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ENERGY DEPENDENCE OF THE KAON-PION RATIO IN THE PHENOMENOLOGICAL MODELS OF BULL AND MAEDA

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Abstract: The phenomenological models of Bull et al. and Maeda have been used to fit the sea level muon spectrum. The derived result is in good agreement with the measured data when the energy dependence of kaon-pion ratio at production is assumed.

1. Introduction

The phenomenological models of Bull et. al.[1] and Maeda[2] have been used by Allkofer et al.[3], Dau et al.[4] and Thompson and Whalley[3] to fit the measured muon intensity data. Earlier Allkofer et al.[3] have assumed that all muons have been originated from pions and gave a best fit muon spectrum. Later Dau et al.[4] have used Maeda's[2] model and have found that the derived muon spectrum agrees with the measured muon spectra at different zenith angles when the kaon-pion ratio was fixed at the value of 0.15. Later Thompson and Whalley[3] have used the diffusion model of Bull et al.[1] and their best estimated muon production spectrum has presented two values of energy exponents in the spectral ranges 30 — 100 GeV and 100 — 700 GeV, respectively.

In the present investigation we have used the models of Bull et al.[1] and Maeda[2] to calculate the sea level muon spectra with assumption of the energy dependence of kaon-pion ratio at production. The energy exponents of the pion and kaon spectra have been assumed to be similar to those of the measured primary proton spectrum of Ramaty et al.[6].

2. Phenomenological models

Model I. The conventional pion and kaon atmospheric diffusion equations after the model of Bull et al.[1] yield the sea level muon spectrum of the following form

$$N(E_\mu)\,dE_\mu = A\,P_\mu\,(E_\mu + \Delta E_\mu)^{-\gamma}\left\{\frac{r_\pi^{\gamma-1} B_\pi}{E_\mu + \Delta E_\mu + B_\pi} + \frac{K}{\pi}\cdot\frac{r_k^{\gamma-1} B_k}{E_\mu + \Delta E_\mu + B_k}\right\} dE_\mu, \tag{1}$$

where the energy degradation factors are $r_\pi = 0.76$, $r_k = 0.52$; the critical energy for pion and kaon decays are $B_\pi = 90$ GeV, $B_k = 452$ GeV.

The muon energy loss from the mean depth of production which is about 100 gcm^{-2} air, reaching sea level atmospheric depth of 1033 g cm^{-2} follows the relation

$$\Delta E_\mu = 2.33 + 0.026\,E_\mu \text{ GeV}. \tag{2}$$

It is assumed that all muons are produced after the penetration of 100 g cm^{-2} air by the parent particles. The survival probability of muons produced at mean depth 100 g cm^{-2} reaching sea level depth of 1033 g cm^{-2} air is calculated by the expression

$$P_\mu = \left(\frac{0.097\,E_\mu}{2.33 + 1.026\,E_\mu}\right)^{1.08/(2.583 + 1.003\,E_\mu)}. \tag{3}$$

Model II. Maeda[2] has developed a phenomenological model for the estimation of sea level muon spectrum. The vertical sea level muon spectrum at energy E_μ at sea level can be evaluated by the following expression

$$I_\mu\,(E_\mu, 1033 \text{ g cm}^{-2}) = A\,(I_{\pi\mu} + [K/\pi]\cdot I_{k\mu})\,P_\mu. \tag{4}$$

The pion and kaon produced intensites $I_{\pi\mu}$ and $I_{k\mu}$ have been calculated from the following expression

$$I_{i\mu} = \frac{B_i}{(1 - r_i^2)}\left[\frac{(E_\mu + \Delta E_\mu)^{-\gamma}}{\gamma B_i + (\gamma+1)(E_\mu + \Delta E_\mu)} - \frac{\{(E_\mu + \Delta E_\mu)/r_i^2\}^{-\gamma}}{\gamma B_i + (\gamma+1)(E_\mu + \Delta E_\mu)/r_i^2}\right],$$

where γ is the energy exponent of the pion and kaon spectra; $i = \pi$ or k; $B_\pi = 121$ GeV; $B_k = 855$ GeV, $r_\pi = 0.76$; $r_k = 0.21$.

