

SENSITIVITY OF ANGULAR DISTRIBUTIONS OF γ -RAYS ON PARAMETERS OF THE DSD CAPTURE MODEL

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Abstract: The angular distributions of γ -rays from the radiative capture of fast neutrons has been calculated using different approaches of the direct-semidirect capture model. The sensitivity of the calculated angular distribution on the main parameters of the model is investigated. It is shown that for low l -value transitions Legendre coefficients are practically independent on a reasonable variation of particle vibration coupling parameters. The sensitivity rises with the angular momentum of the populated states. The comparison of the calculated and measured angular distributions allows the selection of the appropriate approach to the DSD model.

1. Introduction

The refined direct-semidirect (DSD) model of fast neutron capture has proven to be very successful in reproducing the excitation functions of medium-light and heavy nuclei. The calculation of the angular distributions provide a further test of the models and gives further insight on the form factor for the inelastic excitation of the dipole giant resonance.

Recent calculations displayed several general features of the angular distributions of γ -rays related to the DSD model. The most important result is that only nonvanishing Legendre expansion coefficient, a_2 , is not very sensitive to the values of the main parameters of the model.

The aim of this work is to analyse between what limits the a_2 coefficient varies a result of a reasonable change of the main parameters of the model. Only the neutron capture reactions are studied. The considered nuclei are selected so that the radiative transitions to levels with angular momentum $l = 0$ to $l = 6$ are included. For some of the cases the experimental data are already available.

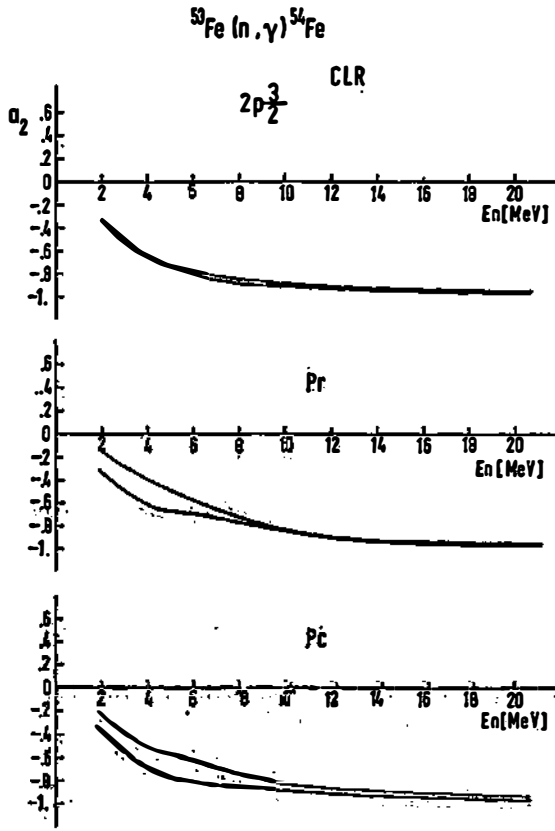


Fig. 1. Limits of the a_2 Legendre coefficient for the angular distribution of γ -rays from the reaction $^{56}\text{Fe}(n, \gamma)^{57}\text{Fe}$. The final single particle state of the capture neutron is $2p_{3/2}$. Rosen optical model parameters were used. The upper figure corresponds to the CLR form factor, the middle one to Pr and the lower to the Pc form factors. Upper and lower limits of the parameter variation are given in Table 1.

2. Theory

The theoretical details of the angular distribution calculations were published elsewhere^{1,2}. Here we only outline the most important points of the formalism.

According to the DSD model which takes into account only dipole transitions the expansion of the differential cross section in Legendre polynomials takes the form

$$\frac{d\sigma}{d\Omega} = A_0 [1 + a_2 P_2(\cos \vartheta)], \quad (1)$$

The coefficient a_2 can be calculated from the following equation

$$a_2 = \frac{1}{2} \frac{a_1 - 2a_0 + a_{-1}}{a_1 + a_0 + a_{-1}},$$

where

$$a_\nu = \left| \sum_{j, j'} i^{j'} \left(2j' + 1 \right) \begin{pmatrix} j & 1 & j' \\ -m & -\nu & \frac{1}{2} \end{pmatrix} \begin{pmatrix} j & 1 & j' \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \langle R_{j', j'} \rangle \right|^2 \quad (2)$$

The symbol $\langle R_{j', j'} \rangle$ stands for the radial single particle integral given by

$$\langle R_{j', j'} \rangle = \langle \varphi_{n, l, j} | r + \frac{V_1(r)}{E_\nu - E_d + i \frac{\Gamma_d}{2}} | \varphi_{j', j'} \rangle. \quad (3)$$

The radial wave functions $\varphi_{j', j'}(r)$ and $\varphi_{n, l, j}(r)$ describe the free neutron moving in optical potential and the neutron bound by the potential of the target nucleus, respectively. The radial part of the form factor describing the inelastic excitation of the giant dipole resonance by the incident neutron is represented by $V_1(r)$.

