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Subject review

Errors in Measuring Height Differences in Engineering

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Abstract: For classic geodetic instruments, sources of errors influencing the total measurement error were determined. In digital levels, the technique of comparing the current image of the graduated rod with the saved picture of the graduated rod is used to read the rod. This reading technique eliminates the subjective influence (operator's error), but an error occurs from comparing two images. Apart from this source of errors, several errors in rod reading are not present at classical levels. Experimental measurements with digital levels have proven that certain sources of errors, which other authors also processed, can be significant concerning the accuracy of measurements. For example, illumination at 45° or steeper angles can cause reading errors more than ten times the measurement error at the station. Or, that inhomogeneous illumination of some parts of the rod causes large measurement errors, up to a few millimetres. One source of error has not been considered so far - this is the error that arises from incorrect sighting on the rod. Experimental measurements confirmed that misalignment caused an error of the order of tenths of a millimeter, and which can be larger than the measurement error at the station. It was also confirmed that this error is not characteristic only for one type of digital level, but it is already being reported in all. This error can be greater than the mean error of measuring the height difference at the station, which is significant in engineering works where height/height difference is often measured or marked using only one station.

Keywords: bar-codes rod; engineering works; reading error; sighting error

1 INTRODUCTION

The development of Prof. Zetsche at Bonn (1966) can be considered as a forerunner of all digital levels. It is also necessary to refer to the research work of the Technical University Dresden in collaboration with Carl Zeiss, Jena (Germany). A development instrumentally based on the Zeiss Ni002 and using a linear CCD array with 1024 elements (pixels) was started in 1982.

The first digital level was produced by Wild in 1990. Two years later, the first precision digital level (Wild NA3000) and the invar bar-code rod followed. A couple of years later, other companies started producing digital levels of different accuracy (Topcon, Zeiss, Sokkia, ...). Each company uses its bar-code design so that the rods cannot be replaced between the instruments of different manufacturers. The technology and the optical-mechanical part in all systems are very similar. The digital levels are automatic levels with extended image acquisition and processing functions. The scheme of measurement procedure is shown in Fig. 1 [9].

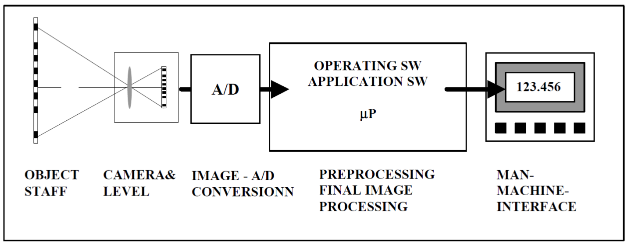

Figure 1 Data capture and processing [9]

Metrological confirmation of measurand and accessories is one of the preconditions for quality assurance of geodetic measurements, especially in high-precision engineering measurements (geodetic, civil engineering, mechanical works). For classic geodetic instruments, individual sources of errors were determined based on which the total measurement error was determined [7, 11]. At digital levels some error sources occur that are identical to those of classical ones, some of them are, in fact, completely new, and some of them have the same effect as in classical instruments, although their mode of occurrence is different.

Digital levels, compared to classical ones, utilize a different measurement technology and different rod, with a bar-code. The difference in the reading technology (image acquisition) eliminates the subjective influence - operator's error. However, an error arises from comparing two images (the current and saved image of the graduated rod). The manufacturers of digital levels do not give the size of the reading error as special data. They give only a total measurement error. The only way to get the size of the reading error is to determine it experimentally.

The difference between the classic and the bar-code rods is in the graduation itself - in the case of classic rods the partition is constant, and on the bar-code is variable. The base to which the partition is applied is the same because the accuracy measurements use an Invar tape. The same is the technique of separation - a black color is applied in the form of a dash of a rectangular shape. In the case of classical they are all the same width, and at the bar-code are variable. The difference in the manufacturing process of the code elements is that the technology is more modern in the case of bar-codes, i.e. the thickness of the code elements is smaller than for the classic ones.

Sometimes during measurement, the center of the graduated rod is not sighting due to the working speed and this can cause a measurement error. This is especially important for high-precision engineering work where spaces are often tightened and must work with short sights.

2 AN OVERVIEW OF LEVELLING ERROR THEORIES

2.1 The Basic Information of Digital Levels

Using digital levels, the reading on the bar-code rods is done by digital image processing so that the observer's eye is replaced by CCD detectors arranged in a row. The CCD detectors convert the image of the bar-code into an electrical signal that is further processed in a microprocessor. By signal processing, the reading on the rods and the length between the rods and the level are obtained.

The basic difference between manufacturers appears in the last step of the "digital image interpretation". Leica uses the cross-correlation method by comparing the known code sequence with the image. Topcon works with Fourier's transformation, and Zeiss and Sokkia use a geometric method.

With new digital level models, bar-code partition and signal processing remained the same, but hardware and software were completely changed. The image is obtained by a new, highly sensitive CCD linear sensor, which is sensitive to the visible part of the spectrum.

Efforts to automate the geometric levelling were made for a long time, i.e. that the rods readings automatically register. Spatial apartness of the level and rod caused significant problems in the realization of such automation. The digital levels are a combination of levels with automatic horizonting, a CCD camera, and an electronic processor for image processing. Modern development has resulted in digital levels which have the following constructive solutions:

- fully automated registration of measured values,
- digital display and control of measured and calculated values (the built-in computer system calculates and controls the height differences at each station, summarizes and stores data in the memory, and all data can be displayed on the screen),
- automatic correction of height differences due to curvature of the Earth, refraction, collimation error, or rod systematic errors, based on previously entered data in the level memory or entered using the keyboard.

Digital levels have some basic parts and functions of conventional levels, but they also have special features and extended capabilities. Electronic components are the basis of the overall function of these levels. The digital display of the reading value on the level screen allows for fast and transparent reading and control, but the biggest advantage is an automatic recording of measurements in the level memory. Automatic registration, in addition, to facilitating the observer, speeds up the process of registering because there is no manual interference in reading and writing.

