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STARK EFFECT AT THE HYDROGEN ATOM IN THE NEW DIRAC FIELD THEORY*

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The weak constant electric field effect at the hydrogen atom is considered in the new Dirac field theory. The difference from the standard theory is found. They begin with the $Z^2\alpha^2$ term.

1. Introduction

The new Dirac field theory[1] applied to the relativistic hydrogen atom[2] gives different the electron space probability distribution from the standard theory. In Ref. 3 we have analysed the influence of this difference on the Zeeman effect. In this article we investigate the same influence on the Stark effect.

A short presentation of the new relativistic hydrogen atom theory is given in Section 2. The interaction with a constant electric field and corresponding perturbation method is considered in Section 3. The energy splitting of the ground state and the first higher states ($2S_{1/2}$, $2P_{1/2}$) is evaluated in Section 4. Conclusions are given in Section 5.

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2. *The new relativistic hydrogen atom theory*

The new Dirac field is defined by the Lagrangian density[1)]

$$\mathscr{L} = \varkappa\,[(-\,\mathrm{i}\,\partial_\alpha\,\bar{\Phi}\gamma^\alpha)\,(\mathrm{i}\,\partial_\beta\,\gamma^\beta\,\Phi) - \varkappa^2\,\bar{\Phi}\Phi], \tag{1}$$

where Φ is a bispinor field and $\varkappa$ is a constant. We use the coordinates $x^\alpha = (x^0, x^1, x^2, x^3)$, the metric

$$g_{00} = -\,g_{11} = -\,g_{22} = -\,g_{33} = 1,\ g_{\alpha\beta} = 0,\ \alpha \neq \beta, \tag{2}$$

the representation of the γ matrices

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix},\quad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\quad \gamma^5 = -\,\mathrm{i}\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3, \tag{3}$$

and units $c = \hbar = 1$.

The new Dirac field interacting with external electromagnetic field is defined by the substitution

$$\mathrm{i}\,\partial_\alpha \to \mathrm{i}\,\partial_\alpha - eA_\alpha,\ e = -\,|e|, \tag{4}$$

and the corresponding Lagrange's density is

$$\mathscr{L} = \varkappa\,\{[(-\,\mathrm{i}\,\partial_\alpha - eA_\alpha)\,\bar{\Phi}\gamma^\alpha]\,[(\mathrm{i}\,\partial_\beta - eA_\beta)\,\gamma^\beta\,\Phi] - \varkappa^2\,\bar{\Phi}\Phi\} + \mathscr{L}_{em} \tag{5}$$

where

$$\mathscr{L}_{em} = -\,\frac{1}{16\pi}\,(\partial_\alpha A_\beta - \partial_\beta A_\alpha)\,(\partial^\alpha A^\beta - \partial^\beta A^\alpha). \tag{6}$$

The corresponding canonical equations are

$$\begin{aligned}
&[(\mathrm{i}\,\partial_\alpha - eA_\alpha)\,\gamma^\alpha - \varkappa]\,\Psi_I = 0,\\
&[(\mathrm{i}\,\partial_\alpha - eA_\alpha)\,\gamma^\alpha + \varkappa]\,\Psi_{II} = 0,\\
&(-\,\mathrm{i}\,\partial_\alpha - eA_\alpha)\,\bar{\Psi}_I\,\gamma^\alpha - \varkappa\bar{\Psi}_I = 0,\\
&(-\,\mathrm{i}\,\partial_\alpha - eA_\alpha)\,\bar{\Psi}_{II}\,\gamma^\alpha + \varkappa\bar{\Psi}_{II} = 0.
\end{aligned} \tag{7}$$