In this work four different radial form factors are considered. In our belief these form factors are the most important cases which define four different approaches to the DSD model.

a) The simplest form factor is of course

$$V_1(r) = 0, \quad (4)$$

where the core polarization effect is neglected. This approach gives the same angular distributions as the form factor

$$V_1(r) = a r, \quad (5)$$

which has been proposed by Brown³⁾.

b) The surface peaked form factor CLR, proposed by Clement, Lane and Rook⁴⁾ is given by

$$V_1(r) = \frac{N}{A} \frac{\hbar^2}{m_r E_d} (1 + 0.8 x) \frac{V_1}{4} \left(- \frac{df_a(r)}{dr} \right). \quad (6)$$

Here $A = N + Z$, m_r is the reduced mass of the neutron, x is the exchange force factor present in the Gell-Mann-Goldberger-Thirring dipole sum rule (we have used $x = 0.5$), V_1 is the strength of the symmetry optical potential while $f_a(r)$ stands for the Saxon-Woods form factor of the nuclear density. The parameters in $f_a(r)$ are the same as in the optical model potential.

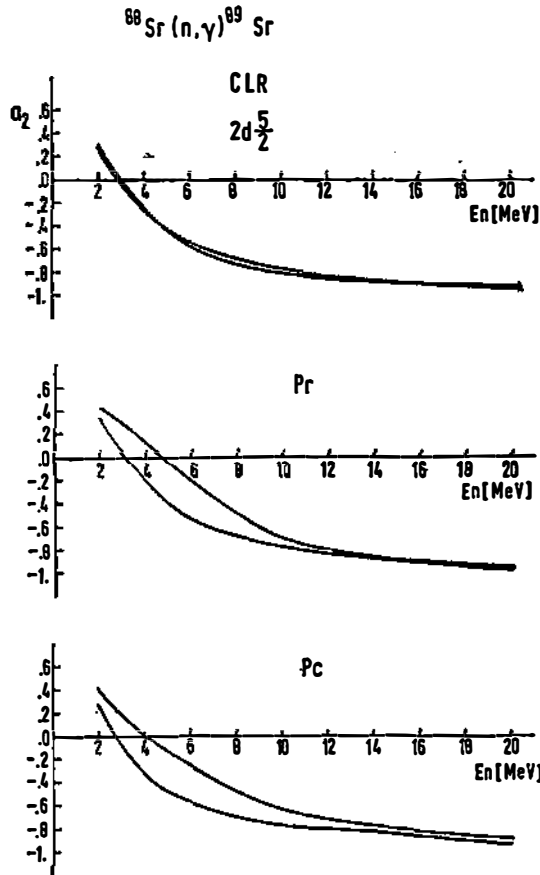


Fig. 2. Same as on Fig. 1 but for the reaction $^{88}\text{Sr}(n,\gamma_0)^{89}\text{Sr}$. The final single particle state of the captured neutron is $2d_{5/2}$.

c) The factor Pr of the volume form^{2,5)} is given by the relation

$$V_1(r) = \frac{2\sigma_{-1}}{\kappa \langle r^2 \rangle 0.096 Z} \frac{V_1}{4} r f_a(r). \tag{7}$$

In the last equation σ_{-1} stands for the bremsstrahlung weighted integrated cross section, $\langle r^2 \rangle$ is the nuclear mean square radius and κ is the correction factor due to several approximations in the derivation of the form factor.

d) The complex form factor Pc has the real part equal to expression (7) and a surface imaginary part

$$V_1(r) = \frac{2\sigma_{-1}}{\kappa \langle r^2 \rangle 0.096 Z} r \left[\frac{V_1}{4} f_a(r) - i \frac{W_1}{4} 4b \frac{df_b(r)}{dr} \right], \tag{8}$$

where W_1 is the strength of the imaginary part of the form factor. The functions $f_a(r)$ and $f_b(r)$ are the same as the Fermi functions of the real and imaginary part of the optical potential, respectively.

In all the above mentioned approaches to the DSD model the main parameters V_1 and W_1 are assumed to be free. They are usually obtained by fitting the experimentally observed excitation functions.

