{3}$$

The Lagrange's density (1) then becomes

$$\mathscr{L}_{free\ D.f.} + \mathscr{L}_{int} = \varkappa\{[(-\mathrm{i}\partial_\alpha - eA_\alpha)\,\bar{\Phi}\,\gamma^\alpha]\,[(\mathrm{i}\partial_\beta - eA_\beta)\,\gamma^\beta\,\Phi] - \varkappa^2\,\bar{\Phi}\,\Phi\} \tag{4}$$

and the total Lagrange's density is

$$\mathscr{L}_{free\ D.f.} + \mathscr{L}_{int} + \mathscr{L}_{em} \tag{5}$$

where

$$\mathscr{L}_{em} = -\frac{1}{16\pi}(\partial_\alpha A_\beta - \partial_\beta A_\alpha)(\partial^\alpha A^\beta - \partial^\beta A^\alpha). \tag{6}$$

The canonical equations of the field (Φ, Φ^+) are now

$$[(\mathrm{i}\partial_\alpha - eA_\alpha)\,\gamma^\alpha - \varkappa]\,\Psi_I = 0,$$

$$[(\mathrm{i}\partial_\alpha - eA_\alpha)\,\gamma^\alpha + \varkappa]\,\Psi_{II} = 0,$$

$$(-\mathrm{i}\partial_\alpha - eA_\alpha)\,\overline{\Psi}_I\,\gamma^\alpha - \varkappa\overline{\Psi}_I = 0, \tag{7}$$

$$(-\mathrm{i}\partial_\alpha - eA_\alpha)\,\overline{\Psi}_{II}\,\gamma^\alpha + \varkappa\overline{\Psi}_{II} = 0,$$

and the Lagrange's equation for A^μ is

$$\partial_\alpha\,\partial^\alpha A^\mu = 4\pi j^\mu, \tag{8}$$

where

$$j^\mu = e\,(\overline{\Psi}_I\,\gamma^\mu\,\Psi_I - \overline{\Psi}_{II}\,\gamma^\mu\,\Psi_{II}). \tag{9}$$

In the presence of the external electromagnetic sources the additional term in $\mathscr{L}$ appears

$$-j_{s\alpha}\,A^\alpha, \tag{10}$$

where $j_{s\alpha}$ is the external source current with

$$\partial_\alpha j_s^\alpha = 0. \tag{11}$$

The Lagrange's equation for A^μ then reads

$$\partial_\alpha\,\partial^\alpha A^\mu = 4\pi\,(j^\mu + j_s^\mu). \tag{12}$$

The general solution of Eq. (12) is

$$A^\mu = A_h^\mu + A_p^\mu, \tag{13}$$

where A_h^μ is a general solution of the homogeneous equation and A_p^μ is a particular solution.

The relativistic hydrogen atom without self-interaction is defined by

$$A_h^\mu = 0,\ A_{pI}^\mu = 0,\ A_{pI_s}^\mu = \left(-\frac{Ze}{r} = A_0, 0, 0, 0\right) \tag{14}$$

and

$$\Psi_I(\vec{r}, t) = \mathrm{e}^{-\mathrm{i}k_0 t}\,\Psi_{Ik_0}(\vec{r}), \Psi_{II}(\vec{r}, t) = \mathrm{e}^{-\mathrm{i}k_0 t}\,\Psi_{II_{k_0}}(\vec{r}) \tag{15}$$

with $0 < k_0 < \varkappa$. The time-independent functions satisfy the equations

$$[k_0 + i\partial_j \alpha^j - eA_0 - \varkappa\gamma^0]\, \Psi_I(\vec{r}) = 0, \tag{16}$$
$$[k_0 + i\partial_j \alpha^j - eA_0 + \varkappa\gamma^0]\, \Psi_{II}(\vec{r}) = 0,$$

or written in the form

$$H\, \Psi(\vec{r}) = k_0\, \Psi(\vec{r}), \tag{17}$$

where

$$\Psi = \begin{pmatrix} \Psi_I \\ \Psi_{II} \end{pmatrix} \tag{18}$$

and

$$H = \begin{pmatrix} -i\partial_j \alpha^j + eA_0 + \varkappa\gamma^0 & 0 \\ 0 & -i\partial_j \alpha^j + eA_0 - \varkappa\gamma^0 \end{pmatrix}. \tag{19}$$