The stationary states of the relativistic hydrogen atom are given by[2)]

$$\begin{aligned}
&[k_0 + \mathrm{i}\,\partial_j\,\alpha^j - eA_0 - \varkappa\gamma^0]\,\Psi_I(\vec{r}) = 0,\\
&[k_0 + \mathrm{i}\,\partial_j\,\alpha^j - eA_0 + \varkappa\gamma^0]\,\Psi_{II}(\vec{r}) = 0
\end{aligned} \tag{8}$$

or written in the form

$$H\Psi(\vec{r}) = k_0 \Psi(\vec{r}),$$

where

$$\Psi = \begin{pmatrix} \Psi_I \\ \Psi_{II} \end{pmatrix}$$

$$H = \begin{pmatrix} -\mathrm{i}\,\partial_j \alpha^j + eA_0 + \varkappa\gamma^0 & 0 \\ 0 & -\mathrm{i}\,\partial_j \alpha^j + eA_0 - \varkappa\gamma^0 \end{pmatrix} \tag{9}$$

and $A_0 = -\dfrac{Ze}{r}$.

The normal solutions we take in this article as in Ref. 3.

3. *The hydrogen atom in constant electric field*

For a constant electric field $\mathscr{E}$ oriented in z-direction

$$A_0 = -\frac{Ze}{r} - \mathscr{E}z = -\frac{Ze}{r} - \mathscr{E}r\cos\vartheta. \tag{10}$$

The canonical equation of the hydrogen atom, according to (7) and (8), are then

$$\begin{aligned} [k_0 + \mathrm{i}\,\partial_j \alpha^j + \frac{Ze^2}{r} + e\mathscr{E}r\cos\vartheta - \varkappa]\,\Psi_I(\vec{r}) = 0, \\ [k_0 + \mathrm{i}\,\partial_j \alpha^j + \frac{Ze^2}{r} + e\mathscr{E}r\cos\vartheta + \varkappa]\,\Psi_{II}(\vec{r}) = 0. \end{aligned} \tag{11}$$

Using the operator (9) it can be written in the form

$$(H_0 + h)\,\Psi = k_0 \Psi, \tag{12}$$

where H_0 is the operator (9) and

$$h = \begin{pmatrix} -e\mathscr{E}r\cos\vartheta & 0 \\ 0 & -e\mathscr{E}r\cos\vartheta \end{pmatrix}. \tag{13}$$

We consider a weak field and apply the perturbation method. Writing

$$\begin{aligned} \Psi = \Psi^{(0)} + \Psi^{(1)} + \ldots, \\ k_0 = k_0^{(0)} + k_0^{(1)} + \ldots \end{aligned} \tag{14}$$

Eq. (12) goes over into

$$H_0 \Psi^{(0)} = k_0^{(0)} \Psi^{(0)}. \tag{15}$$

$$H_0 \Psi^{(1)} + h\Psi_l^{(0)} = k_0^{(0)} \Psi^{(1)} + k_0^{(1)} \Psi^{(0)}, \tag{16}$$

$$\vdots \quad \cdot$$

The first order correction to k_0 for nondegenerate states is

$$\int \Psi_l^{(0)+} \tau_+ h\Psi_l^{(0)} \, \mathrm{d}^3 x = k_{0l}^{(1)}, \tag{17}$$

where l stands for indices of the nonperturbed states.

For degenerate states the corrections is given by

$$\begin{vmatrix} \langle l1|\tau_+ h|l1\rangle - k_0^{(1)} & \langle l1|\tau_+ h|l2\rangle \ldots \langle l1|\tau_+ h|lf_l\rangle \\ \vdots & \\ \langle lf_l|\tau_+ h|l1\rangle \ldots & \langle lf_l|\tau_+ h|lf_l\rangle - k_0^{(1)} \end{vmatrix} = 0, \tag{18}$$

where f_l denotes degree of degeneracy. The higher order corrections we do not consider here.