Partial processing of measurement data on-site enables direct testing of results achieved. With the ability to control the data during measurement, subsequent measurements and re-entry to the field are prevented, significantly increasing the measurement's cost-effectiveness. Instrument errors can be stored to automatically correct the measurement results.

2.2 Basic Formulas for an Analysis of Levelling Accuracy

One of the first official theories concerning levelling error propagation was presented by Ch. Lallemand. He separated random and systematic influence in levelling measurements between two points. In 1912, the IUGG (International Union of Geodesy and Geophysics) accepted his theory and registered the official formulas for analyzing levelling accuracy.

In the analysis of the accuracy of levelling data, well-known formulas are applied, which are adapted to new levelling technologies. The basic formulas of classical theories of accuracy are based on the definition of the source of levelling errors, the representation of the components of random and systematic influences, and the theory of error propagation.

Lallemand differentiated the terms random and systematic error in levelling measurement and estimated the variable systematic errors from the cumulative frequency of two levelling routes. The estimation process was based on a hypothesis that assumes an upward or downward tendency of linear regression, possibly caused by the systematic effect. His theory of accuracy is based on the height differences in two-way levelling routes adjusted to the length of one kilometer of the levelling route. The reliable estimation of the accuracy of a one-kilometer-long levelling route is derived by using the law of error propagation of levelling errors. It is given by the quadratic mean of double levelling, known as unit standard deviation. Lallemand recalculated height differences in two-way levelling on kilometer differences by dividing by root squared of the quantity R, representing the length of the levelling route in km. Then the most used accuracy estimation of a one-kilometer-long levelling route follows from the law of error propagation and is given by the quadratic mean of double levelling. This accuracy characteristic is known as the unit standard deviation σ_0 and is given by the following formula [9]:

$$\sigma_0 = \frac{1}{2}\sqrt{\frac{1}{n_R}\sum \frac{\rho^2}{R}} \tag{1}$$

where ρ is the difference between height differences computed in each direct and reverse levelling section with length R given in kilometers.

Over time, criticisms related to the use of Lallemand's formulas appear. His main opponent was Wignall, whose theory consisted of determining the limited length of the levelling route, within which the unit standard deviation represented in Eq. (1) includes only the effect of random errors and short-period variable systematic errors.

According to Wignall's theory for analyzing the accuracy of precise levelling, deviations of cumulative height differences in two-way levelling are caused not only by systematic errors but also by random noise. Also, the influence of systematic errors defined by Lallemand is valid only within a certain distance (limited length) of the levelling route, beyond which they behave as variable systematic errors dispersed around the mean systematic error.

According to Wignall's theory, the levelling variance is expressed as the square root of the total variance:

$$\tau^2 = \eta^2 + \xi^2 \tag{2}$$

Random errors η in the levelling route increase in proportion to the number of stations, and the occurrence of systematic error ξ depends on the square of the number of stations. The behaviour of the random and systematic component varies in the frame of limit length, and beyond, it behaves as a constant. The systematic components can be determined from the levelling closures or using differences in the endpoints of the regression line, estimated from the cumulative height differences in bidirectional levelling.

Different types of errors occur during levelling. These errors can affect the accuracy and reliability of height measurements. These can arise from various sources, including instrumental imperfections, atmospheric conditions, temperature and pressure variations, human factors, and improper surveying techniques.

Instrumental errors are caused by imperfections or malfunctions in the levelling instrument itself. Examples include incorrect calibration, misalignment of the levelling bubble, or faulty compensator in an automatic level.

A human factor causes reading errors. At classic levels, reading errors occur when the observer misreads or misinterprets the measurements on the levelling rod. Parallax, where the observer's eye is not directly in line with the measurement, can lead to reading errors. With digital levels, the error of matching and interpreting the rod image is equivalent to this error with classic levels.

Collimation error (instrumental errors) in levelling occurs when the line of sight between the levelling instrument and the levelling rod is not perfectly horizontal. This can be caused by misalignment of the telescope or a tilted levelling rod.

Refraction (atmospheric conditions) refers to the bending of light as it passes through different mediums, such as air layers of different densities. This can cause the levelling line of sight to deviate from the horizontal plane, resulting in errors.

Other atmospheric conditions errors are temperature errors. Temperature changes can cause the levelling instrument or the levelling rod to expand or contract, leading to measurement errors. Thermal gradients in the air can also cause refraction and affect the accuracy of the levelling measurements.

Assessment of the accuracy of height difference measurements between benchmarks is performed according to developed and internationally accepted formulas. On the other hand, the assessment of the accuracy of height or height difference measurements in engineering works can vary significantly and should be approached very seriously. The specificity of work in engineering tasks, especially during the assembly of machine elements (machines, elevators, ...), lies in the fact that it is done with very short lines of sight and very often with only one station. In such cases, some sources of errors may come into play that can be eliminated or rendered insignificant in network measurements (due to their random nature).

In measurements between two benchmarks, the basic formula for assessing the accuracy of the height difference is [7, 11]:

$$\begin{aligned}(\sigma_H)^2_{\tau 0} &= \frac{n_h}{2n}\left[\frac{\sigma^2_{RM}}{2} + 2\sigma^2_Z + d^2_0\left(2\sigma^2_r + \sigma^2_O\right)\right] + \\ &+ H^2\left[\sigma^2_M + \gamma^2_{PNL} S\left(\Delta t\right)^2_{2S}\right]\end{aligned} \tag{3}$$

The formula for assessing the accuracy of the height difference in one levelling station is:

$$\begin{aligned}(\sigma_h)^2_{\tau} &= \frac{1}{2n}\left[\frac{\sigma^2_{RM}}{2} + d^2\left(2\sigma^2_r + \sigma^2_O\right)\right] + \\ &+ h^2\left(\sigma^2_M + \gamma^2_{PNL}\left(\Delta t\right)^2_{2SR}\right) + 2\sigma^2_Z\end{aligned} \tag{4}$$

where are:

n_h - number of stations between benchmarks, n - number of height difference measurements (for two-way measurement, or two height differences at the station, this number is 1), σ_{RM} - error of the working measure (error of the scale values of the rod and error of the scale values of the optical micrometer in classic levels), σ_Z - rounding error when reading, d_0 - the average length of sight, σ_r - error of refraction, σ_O - rod read error, σ_M - error of the mean meter of a pair of rods (or one rod), γ_{PNL} - temperature coefficient of expansion of the rods (the base of the code lines, most often the invar tape), $S\left(\Delta t\right)_{2S}$, $\left(\Delta t\right)_{2SR}$ - the difference in mean temperature during rod calibration and measurement, *H*, *h* - corresponding height differences.