The normal solutions of this eigenvalue problem are*

$$\Psi_I(\vec{r}) = \begin{pmatrix} f_\varkappa(r)\; \Omega_{jlm} \\ i g_\varkappa(r)\; \Omega_{jl'm} \end{pmatrix}, \qquad l' = 2j - l, \tag{20}$$
$$\Psi_{II}(\vec{r}) = \begin{pmatrix} f_{-\varkappa}(r)\; \Omega_{jlm} \\ i g_{-\varkappa}(r)\; \Omega_{jl'm} \end{pmatrix},$$

where Ω_{jlm} are the spherical spinors:

$$\Omega_{l+\frac{1}{2},l,m} = \begin{pmatrix} \sqrt{\dfrac{j+m}{2j}}\; Y_{l,m-\frac{1}{2}} \\ \sqrt{\dfrac{j-m}{2j}}\; Y_{l,m+\frac{1}{2}} \end{pmatrix}, \quad \Omega_{l-\frac{1}{2},l,m} = \begin{pmatrix} -\sqrt{\dfrac{j-m+1}{2j+2}}\; Y_{l,m-\frac{1}{2}} \\ \sqrt{\dfrac{j+m+1}{2j+1}}\; Y_{l,m+\frac{1}{2}} \end{pmatrix} \tag{21}$$

$$\left(j = l + \frac{1}{2}\right) \qquad\qquad \left(j = l - \frac{1}{2}\right)$$

and

$$f_\varkappa(r) = A_{n_r k}(\varkappa + k_0)\, e^{-\frac{\varrho}{2}}\, \varrho^{\gamma-1}\, [F(-n_r, 2\gamma+1, \varrho) + \\ + \frac{n_r}{k - \dfrac{Z\alpha\varkappa}{\lambda}} F(1 - n_r, 2\gamma + 1, \varrho)], \tag{22}$$

* The factor i is taken in front of $g(r)$ instead of i^{1+l-l}, as in Ref. 2 in order to avoid some misunderstanding in the usage of the spherical harmonics. In this article we use the spherical harmonics as in Ref. 3.

$$g_{\varkappa}(r) = -A_{n_r k}\sqrt{\varkappa^2 - k_0^2}\, e^{-\frac{\varrho}{2}}\, \varrho^{\gamma-1}\, [F(-n_r, 2\gamma+1, \varrho) - \\ - \frac{n_r}{k - \frac{Z\alpha\varkappa}{\lambda}} F(1-n_r, 2\gamma+1, \varrho)] \tag{23}$$

$$n_r = \begin{cases} 0, 1, 2, 3, \ldots, & k < 0 \\ 1, 2, 3, \ldots, & k > 0, \end{cases}$$

$$f_{-\varkappa}(r) = A_{n_r k}(k_0 - \varkappa)\, e^{-\frac{\varrho}{2}}\, \varrho^{\gamma-1}\, [F(-n_r, 2\gamma+1, \varrho) + \\ + \frac{n_r}{k + \frac{Z\alpha\varkappa}{\lambda}} F(1-n_r, 2\gamma+1, \varrho)], \tag{24}$$

$$(n_r \neq 0),$$

$$g_{-\varkappa}(r) = -A_{n_r k}\sqrt{\varkappa^2 - k_0^2}\, e^{-\frac{\varrho}{2}}\, \varrho^{\gamma-1}\, [F(-n_r, 2\gamma+1, \varrho) - \\ - \frac{n_r}{k + \frac{Z\alpha\varkappa}{\lambda}} F(1-n_r, 2\gamma+1, \varrho)], \quad (n_r \neq 0), \tag{25}$$

$$(n_r = 0,\ f_{-\varkappa}(r) = g_{-\varkappa}(r) = 0),$$

$$k = \begin{cases} -\left(j + \frac{1}{2}\right) = -(l+1), & j = l + \frac{1}{2} \\ \left(j + \frac{1}{2}\right) = l, & j = l - \frac{1}{2}, \end{cases} \tag{26}$$