4. *Energy splitting of the ground and first excited states*

The general matrix element of the perturbation, according to Ref. 3 and (13) is

$$\begin{aligned} \langle n_r kjlm|\tau_+ h|n_r' k' j' l' m'\rangle = & = e\mathscr{E} \int r^3 \cos\vartheta \sin\vartheta \, \mathrm{d}r \, \mathrm{d}\vartheta \, \mathrm{d}\varphi. \\ & \{(f^*_{\varkappa n_r k} f_{\varkappa n_r' k'} - f^*_{-\varkappa n_r k} f_{-\varkappa n_r' k'}) \Omega^+_{jlm} \Omega_{j'l'm'} + \\ & + (g^*_{\varkappa n_r k} g_{\varkappa n_r' k'} - g^*_{-\varkappa n_r k} g_{-\varkappa n_r' k'}) \Omega^+_{jl'm} \Omega_{j'l''m'}\}. \end{aligned} \tag{19}$$

Two states belong to the ground state energy of the unperturbed atom:

$$|n_r kjlm\rangle = \begin{cases} |0 \; -1 \; \frac{1}{2} \; 0 \; \frac{1}{2}\rangle. \\ |0 \; -1 \; \frac{1}{2} \; 0 \; -\frac{1}{2}\rangle. \end{cases} \tag{20}$$

Since for these states $\Psi_{II} = 0$ the energy correction due to the perturbation is the same as in the standard theory. As it is well known there are no the first order energy correction. Indeed,

$$\langle 0\,-1\,\tfrac{1}{2}\,0\,\pm\tfrac{1}{2}\,|\,\tau_+ h|\,0\,-1\,\tfrac{1}{2}\,0\,\pm\tfrac{1}{2}\rangle = \langle 0\,-1\,\tfrac{1}{2}\,0\,\tfrac{1}{2}\,|\,\tau_+ h\,|$$

$$|\,0{-}1\tfrac{1}{2}\,0\,-\tfrac{1}{2}\rangle = \langle 0\,-1\,\tfrac{1}{2}\,0\,-\tfrac{1}{2}\,|\,\tau_+ h|\,0{-}1\,\tfrac{1}{2}\,0\,\tfrac{1}{2}\rangle = 0 \qquad (21)$$

because of

$$\int \cos\vartheta \sin\vartheta\, d\vartheta\, d\varphi\, \Omega^{\dagger}_{\frac{1}{2}0\pm\frac{1}{2}}\, \Omega_{\frac{1}{2}1\pm\frac{1}{2}} = \int \cos\vartheta \sin\vartheta\, d\vartheta\, d\varphi\, \Omega^{\dagger}_{\frac{1}{2}0\frac{1}{2}}\, \Omega_{\frac{1}{2}1-\frac{1}{2}} =$$

$$= \int \cos\vartheta \sin\vartheta\, d\vartheta\, d\varphi\, \Omega^{\dagger}_{\frac{1}{2}0-\frac{1}{2}}\, \Omega_{\frac{1}{2}1\frac{1}{2}} = 0. \qquad (22)$$

The first higher energy level contains four states:

$$n_r = 1,\ k = \begin{cases} -1\ j=\frac{1}{2} & l=1\ m=\begin{cases} \frac{1}{2} \\ -\frac{1}{2} \end{cases} \\ +1\ j=\frac{1}{2} & l=0\ m=\frac{1}{2} \end{cases}. \qquad (23)$$

The nonzero matrix elements of the perturbation for these states are

$$\langle 1\,-1\,\tfrac{1}{2}\,0\,\tfrac{1}{2}|\,\tau_+ h\,|\,1\,1\,\tfrac{1}{2}\,1\,\tfrac{1}{2}\rangle =$$

$$= -\,e\mathscr{E}\left(-\frac{1}{4\varkappa Za}\right)\sqrt{z^2-1}\,z^2\,\frac{1-\dfrac{2-z}{2+z}}{\left\{1-2\dfrac{2-z}{2+z}\dfrac{z^2+1}{z^2-1}+\left(\dfrac{2-z}{2+z}\right)^2\right\}^{1/2}} = a, \qquad (24)$$