Given that in engineering works during the assembly of certain elements, height differences/heights are mostly measured from a single station, only Eq. (4) will be considered. Additionally, only measurements with digital levels will be considered, as they are most frequently used.

Table 1 Values of certain sources of errors - general data

Error name	Label	Value
error of the working measure = error of the scale values of the rod	σ_{RM}	±12 μm
refraction	σ_r	±0,05″
rod read error	σ_0	±0,38″
error of the mean meter of a pair of rods	σ_M	±7 μm
rounding error when reading	σ_Z	±30 μm

Based on a large number of experiments conducted long ago, as well as studies that dealt with specific sources of errors in leveling (primarily related to rod calibration errors), general data on the values of specific sources of errors have been obtained. The values of specific sources of errors are shown in Tab. 1.

With digital levels, the error of the working measure is equal to the error of the division of the rod because there is no optical micrometer. Also, the reading error consists of the error of bringing the sight to a horizontal position (centering of the bubble in classic levels, i.e. the compensator error in digital levels) and the sighting error of the code elements on the rod. With digital levels, there is no sighting error, but there is an error of matching the current image of the bar with its image in the level's memory.

2.3 Calibration

Level calibration is crucial for ensuring the accuracy and reliability of measurements in geodetic, construction, mechanical, and other works. Some of the reasons why it is necessary to calibrate the level are:

1. Ensuring accuracy: over time, from use, levels can become inaccurate due to wear, mechanical damage, or environmental changes. Calibration helps confirm that the instrument can still measure height differences within acceptable tolerances.

2. Compliance with Standards: in many engineering-technical projects, compliance with international standards

and regulations is necessary. Regular calibration ensures that instruments meet these standards.

3. Minimizing errors: calibration identifies and corrects systematic errors that may arise due to improper operation of the instrument. This minimizes the risk of measurement errors, which is especially important for precision work.

4. Life extension: regular maintenance and calibration of the level can extend the instrument's life as problems can be detected and solved before they lead to major breakdowns.

5. Confidence in the results: when measurement results are critical for decision-making, as is the case in engineering projects and especially in mechanical engineering, users must have confidence in the accuracy of their instruments. Calibration provides that confidence because it verifies that the instrument is working correctly.

6. Quality control: in many companies and organizations, quality control requires regular calibration of equipment as part of quality assurance procedures. This is especially important in industries where accuracy and reliability are key.

7. Legal requirements: in almost all countries, laws and regulations require regular calibration of geodetic equipment. Failure to comply with these requirements may lead to legal consequences or invalidity of the measurement.

Calibration of a level includes checking and adjusting the instrument to ensure that all parts are functioning correctly and that the measurements are accurate. This is usually done in specialized laboratories or service centers that have the necessary equipment and expertise for precise adjustment of instruments.

The calibration of the level is carried out according to the international standard ISO 17123-2:2001. Before starting measurements, the operator should check whether the accuracy of the measuring equipment used for calibration is appropriate.

According to the standard, the level and its ancillary equipment must be adjusted following the methods specified in the manufacturer's manual and used with tripods and appropriate rods. Additionally, the conditions during calibration should match as closely as possible those expected during the actual measurements.

ISO 17123-2, in chapters 5 and 6, describes two different procedures - simplified and full. The procedure that corresponds to the required accuracy of the level is chosen from these two.

The simplified calibration procedure is usually intended for checking the accuracy of an optical level used for surface levelling, i.e. for levelling with unequal sight lengths, e.g. construction sites.

The procedure is based on determining the height difference between two points from the middle and from the end. The height difference obtained with approximately equidistant sights is adopted as the true value. The difference between the height difference measured with unequal sight lengths and the value measured between the same two points obtained by levelling with equal sight lengths indicates whether the level meets the permissible deviation (according to ISO 4463-1) for the intended measurement task.

"The full test procedure shall be adopted to determine the best achievable measure of the precision of a particular level and its ancillary equipment under field conditions and requires the adoption of equal sight lengths (maximum variation 10%). It is normally intended for field trials of levels to be used for more precise levelling, linear applications, and other major surveys, e.g. civil engineering." (ISO 17123-2:2001). For this test, sight lengths of 30 m are recommended. The full test procedure is based only on equal sight lengths. A displacement of the collimation axis of the level cannot be detected by this procedure. This test procedure is intended for determining the measure of precision in the use of a particular level and it is expressed in terms of the experimental standard deviation of a 1-km double-run levelling.

3 ANALYSIS OF EXPERIMENTAL DATA

3.1 Levels Calibration According to ISO

Before calibration, all the levels were checked before commencing levelling by the manufacturer's handbook to determine the collimation error. Checking was carried out at the calibration site of the accredited laboratory of the Association Transverzala, Belgrade. For all used levels collimation error was 3" (0,01 mm at 1 m) or less.

Each level then was calibrated according to the ISO 17123-2:2001 standard, the full test procedure. The configuration of the test line is according to the standard - the rods were at a distance of 30 m (Fig. 2).

Two sets of readings were carried out. The first set consists of twenty pairs of readings with each measurement comprising one backward reading, $x_{\mathrm{A},j}$, to the levelling rod at point A $(j=1, 2, \dots 20)$ and one forward reading, $x_{\mathrm{B},j}$, to the levelling rod at point B $(j=1, 2, \dots 20)$. Between each pair of readings, the level has been lifted and placed at a slightly different position. After ten measurements $(x_{\mathrm{A},1}, x_{\mathrm{B},1}, \dots x_{\mathrm{A},10}, x_{\mathrm{B},10})$, the backward and forward readings were reversed for another ten measurements $(x_{\mathrm{B},11}, x_{\mathrm{A},11}, \dots x_{\mathrm{B},20}, x_{\mathrm{A},20})$.