$\left(k = \pm 1, \pm 2, \pm 3, \ldots;\right.$ the positive values correspond to $j = l - \frac{1}{2}$ and negative to $\left. j = l + \frac{1}{2}\right)$,

$$k_0 = \left\{\frac{\varkappa^2}{1 + \left(\frac{Z\alpha}{\gamma + n_r}\right)^2}\right\}^{1/2}, \quad \alpha = e^2, \tag{27}$$

$$\lambda = \sqrt{\varkappa^2 - k_0^2}, \quad \gamma = \sqrt{k^2 - Z^2\alpha^2}.$$

The functions (18) are orthonormal according to

$$\int \Psi^{+}_{n_r kjlm}\, \tau_+\, \Psi_{n_r' k' j' l' m'}\, \mathrm{d}^3 x = \delta_{n_r n_r'}\, \delta_{kk'}\, \delta_{jj'}\, \delta_{ll'}\, \delta_{mm'}, \tag{28}$$

where

$$\tau_+ = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{29}$$

3. *Hydrogen atom in a constant magnetic field*

For a constant magnetic field

$$\vec{A} = \frac{1}{2} \vec{\mathscr{H}} \times \vec{r}. \tag{30}$$

Taking the z-axis in the direction of the magnetic field the expression (30) becomes

$$\vec{A} = \frac{\mathscr{H}}{2} r \sin \Theta \, (-\vec{i} \sin \varphi + \vec{j} \cos \varphi). \tag{31}$$

The canonical equations of the hydrogen atom in the constant magnetic field, according to (7) and (15), are

$$[k_0 + \mathrm{i}\partial_j a^j - eA_0 - eA_j a^j - \varkappa \gamma^0]\, \Psi_I (\vec{r}) = 0, \tag{32}$$

$$[k_0 + \mathrm{i}\partial_j a^j - eA_0 - eA_j a^j + \varkappa \gamma^0)\, \Psi_{II} (\vec{r}) = 0,$$

where $\vec{A}$ is given by (31). Using the operator (19) it can be written in the form

$$(H_0 + h)\, \Psi = k_0\, \Psi, \tag{33}$$

where H_0 is the operator (19) and

$$h = \begin{pmatrix} -e\vec{A}\,\vec{a} & 0 \\ 0 & -e\vec{A}\,\vec{a} \end{pmatrix} =$$

$$= -e \frac{\mathrm{i}\mathscr{H}}{2} r \sin \Theta \begin{pmatrix} 0 & \hat{d} & 0 & 0 \\ \hat{d} & 0 & 0 & 0 \\ 0 & 0 & 0 & \hat{d} \\ 0 & 0 & \hat{d} & 0 \end{pmatrix}, \tag{34}$$

where

$$\hat{d} = \begin{pmatrix} 0 & -e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix}. \tag{35}$$

We consider a weak field and apply the perturbation method. Writing

$$\Psi = \Psi^{(0)} + \Psi^{(1)} + \ldots, \tag{36}$$

$$k_0 = k_0^{(0)} + k_0^{(1)} + \ldots \tag{37}$$

Eq. (33) goes over into

$$H_0\, \Psi^{(0)} = k_0^{(0)}\, \Psi^{(0)}, \tag{38}$$

$$H_0\, \Psi^{(1)} + h\Psi^{(0)} = k_0^{(0)}\, \Psi^{(1)} + k_0^{(1)}\, \Psi^{(0)}, \tag{39}$$

$$\vdots$$

The first order correction to k_0 for nondegenerate states is

$$\int \Psi_l^{(0)+}\, \tau_+\, h\, \Psi_l^{(0)}\, \mathrm{d}^3x = k_{0l}^{(1)}, \tag{40}$$

where l stands for indices of the nonperturbed states.