$$\langle 1\,1\,\tfrac{1}{2}\,1\,\tfrac{1}{2}\,|\,\tau_+ h\,|\,1\,-1\,\tfrac{1}{2}\,0\,\tfrac{1}{2}\rangle = a,$$

$$\langle 1\,-1\,\tfrac{1}{2}\,0\,-\tfrac{1}{2}\,|\,\tau_+ h\,|\,1\,1\,\tfrac{1}{2}\,1\,-\tfrac{1}{2}\rangle = -\,a, \qquad (25)$$

$$\langle 1\,1\,\tfrac{1}{2}\,1\,-\tfrac{1}{2}\,|\tau_+ h|\,1\,-1\,\tfrac{1}{2}\,0\,-\tfrac{1}{2}\rangle = -\,a$$

where

$$\gamma = \sqrt{1-Z^2\alpha^2}, \quad \lambda = \sqrt{\varkappa^2 - k_0^2}, \quad k_0 = \frac{\varkappa}{\sqrt{1+\dfrac{Z^2\alpha^2}{(\gamma+1)^2}}},$$

$$z = \frac{Z\alpha\varkappa}{\lambda}.$$

Eq. (18) is then

$$\begin{vmatrix} -k_0^{(1)} & 0 & a & 0 \\ 0 & -k_0^{(1)} & 0 & -a \\ a & 0 & -k_0^{(1)} & 0 \\ 0 & -a & 0 & -k_0^{(1)} \end{vmatrix} = 0. \tag{26}$$

From here one finds

$$k_0^{(1)} = \pm a. \tag{27}$$

In the standard theory[4,5] ($a \to a_{st}$)

$$a_{st} = -e\mathscr{E}\left(-\frac{1}{4\varkappa Z a}\right)\sqrt{z^2-1}\,z^2. \tag{28}$$

Making expansion of a and a_{st} with respect to $Z^2\alpha^2$ and keeping the first two elements one gets

$$a = \sqrt{3}\,e\mathscr{E}\frac{1}{\varkappa Z a}\left(1 - \frac{9}{24}Z^2\alpha^2\right), \tag{29}$$

$$a_{st} = \sqrt{3}\,e\mathscr{E}\frac{1}{\varkappa Z a}\left(1 - \frac{5}{12}Z^2\alpha^2\right). \tag{30}$$

The difference between the new and the standard theory is then

$$\Delta a = a - a_{st} = \sqrt{3}\,e\mathscr{E}\frac{1}{\varkappa Z a}\frac{1}{24}Z^2\alpha^2. \tag{31}$$

5. *Conclusions*

Eq. (31) shows that the first difference between the new and the standard theory of the Stark effect at the hydrogen atom appears in the terms $Z^2\alpha^2$. The new theory gives slightly larger this term. Consequently the energy splitting is larger.

The experimental verification of this difference is difficult as in the cas of the Zeeman effect[3]. Due to the first order approximation the electric field strenght is restricted to the energy effects much less then the energy difference of the $2P_{3/2}$ and $2P_{1/2}$ states. The corresponding frequency is $10\,969.13 \pm 0.10$ MHz[6,7]. For the hydrogen atom $Z^2\alpha^2 \doteq 5 \cdot 10^{-5}$. The difference Δa is then much less then 0.02 MHz and this is within experimental errors at the present time.

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STARKOV EFEKT KOD VODIKOVOG ATOMA U NOVOJ TEORIJI DIRACOVOG POLJA

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Razmatran je utjecaj slabog konstantnog električnog polja na energetske nivoe relativističkog vodikovog atoma u novoj teoriji Diracovog polja. Proračunate su nove energije u prvoj aproksimaciji osnovnog i prvog pobuđenog stanja. Nađeno je da su razlike u odnosu na standardnu teoriju manje od 10^{-6} relativističkih popravki energija.