Then, the two levelling rods at the points A and B were interchanged and the procedure was repeated another twenty times in the same manner as described for the first set of measurements.

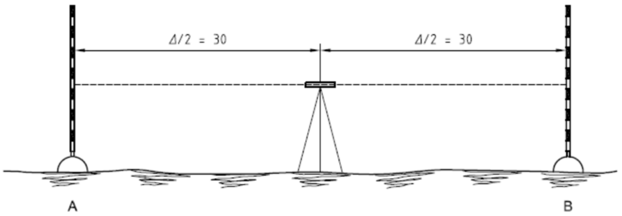

Figure 2 Configuration of the test line for the calibration in full test procedure

After measurement, it is calculated the difference, d_j, between the backward reading, $x_{\mathrm{A},j}$, and the forward reading, $x_{\mathrm{B},j}$, for all measurements:

$$d_j = x_{\mathrm{A},j} - x_{\mathrm{B},j}, \quad j=1, \dots 40 \tag{5}$$

Then the arithmetic means of the differences were calculated separately for each set of measurements:

$$\bar{d}_1 = \frac{\sum_{j=1}^{20} d_j}{20}; \quad \bar{d}_2 = \frac{\sum_{j=21}^{40} d_j}{20} \tag{6}$$

Difference of arithmetic means:

$$\delta = \bar{d}_1 - \bar{d}_2 \tag{7}$$

does not influence the experimental standard deviation, but it is an indicator of a difference in the zero-point offsets of the two levelling rods. According to ISO 17123-2, a statistical test is:

$$\begin{aligned} |\delta| &\le s_\delta \cdot t_{1-\alpha/2} \ (f) \\ |\delta| &\le s_\delta \cdot t_{0,875} \ (38) \end{aligned} \tag{8}$$

where *f* is the number of degrees of freedom and:

$$s_\delta = \frac{s}{\sqrt{10}} \tag{9}$$

is experimental standard deviation calculated from measurement - the Eq. (13). For all levels and rods, statistical tests indicate that there is no difference in the zero-point offsets of the two used levelling rods.

The next step was to calculate residuals of the corresponding measured height difference between the two levelling points A and B, as follows:

$$\begin{aligned} r_j &= \bar{d}_1 - d_j; \quad j = 1, \ ..., \ 20 \\ r_j &= \bar{d}_2 - d_j; \quad j = 21, \ ..., \ 40 \end{aligned} \tag{10}$$

As a control, for all levels, the sums of residuals calculated according to Eq. (10) were zero.

The sum of the squares of all residuals r_j is:

$$\sum_{j=1}^{40} r_j^2 = \sum_{j=1}^{20} r_j^2 + \sum_{j=21}^{40} r_j^2 \tag{11}$$

with the number of degrees of freedom:

$$f = 2 \cdot (20 - 1) = 38 \tag{12}$$

The experimental standard deviation, *s*, for a height difference at a distance of 60 m is:

$$s = \sqrt{\frac{\sum_{j=1}^{40} r_j^2}{f}} = \sqrt{\frac{\sum_{j=1}^{40} r_j^2}{38}} \tag{13}$$

The experimental standard deviation for 1 km double-run levelling is:

$$s_{ISO-LEV} = \frac{s}{\sqrt{2}} \cdot \sqrt{\frac{1000 \text{ m}}{60 \text{ m}}} = s \cdot 2,89 \tag{14}$$

After that, it was tested whether the experimental standard deviation, *s*, is less than or equal to a predetermined value, *σ*, given by manufacturers:

$$s \le \sigma \cdot \sqrt{\frac{\chi^2_{1-\alpha}\ (f)}{f}} = \sigma \cdot \sqrt{\frac{\chi^2_{0,95}\ (38)}{38}} \tag{15}$$

Value of $\chi^2_{0,95}\ (38)$ is 53,38 and, definitely is:

$$s \le \sigma \cdot \sqrt{\frac{53,38}{38}} = \sigma \cdot 1,19 \tag{16}$$

For all tested levels it was a value less than estimated, i.e. all levels satisfy the null hypothesis. This means that all levels achieved accuracy by the declared (by the manufacturers).

3.2 Determining the Instrument and Rod Error

Instrument errors are determined and corrected during their calibration.

Many authors have addressed the errors in rod graduations and the errors that occur when reading bar-code rods in their works. To confirm certain results related to the sources of height difference measurement errors, experimental measurements were conducted. The level used was a Leica DNA03, with an invar bar-code rod. It was decided to use this level because there were the most published papers on various measurement errors related to the level itself and its associated rod. Some of these tests were later repeated with other digital levels.

By determining the errors of bar-code rod graduations, for example [1, 4] and [12], but also in many other studies, the values of errors obtained were several micrometers smaller compared to traditional levels. For example, the error in determining the mean meter of a pair of conventional invar rods is 10 μm [11], while for bar-code rods it is 7 μm [7].

The testing of various influences of bar-code rod errors on the measuring error of digital levels has been carried out, namely:

- influence of the dash thickness when lighting the rod [2, 3];
- influence of the brightness of the rod [4];
- projected the thinnest code line into a different number of pixels [2];
- inhomogeneous (non-monochrome) illumination [5].

One of the tests performed was the influence of the angle at which light falls on the rod. Readings were taken when the rod was illuminated with artificial light at an angle of 90°, i.e., perpendicular to the rod, at an angle of 45°, and at sharper angles, down to 15°. Approximately 20 readings were taken in each position, and previous research was confirmed - illumination at an angle of 90° to 45° does not affect the value of the results, i.e., the rod readings, while illumination at steeper angles can cause reading errors of up to (0.8-0.9) mm, which is more than ten times the measurement error at the station.