For degenerate states the correction is given by

$$\begin{vmatrix} \langle l1\, |\tau_+ h|\, l1\rangle - k_0^{(1)} & \langle l1\, |\tau_+ h|\, l2\rangle \ldots & \langle l1\, |\tau_+ h|\, lf_l\rangle \\ \vdots & & \\ \langle lf_l\, |\tau_+ h|\, l1\rangle & \ldots & \langle lf_l\, |\tau_+ h|\, lf_l\rangle - k_0^{(1)} \end{vmatrix} = 0, \tag{41}$$

where f_l denotes degree of the degeneracy. The higher order corrections we do not consider here.

4. *Energy splitting of the ground and first excited states*

The general matrix element of the perturbation, according to (18, 20, 22—25), is

$$\langle n_r k\, j\, l\, m\, |\tau_+\, h|\, n_r'\, k'\, j'\, l'\, m'\rangle = \frac{e\mathscr{H}}{2} \int r^3 \sin^2\Theta\, \mathrm{d}r\, \mathrm{d}\Theta\, \mathrm{d}\varphi\, \{(f_{\varkappa\, n_r k}\, g_{\varkappa\, n_r' k'} -$$

$$- f_{-\varkappa\, n_r k}\, g_{-\varkappa n_r' k'})\, \Omega_{jlm}^{+}\, \hat{\mathrm{d}}\Omega_{j'(l')m'} - \tag{42}$$

$$- (g_{\varkappa\, n_r' k}\, f_{\varkappa\, n_r' k'} - g_{-\varkappa n_r k}\, f_{-\varkappa\, n_r' k'})\, \Omega_{jl'm}^{+}\, \hat{\mathrm{d}}\Omega_{j' l' m'}\}.$$

Two states belongs to the ground state energy of the unperturbed hydrogen atom:

$$|n_r k j l m\rangle = \begin{cases} |0 -1 \frac{1}{2} 0 \frac{1}{2}\rangle \\ |0 -1 \frac{1}{2} 0 -\frac{1}{2}\rangle. \end{cases} \tag{43}$$

Since for these states $\Psi_{II} = 0$ the first order energy correction $k_0^{(1)}$ is the same as in the standard theory[4]. Indeed,

$$\langle 0 - 1 \frac{1}{2} 0 \pm \frac{1}{2} |\tau_+ h| 0 - 1 \frac{1}{2} 0 \pm \frac{1}{2}\rangle =$$

$$= \pm \frac{2}{3} e \mathscr{H} \int r^3 f_{\varkappa 0-1} g_{\varkappa 0-1} \, \mathrm{d}r = \mp \frac{1}{6} e \mathscr{H} \frac{2\gamma + 1}{\varkappa} \quad ,$$

$$\langle 0 - 1 \frac{1}{2} 0 \frac{1}{2} |\tau_+ h| 0 - 1 \frac{1}{2} 0 - \frac{1}{2}\rangle = \tag{44}$$

$$= \langle 0 - 1 \frac{1}{2} 0 - \frac{1}{2} | \tau_+ h | 0 - 1 \frac{1}{2} 0 \frac{1}{2}\rangle = 0$$

and

$$k_0^{(1)} = \mp \frac{1}{6} e \mathscr{H} \frac{2\gamma + 1}{\varkappa}, \tag{45}$$

where $\gamma = \sqrt{1 - Z^2 \alpha^2}$.

Making expansion with respect to $Z^2 \alpha^2$ and keeping the terms up to the first order one gets

(46)

Using the number $m \left(= \pm \frac{1}{2}\right)$, the zero order term is

$$k_0^{(1)} = -\frac{e}{\varkappa} m \mathscr{H} \tag{47}$$

in agreement with the standard theory and experiment[4,5,6].