3. *Results and discussion*

On assuming the energy exponents of pion and kaon spectra similar to that of the measured primary proton spectrum of Ramaty et al.[6] the calculated muon spectra from the Models I and II have been plotted in the Fig. 1 along with the measured absolute muon intensity data of Allkofer et al.[3], Ayre et al.[7] and ours (Bhattacharyya[8]). It is found that the derived muon spectrum from the Model-I

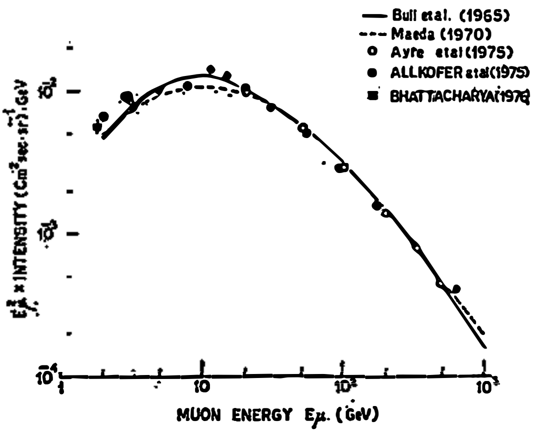

Fig. 1. The calculated sealevel muon spectra after Bull et al.[1] — full curve, and after Maeda[2] — broken curve. Experimental data — • — Allkofer et al.[3]; o Ayre et al.[4] and Bhattacharyya[4].

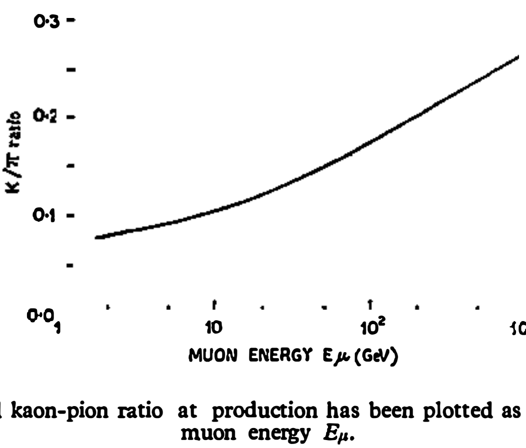

Fig. 2. The assumed kaon-pion ratio at production has been plotted as a function of sealevel muon energy E_μ.

fits the data for $\gamma = 2.64$ and $A = 0.195$ and that from the Model-II agrees with the data for $\gamma = 2.64$ and $A = 0.16$. The assumed energy dependence of the kaon -pion ratio in relations (1) and (4) for the fit to the experimcntal data has been presented in the Fig. 2. It is evident from the Fig. 1 that the derived sea level muon spectra differ from each other especially in the energy region below 20 GeV. From 20 to 600 GeV the calculated spectral shapes are similar and fit well the data. Above 600 GeV muon energy the calculated spectral shapes are different.

The estimated kaon-pion ratio from the Intersecting Storage Ring data of Antinucci et al.[9] changes from 5—10% in the muon energy range 5—200 GeV. The present survey indicates that the change in the kaon-pion ratio at production is 9—20% in the same spectral range. These phenomenological models indicate that the increase of kaon-pion ratio is higher for almost a factor of 2 at high energies than that expected from the extrapolated ISR results of Antinucci at al.[9]. This observation is also supported by our previous investigation[10] with Bose type model which reveals the fact that the muon that intensity comes from $K \rightarrow \mu_2$ decay increases with muon energy.

4. *Conclusion*

The present study indicates that a single energy exponent ($\gamma = 2.64$) can fit the measured muon energy spectrum derived from the models of Bull et al. and Maeda. The model of Bull et al. is satisfactorily in agreement with the data even at energies below 20 GeV.

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