Lighting tests showed that the brightness threshold, minimum brightness, is not the same for all rods or digital levels. In this part, the Leica DNA03 level was shown as the best because even with a minimum brightness of only 6 lux, which is virtually dark, it performed accurate readings (the reading difference in normal light and minimal light is insignificant). At other levels, much more light (20-50) lux was needed, which roughly corresponds to twilight. When the illumination is too strong, all the levels have a problem and cannot read the rod.

Research has also been carried out to project the thinnest code line into a different number of pixels [10]. When crossing the size of a code line of three to two or two to one pixel, a significant error in rod reading occurs, greater than the declared accuracy. This test was done only for the Leica level because we found the characteristic values of the lengths of transitions from one to two and then to three pixels only for it. Series of measurements were made at 8.9 m, 13.35 m, and 26.7 m as well as at 1-2 cm before and after these lengths, without moving the support. Similar results were obtained as in the literature [2] - reading differences were in the range (20-40) μm, which is within or at the limit of measurement accuracy.

It has been experimentally proved that the inhomogeneous (non-monochromatic) illumination of some parts of the rod leads to significant errors in the rod reading. Under this illumination, the errors even go into gross measurement errors and have values up to several millimeters.

Determining the reading error

Because the manufacturers do not provide values of bar code reading errors, it was determined experimentally.

To determine the bar-code reading error, 10 readings were performed on four different sighting distances. The measurements were made in the laboratory under stable temperature conditions, so that environmental influences were eliminated. The rods were fixed so that the impact of their movement was eliminated. Because of the use of this procedure in reading differences, only the rod reading errors remain. From the statistical parameters, average and standard deviation of reading σ_h, calculated from the deviations of individual measurements from the average are given, in millimeters and seconds:

$$\bar{h} = \frac{\sum h_i}{10} \tag{17}$$

$$\sigma_h = \sqrt{\frac{\sum \left(\bar{h} - h_i\right)^2}{9}} \tag{18}$$

Based on the measurement data, the reading errors for each series of measurements have been determined separately. The reading error in seconds was determined by the expression:

$$\sigma_O = \frac{\sigma_h}{d}\rho'' \tag{19}$$

where: σ_h - standard deviation h in millimeters; d - sighting distance in millimeters; $\rho'' = (180/\pi)\cdot 3600$.

3.3 Used Levels

Three digital levels were used in the experimental part: Leica DNA03, Topcon DL-101C, and Sokkia SDL30 (Fig. 3). Invar rods were utilized with Leica DNA03 and Topcon DL-101C, while the fiberglass rods were available for Sokkia SDL30. The specifications of the instruments are given in Tab. 2.

Figure 3 Used levels: Leica DNA03, Topcon DL-101C and Sokkia SDL30

Table 2 Levels under test

Characteristic	Leica DNA03	Topcon DL-101C	Sokkia SDL30
Accuracy (for double-run leveling):			
with Invar rods	0,3 mm	0,4 mm	0,4 mm
with fiberglass rods	0,6 mm	0,6 mm	0,6 mm
Distance measurement accuracy	1 cm/20 m	1 cm to 5 cm	1 cm to 20 cm
Measuring range	1,8 m to 110 m	2 m to 100 m	1,6 m to 100 m
Compensator setting accuracy	0,3"	0,3"	0,3"
Measuring time	3 s	4 s	< 3 s

The determination of the reading error was made for three used digital levels, on a number of different sight lengths. The mean reading error for Leica is 0,39" (the range from 0,11" to 0,67"), for Topcon is 0,29" (the range from 0,22" to 0,36"), and for Sokkia is is 0,46" (the range from 0,32" to 0,52"). The mean reading error for all instruments is 0,39", i.e. it has the same value as with classic levels.

Using the data from Tab. 1 for the height difference measurement error at the station (Eq. (4)), the values shown in Tab. 3 are obtained. The calculation was performed for the invar rod, i.e. for the temperature coefficient of expansion of the invar tape $\gamma_{PNL} = 1\ \mu\text{m}/(\text{m}\cdot{}^\circ\text{C})$, temperature difference during calibration of the rod and measurement of 10 °C and height difference at the station of 1 m.

Table 3 The values of the measurement errors at the station

	Length / m					
	2	5	10	15	20	30
Error value / μm	43	44	45	47	51	58

During these experiments, another source of errors was identified that has not been addressed so far - incorrect sighting of the rod. This error occurs when the middle of the rod is not sighted, but the sight line is directed towards the ends of the rod. This happens especially with shorter sight lengths and when it is necessary to measure the height difference in the shortest possible time because the operator takes the reading as soon as the sight line is 'somewhere' on the rod.

The error due to improper sighting was 'discovered' during experiments with the Leica level. An experiment was then designed to determine this source of error (Fig. 4), and measurements were taken at only four sight lengths to confirm whether this source of error affects the reading values. The measurement results are shown in Tab. 4. Depending on whether the center or edge of the graduation is sighting (Fig. 4), reading differences are detected when the center of the graduation is visible and when the left or right edges of the graduation are visible. For each sighting distance ten readings of the middle, left, and right edges of the graduation were done. In Tab. 3 h_L, h_M and h_R are the readings of the graduation to the left edge, in the middle, and on the right edge respectively.

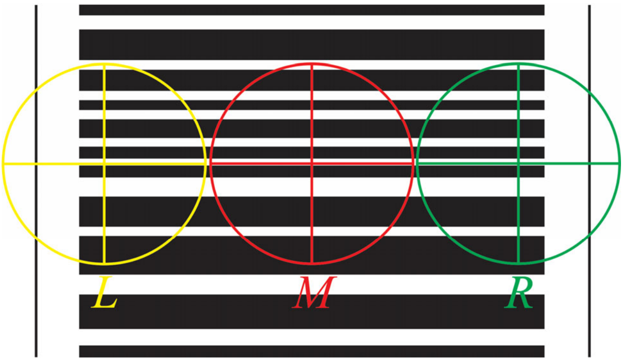

Figure 4 Bar-code sighting method: a) middle of the rod, b) left edge of the rod, c) right edge of the rod

Leica DNA03

Within the experiment scope, the measurements were carried out with the level at four different sighting distances to determine the influence of the rod sighting method in a horizontal sense on the reading error. Depending on whether the center or edge of the graduation is sighting (Fig. 4), reading differences are detected when the center of the graduation is visible and when the left or right edges of the graduation are visible. For each sighting distance ten readings of the middle, left and right edges of the graduation were done. In Tab. 4 h_L, h_M and h_R are the readings of the graduation to the left edge, in the middle and on the right edge respectively.