The first higher energy level contains four states:

$$n_r = 1, k = \left\langle \begin{array}{l} -1\ j = \frac{1}{2}\ l = 0\ m = \left\langle \begin{array}{l} \frac{1}{2} \\ -\frac{1}{2} \end{array} \right. \\ +1\ j = \frac{1}{2}\ l = 1\ m = \left\langle \begin{array}{l} -\frac{1}{2} \\ \frac{1}{2} \end{array} \right. \end{array} \right. . \tag{48}$$

The matrix elements of the perturbation for these states are

$$\langle 1\ -1\ \frac{1}{2}\ 0\ \frac{1}{2}\ |\tau_+ h|\ 1\ -1\ \frac{1}{2}\ 0\ \frac{1}{2}\rangle =$$

$$= -\langle 1\ -1\ \frac{1}{2}\ 0\ -\frac{1}{2}\ |\tau_+ h|\ 1\ -1\ \frac{1}{2}\ 0\ -\frac{1}{2}\rangle =$$

$$= \frac{2e\mathscr{H}}{3}\frac{1}{4\varkappa}\,\frac{-\left[\frac{2\gamma+3}{2\gamma+1}-\frac{2\gamma+1}{(1+z)^2}\right]+\frac{k_0-\varkappa}{k_0+\varkappa}\left[\frac{2\gamma+3}{2\gamma+1}-\frac{2\gamma+1}{(1-z)^2}\right]}{\frac{1}{2\gamma+1}+\frac{1}{(1+z)^2}+\frac{k_0-\varkappa}{k_0+\varkappa}\left[\frac{1}{2\gamma+1}+\frac{1}{(1-z)^2}\right]} =$$

$$= -\frac{e\mathscr{H}}{2\varkappa}\frac{1}{3}(z+1)\frac{1+\frac{z-2}{z+2}}{1+\frac{z-2}{z+2}\frac{z+1}{z-1}} = a, \tag{49}$$

$$\langle 1\ 1\ \frac{1}{2}\ 1\ \frac{1}{2}\ |\tau_+ h|\ 1\ 1\ \frac{1}{2}\ 1\ \frac{1}{2}\rangle = -\langle 1\ 1\ \frac{1}{2}\ 1\ -\frac{1}{2}\ |\tau_+ h|\ 1\ 1\ \frac{1}{2}\ 1\ -\frac{1}{2}\rangle =$$

$$= -\frac{2e\mathscr{H}}{3}\frac{1}{4\varkappa}\,\frac{-\left[\frac{2\gamma+3}{2\gamma+1}-\frac{2\gamma+1}{(1-z)^2}\right]+\frac{k_0-\varkappa}{k_0+\varkappa}\left[\frac{2\gamma+3}{2\gamma+1}-\frac{2\gamma+1}{(1+z)^2}\right]}{\frac{1}{2\gamma+1}+\frac{1}{(z-1)^2}+\frac{k_0-\varkappa}{k_0+\varkappa}\left[\frac{1}{2\gamma+1}+\frac{1}{(z+1)^2}\right]} =$$

$$= -\frac{e\mathscr{H}}{2\varkappa}\frac{1}{3}(z-1)\frac{1+\frac{z-2}{z+2}}{1+\frac{z-2}{z+2}\frac{z-1}{z+2}} = b, \tag{50}$$

where

$$z = \frac{Z\alpha\varkappa}{\lambda} \text{ and } 2\gamma + 1 = z^2 - 1, \frac{k_0}{\varkappa} = \frac{z}{2}.$$

All other matrix elements are zero.

The determinant (41) is then

$$\begin{vmatrix} a - k_0^{(1)} & 0 & 0 & 0 \\ 0 & -a - k_0^{(1)} & 0 & 0 \\ 0 & 0 & b - k_0^{(1)} & 0 \\ 0 & 0 & 0 & -b - k_0^{(1)} \end{vmatrix} = 0. \tag{51}$$

Consequently

$$k_0^{(1)} = \pm a, \pm b. \tag{52}$$

The corresponding energy correction in the standard theory one gets from the determinant (41) with the matrix elements

$$\langle n_r kjlm \,|h|\, n_r kjlm \rangle \tag{53}$$

where the state functions are given by

$$\Psi_{n_r kjlm} = \begin{pmatrix} f(r)\,\Omega_{jlm} \\ \mathrm{i}g(r)\,\Omega_{jl'm} \end{pmatrix} \tag{54}$$

with the normalization

$$\int (f^2 + g^2)\, r^2 \mathrm{d}r = 1. \tag{55}$$

These corrections are

$$k_0^{(1)} = \pm a_{st} \pm b_{st}, \tag{56}$$

where

$$a_{st} = -\frac{e\mathscr{H}}{2\varkappa}\frac{1}{3}(z + 1), \tag{57}$$

$$b_{st} = -\frac{e\mathscr{H}}{2\varkappa}\frac{1}{3}(z - 1). \tag{58}$$

Making expansion of a and b with respect to $Z^2\alpha^2$ and keeping the first two elements one gets

$$\begin{aligned} a &= -\frac{e\mathscr{H}}{2\varkappa}\left(1 + \frac{1}{24} Z^2\alpha^2\right), \\ b &= -\frac{e\mathscr{H}}{6\varkappa}\left(1 - \frac{7}{24} Z^2\alpha^2\right). \end{aligned} \tag{59}$$

The corresponding expressions in the standard theory are

$$\begin{aligned} a_{st} &= -\frac{e\mathscr{H}}{2\varkappa}\left(1 - \frac{1}{12} Z^2\alpha^2\right), \\ b_{st} &= -\frac{e\mathscr{H}}{6}\left(1 - \frac{1}{4} Z^2\alpha^2\right). \end{aligned} \tag{60}$$

The difference between the new and the standard theory is then

$$\Delta a = a - a_{st} \doteq -\frac{e\mathcal{H}}{2\varkappa}\frac{Z^2\alpha^2}{8},$$

$$\Delta b = b - b_{st} = \frac{e\mathcal{H}}{6\varkappa}\frac{Z^2\alpha^2}{24}. \qquad (61)$$

5. *Conclusions*

Eqs. (61) show that the first differencies between the new and the standard theory appear in terms $Z^2\alpha^2$. The differencies in the space probability distribution of the new and the standard theory begin also with term $Z^2\alpha^2$. These results are therefore consistent.

The experimental verification of these differencies is difficult at the present time. The estimation of the differencies (61) one may get on the following way. Due to the first order approximation the magnetic field strength is restricted to the energy effects much less then the energy difference of the 2 $P_{3/2}$ and 2 $P_{1/2}$ states. The corresponding frequency is 10 969.13 $\pm$ 0.10 MHz[7,8]. Therefore, $\frac{e\mathcal{H}}{2mc} \leqslant 10\,969$ MHz. For the hydrogen atom $Z^2\alpha^2 \approx 5 \cdot 10^{-5}$. The corresponding frequency differencies (61) are then much less then 0.06 MHz. From here it follows that the difference (61) are within experimental errors. Thus, one come to the conclusion that the difference between the new and standard theory of the Zeeman effect of the hydrogen atom exists but can not be measured at the present time.

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ZEEMANOV EFEKT KOD VODIKOVOG ATOMA U NOVOJ TEORIJI DIRACOVOG POLJA

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Razmatran je utjecaj slabog i konstantnog magnetskog polja na energetske nivoe relativističkog vodikovog atoma u novoj teoriji Diracovog polja. Proračunate su nove energije u prvoj aproksimaciji osnovnog i prvog pobuđenog stanja. Nađene su razlike u odnosu na standardnu teoriju koje počinju sa članom reda $Z^2\alpha^2$. Ove razlike su manje od 0.06 MHz, pa se nalaze u sadašnjem vremenu unutar eksperimentalnih pogrešaka.