These differences were statistically tested. The limit value used to compare the maximum reading difference is $D_{\max}$. If they are higher than the limit value, the differences are significant. $D_{\max}$ is determined based on the expression:

$$D_{\max} = t_{p,f} \cdot \sigma_D \tag{20}$$

where: $t_{p,f}$ - the critical value of the Student's distribution for the accepted probability of 95% and the number of degrees of freedom of $f = 9$; σ_D - value obtained based on standard deviations $\sigma_{\bar{h}}$ given in Tab. 4 and based on the expressions $\sigma_D^2 = \sigma_L^2 + \sigma_M^2$ and $\sigma_D^2 = \sigma_R^2 + \sigma_M^2$.

Each standard deviation (σ_L^2, σ_M^2, σ_R^2) is calculated based on the expression:

$$\sigma_{\bar{h}} = \frac{\sigma_h}{\sqrt{10}} \tag{21}$$

where σ_h is calculated through the Eq. (18).

Table 4 The readings of the rod and statistical parameters for *Leica*

No.	Series 1: d = 2,13 m			Series 2: d = 5,61 m		
	h_L / m	h_M / m	h_R / m	h_L / m	h_M / m	h_R / m
1	1,21271	1,21273	1,21279	1,15355	1,15349	1,15345
2	1,21270	1,21274	1,21278	1,15356	1,15348	1,15345
3	1,21271	1,21274	1,21278	1,15355	1,15349	1,15344
4	1,21271	1,21274	1,21278	1,15356	1,15348	1,15345
5	1,21271	1,21274	1,21278	1,15356	1,15348	1,15344
6	1,21271	1,21274	1,21278	1,15355	1,15349	1,15344
7	1,21271	1,21274	1,21278	1,15357	1,15349	1,15344
8	1,21271	1,21274	1,21278	1,15356	1,15349	1,15345
9	1,21271	1,21274	1,21279	1,15355	1,15348	1,15345
10	1,21271	1,21274	1,21278	1,15355	1,15349	1,15344
$\bar{h}$ / m	1,21271	1,21274	1,21278	1,15356	1,15349	1,15345
$\sigma_{\bar{h}}$ / mm	0,001	0,001	0,001	0,002	0,002	0,002
No.	Series 3: d = 8,69 m			Series 4: d = 19,61 m		
	h_L / m	h_M / m	h_R / m	h_L / m	h_M / m	h_R / m
1	1,15691	1,15687	1,15682	1,25111	1,25118	1,25108
2	1,15692	1,15687	1,15682	1,25112	1,25109	1,25111
3	1,15692	1,15688	1,15683	1,25110	1,25104	1,25108
4	1,15692	1,15688	1,15682	1,25119	1,25117	1,25112
5	1,15692	1,15687	1,15682	1,25111	1,25102	1,25104
6	1,15691	1,15688	1,15683	1,25106	1,25110	1,25111
7	1,15690	1,15688	1,15682	1,25103	1,25120	1,25105
8	1,15691	1,15688	1,15682	1,25100	1,25106	1,25108
9	1,15691	1,15688	1,15683	1,25111	1,25104	1,25114
10	1,15690	1,15688	1,15683	1,25106	1,25111	1,25104
$\bar{h}$ / m	1,15691	1,15688	1,15682	1,25109	1,25110	1,25109
$\sigma_{\bar{h}}$ / mm	0,002	0,002	0,002	0,017	0,020	0,011

Table 5 Maximum differences in the rod reading - *Leica*

d / m	Maximum and allowed differences					
	$h_L - h_M$ / μm	$D_{\max}$ / μm	+ / –	$h_R - h_M$ / μm	$D_{\max}$ / μm	+ / –
2,13	–30	3	+	43	3	+
5,61	70	5	+	-41	5	+
8,69	35	6	+	-53	4	+
19,61	–12	53	–	-16	46	–

Legend: +: $|D| > D_{\max}$ significant; -: $|D| < D_{\max}$ insignificant

For sighting distances up to 9 m, the maximum reading difference is 0,07 mm and according to the test stats, they are all significant. For a distance of 19,61 m, the differences are insignificant. Differences in reading and limit values $D_{\max}$ are given in Tab. 5.

Topcon DL-101C

With this level has been measured at several different sighting distances in the series of 10 readings. Depending on whether the center or edge of the bar-code bar is sighting, reading differences are determined, and then the differences are tested for significance the same as Leica. Values are shown in Tab. 6.

Tab. 6 gives the values of reading the graduated rod on individual sighting distances, and the same labels are used as in Tab. 4. Tab. 7 presents the differences in rod reading at all measured sighting distances as well as the limit values to which they are compared. The limit value is determined according to expression (20), with standard deviations σ_h from Tab. 6.

By comparing the values of the differences with limit values $D_{\max}$ from Tab. 7, it can be concluded that almost all (16 of 20) reading differences are significant. This means that it is necessary to sight and read only the middle of the rod, even at larger sighting distances (and at larger sighting distances differences are close to the boundary values).

Table 6 The readings of the rod and statistical parameters for *Topcon*

No.	Series 1: d = 2,01 m			Series 2: d = 6,03 m		
	h_L / m	h_M / m	h_R / m	h_L / m	h_M / m	h_R / m
1	1,20973	1,20975	1,20979	1,19777	1,19773	1,19765
2	1,20973	1,20975	1,20979	1,19778	1,19773	1,19764
3	1,20973	1,20975	1,20979	1,19779	1,19772	1,19765
4	1,20973	1,20975	1,20979	1,19778	1,19771	1,19764
5	1,20973	1,20975	1,20979	1,19778	1,19771	1,19765
6	1,20973	1,20975	1,20979	1,19778	1,19772	1,19765
7	1,20972	1,20975	1,20979	1,19777	1,19773	1,19765
8	1,20973	1,20976	1,20979	1,19777	1,19771	1,19764
9	1,20973	1,20975	1,20979	1,19777	1,19771	1,19763
10	1,20973	1,20975	1,20979	1,19777	1,19772	1,19764
$\bar{h}$ / m	1,20973	1,20975	1,20979	1,19778	1,19772	1,19764
$\sigma_{\bar{h}}$ / mm	0,001	0,001	0	0,002	0,003	0,002
No.	Series 3: d = 7,97 m			Series 4: d = 20,02 m		
	h_L / m	h_M / m	h_R / m	h_L / m	h_M / m	h_R / m
1	1,20996	1,20995	1,20991	1,40315	1,40313	1,40304
2	1,20998	1,20994	1,20994	1,40322	1,40314	1,40309
3	1,20998	1,20995	1,20991	1,40319	1,40316	1,40307
4	1,20997	1,20996	1,20994	1,40313	1,40314	1,40313
5	1,20995	1,20994	1,20993	1,40318	1,40313	1,40315
6	1,20998	1,20995	1,20995	1,40322	1,40316	1,40305
7	1,20998	1,20995	1,20994	1,40318	1,40317	1,40305
8	1,20995	1,20993	1,20991	1,40326	1,40311	1,40305
9	1,20998	1,20993	1,20993	1,40321	1,40311	1,40309
10	1,20995	1,20994	1,20991	1,40323	1,40312	1,40305
$\bar{h}$ / m	1,20997	1,20994	1,20991	1,40320	1,40314	1,40308
$\sigma_{\bar{h}}$ / mm	0,004	0,003	0,005	0,012	0,007	0,012

Table 7 Maximum differences in the rod reading - *Topcon*

d / m	Maximum and allowed differences					
	$h_L - h_M$ / µm	$D_{\max}$ / µm	+ / –	$h_R - h_M$ / µm	$D_{\max}$ / µm	+ / –
2,01	–22	3	+	39	2	+
4,03	48	7	+	30	5	+
6,03	57	7	+	–75	7	+
7,97	24	11	+	–17	12	+
10,02	60	13	+	–122	14	+
11,99	62	18	+	–12	19	–
13,99	–76	29	+	106	29	+
16,00	28	31	–	–7	27	–
18,04	–17	25	–	102	38	+
20,02	60	28	+	–60	27	+

Legend: +: $|D| > D_{\max}$ significant; -: $|D| < D_{\max}$ insignificant

Sokkia SDL30

To determine the influence of the method of horizontal sighting of the rod, readings were made as in the previous instruments. The values of individual readings of the graduated rod at some sighting distances are given in Tab. 8. Their differences were determined according to the procedure described above for testing significance (Tab. 10).

The standard deviations of some height difference (marked with *) are calculated by reading error (0.46") by changing the left and right sides of the Eq. (3). From the significance test it can be concluded that approximately the same number of significant and insignificant (11:9) differences are found in the rod reading. This indicates the same conclusion as the Topcon level - only the center of the bar-code rod should be sighting and read, even at the larger sighting distances.

Table 8 The readings of the rod and statistical parameters for *Sokkia*

No.	Series 1: d = 2.01 m			Series 2: d = 5.96		
	h_L / m	h_M / m	h_R / m	h_L / m	h_M / m	h_R / m
1	1.2246	1.2247	1.2247	1.2343	1.2343	1.2343
2	1.2246	1.2247	1.2247	1.2342	1.2343	1.2343
3	1.2246	1.2247	1.2247	1.2343	1.2343	1.2342
4	1.2246	1.2247	1.2247	1.2343	1.2343	1.2343
5	1.2246	1.2247	1.2247	1.2343	1.2343	1.2343
6	1.2246	1.2247	1.2247	1.2343	1.2343	1.2343
7	1.2246	1.2247	1.2247	1.2343	1.2343	1.2342
8	1.2246	1.2247	1.2247	1.2343	1.2343	1.2342
9	1.2246	1.2247	1.2247	1.2343	1.2343	1.2342
10	1.2246	1.2247	1.2247	1.2343	1.2343	1.2342
$\bar{h}$ / m	1.22460	1.22470	1.22470	1.23429	1.23430	1.23425
$\sigma_{\bar{h}}$ / mm	0	0	0	0,010	0	0,017
No.	Series 3: d = 9.95			Series 4: d = 20.02		
	h_L / m	h_M / m	h_R / m	h_L / m	h_M / m	h_R / m
1	1.2658	1.2658	1.2658	1.4755	1.4756	1.4756
2	1.2658	1.2658	1.2658	1.4755	1.4756	1.4755
3	1.2658	1.2658	1.2658	1.4755	1.4756	1.4756
4	1.2658	1.2658	1.2658	1.4756	1.4756	1.4756
5	1.2658	1.2658	1.2658	1.4756	1.4756	1.4756
6	1.2658	1.2658	1.2658	1.4756	1.4756	1.4756
7	1.2658	1.2658	1.2658	1.4756	1.4756	1.4756
8	1.2658	1.2658	1.2658	1.4756	1.4756	1.4755
9	1.2658	1.2658	1.2658	1.4755	1.4756	1.4755
10	1.2658	1.2658	1.2658	1.4756	1.4756	1.4755
$\bar{h}$ / m	1.26580	1.26580	1.26580	1.47556	1.47560	1.47556
$\sigma_{\bar{h}}$ / mm	0	0	0	0,016	0	0,016

Table 9 Maximum differences in the rod reading - *Sokkia*

d / m	Maximum and allowed differences					
	$h_L - h_M$ / µm	$D_{\max}$ / µm	+ / –	$h_R - h_M$ / µm	$D_{\max}$ / µm	+ / –
2,01	100	4*	+	0	4*	–
4,00	100	8*	+	–100	8*	+
5,96	010	22*	–	–50	39	+
8,03	0	16*	–	–100	16*	+
9,95	0	20*	–	0	20*	–
12,04	–60	42*	+	90	41*	+
13,97	–70	36*	+	–100	43	+
16,01	–50	41	+	–50	38	+
18,00	10	33	–	–30	42	–
20,02	–40	43	–	–40	46	–

Legend: +: $|D| > D_{\max}$ significant; -: $|D| < D_{\max}$ insignificant

The same experiment was performed with this instrument but with aluminum telescopic rods and almost identical results were obtained (no measurement and processing will be made) - the rod should be sighting and reading only in the middle of the bar-code.

For levels, Topcon (Fig. 5) and Sokkia (Fig. 6), diagrams of reading differences on the rod to the left and right relative to the center were made using the values from Tabs. 7 and 9. On the same diagram, the values of the mean errors of height difference measurements at the station (Tab. 3) are plotted for comparison with the difference values. In diagrams 5 and 6, the values s+ and s– represent the positive and negative values of the mean error of height difference measurement at the station calculated according to Eq. (4).

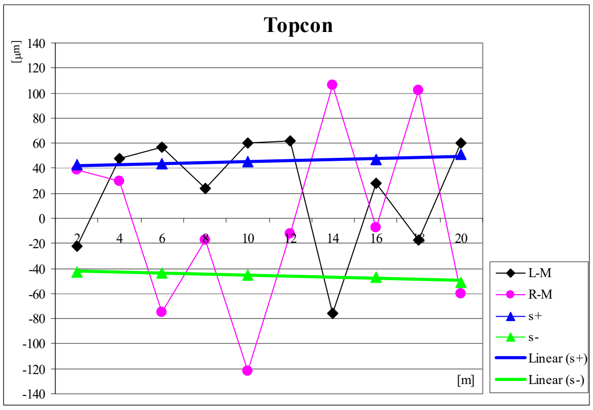

Figure 5 Differences between correct and incorrect reading of the rod - Topcon

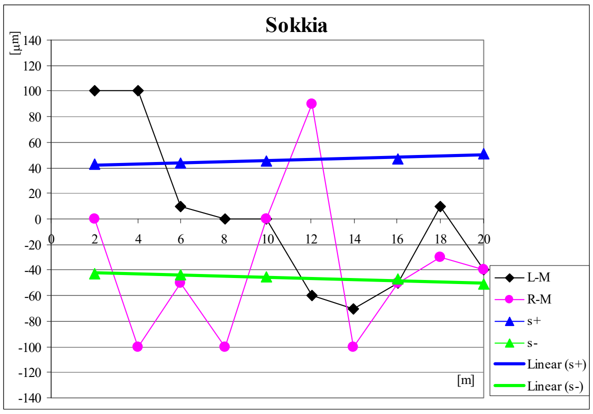

Figure 6 Differences between correct and incorrect reading of the rod - Sokkia

The presented results of the research have proven that improper sighting of the rod with digital levels has a significant impact on the accuracy of measuring height differences at the station. From Tabs. 5, 7 and 9 it can be concluded that the impact of improper sighting is significant because in most cases it exceeds the value of the measurement error. Also, from Figs. 5 and 6, it can be concluded that the values of the reading difference exceed the values of the measurement errors at the station.

4 DISCUSSION AND CONCLUSIONS

Achieving the desired results and safety for any engineering project requires regularly reviewing and checking the technical specifications and accuracy of the geodetic instruments. Standard calibration models and procedures exist for all geodetic instruments but they must be developed and modified to meet the standard for advanced precise digital level, especially for deformation measurements. Digital levels are widely used for setting out engineering structures and monitoring structural deformation because of their accuracy, in addition to the possibility of automatically collecting and storing data, which saves the time and effort required for observations.

Digital levels have made levelling faster and more reliable. The process of reading the rod is simpler and quicker, eliminating operator error. Built-in programs enable automatic control of measurements at the station through predefined values for permissible differences. Data registration is automatically blocked if the differences exceed the defined limits. In addition to the station, data between benchmarks can also be immediately controlled, providing additional security and reducing processing time.

Despite the undeniable advantages of digital levels over traditional ones in terms of speed and reliability, some new measurement errors are present with digital levels. Therefore, research has been conducted to determine specific sources of errors in levels and bar-code rods. New systems for calibrating digital levels using vertical comparators have been investigated [16]. New mathematical calibration models have been used to determine specific characteristics of precise digital levels [15]. Digital levels should be protected from the effects of changes in atmospheric conditions because their internal components are thermally stabilized. Therefore, the impact of temperature changes on the characteristics of the level, primarily on collimation error, has been studied [13, 14]. However, most of the research has been dedicated to leveling rods.

Conducted experiments in this paper have proved the existence of some sources of errors that are not characteristic of classical levels - the brightness of the rod, the impact of the thickness of the bars in the lighting of the rod, the non-monochromatic lighting, ... All these effects were previously discovered and published. However, one source of error has been noticed and has not yet been considered, and it is not characteristic of classical levels - incorrect sighting of the rod. Namely, during the measurement, the operator does not pay attention to the visibility of the rod, it is only important for him to see it in the field of view. Unfortunately, this approach can cause a measurement error that is greater than the accuracy of measuring the height differences at the station. It can also be higher than the declared accuracy, which cannot be tolerated. Therefore, a general conclusion can be drawn for all digital levels, especially in precise measurements - when reading the bar-code, the rod must be sighting in the middle, as this eliminates the origin of the error whose influence may be greater than the error measurement value.

This error is likely due to a direct effect of the manufacturing method of drawing bar-code elements, which are not perfectly shaped and do not have perfectly straight edges, depending on the rod. An additional effect comes from the curvature of the tape in the guideways, especially when the rod housing expands differently than the tape, yielding to the situation where the bar code is slightly tilted from left to right concerning the setup is normal. Another effect probably comes from the camera system itself [8].

These investigations have revealed a new source of error in digital level measurements, which, I hope, has made an original research contribution to a better understanding of the digital level measurement process. This source of error can help operators avoid making reading errors when making accurate measurements. This is very important in engineering works where frequently height differences are measured at only one station and where accuracy is required in hundredths of a millimeter. It is essential to consider how the rods are aimed when measuring in narrow myopic conditions and when installing machine elements, where high accuracy is most often required.

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