3. Results and discussion

The sensitivity of the coefficient a_2 on the parameters V_1 and W_1 was studied for four different reactions, where the values of the orbital angular momentum quantum numbers of the final single particle states cover the range $0 \leq l \leq 6$. All the mentioned expressions of the form factor were considered. Parameters other than V_1 and W_1 were kept fixed during the calculations (see Table 1). The Rosen³⁾ optical model parameters were used throughout the calculations.

Table 1
PARAMETERS OF THE FORM FACTORS

Reaction	σ_{-1} [mb]	r^2 [f ²]	E_d [MeV]	Γd [MeV]
⁵³ Fe (n, γ_0) ⁵⁴ Fe	47	13.9	21.0	4.2
⁸⁸ Sr (n, $\gamma_{0,1}$) ⁸⁹ Sr	75.8	18.3	16.7	4.2
⁴⁰ Ca (n, γ_0) ⁴¹ Ca	30.0	11.1	18.0	4.0
²⁰⁸ Pb (n, $\gamma_{0,1}$) ²⁰⁹ Pb	251	29.8	13.42	4.5

The main parameters V_1 and W_1 were varied in intervals (see Table 2) obtained in the following way. First the best fit of the corresponding experimental excitation function was obtained for the chosen form factor to get the best fit parameters V_1 and W_1 . If there was no experimental excitation function to compare with, best fit parameters of the neighbouring nuclei were adopted. The interval of the parameter variations was chosen so that its lower limit was roughly one half of the value of the best fit parameter and its upper limit for about 100 MeV above it. The numbers were rounded, however, to 25 MeV for lower limits and 50 MeV for the upper ones.

After that the a_2 Legendre coefficient was calculated as function of the incoming neutron energy using different parameters V_1 and W_1 . For a given neutron energy and a given choice of the form factor only the lowest and the highest values of the calculated a_2 coefficients were plotted in the figures. The upper and lower points were then connected with lines covering the area of possible a_2 coefficients according to the interval of parameter variations.

Table 2

INTERVALS OF VARIATION OF THE PARAMETERS V_1 AND W_1

Reaction	Limit	V_1 CLR	V_1 Pr	V_1 Pc	W_1 Pc
$^{56}\text{Fe}(n, \gamma_0)^{57}\text{Fe}$	upper	200	200	150	150
	best fit	—	—	—	—
	lower	50	50	25	25
$^{88}\text{Sr}(n, \gamma_{0,1})^{89}\text{Sr}$	upper	250	250	150	200
	best fit	160	135	68	102
	lower	50	50	25	50
$^{40}\text{Ca}(n, \gamma_0)^{41}\text{Ca}$	upper	200	200	150	150
	best fit	108	107	77	46.2
	lower	50	50	25	25
$^{208}\text{Pb}(n, \gamma_{0,1})^{209}\text{Pb}$	upper	400	250	200	250
	best fit	254	133	72	147.6
	lower	100	50	25	50

The results are shown in Figs. 1—5. In Fig. 1 the results of the reaction $^{56}\text{Fe}(n, \gamma_0)^{57}\text{Fe}$ are presented. The final single particle state is a lp state. The weak deformation of the final nucleus was neglected in this treatment. The excitation function for this state is not yet experimentally measured, so that the best fit parameters V_1 and W_1 were taken from the analysis of the reaction $^{88}\text{Sr}(n, \gamma)^{89}\text{Sr}$. The displayed strips containing possible values of the a_2 coefficients are very narrow in all cases indicating a generally weak sensitivity of the a_2 coefficient on the choice of these parameters.

This sensitivity is somewhat more pronounced in the case of reactions $^{88}\text{Sr}(n, \gamma_0)^{89}\text{Sr}$ and $^{40}\text{Ca}(n, \gamma_0)^{41}\text{Ca}$, especially in the cases of real volume and complex form factors (see Figs. 2 and 3). The single particle states are $2d_{5/2}$ in the case of the capture by ^{88}Sr and $1f_{7/2}$ in the neutron capture by ^{40}Ca .

The results for the reaction $^{208}\text{Pb}(n, \gamma_0)^{209}\text{Pb}$ are displayed in Figs. 4 and 5. The results for the $2g_{9/2}$ single particle state of ^{209}Pb are shown in Fig. 4 while those for the first excited single particle $1i_{11/2}$ state of the same nucleus are shown in Fig. 5. One can observe the strongest sensitivity of the a_2 coefficient in Fig. 5, where we have the highest orbital momentum quantum number value $l = 6$ and the complex form factor used in the calculation.

There was one more calculation performed, namely for the reaction $^{88}\text{Sr}(n, \gamma_1)^{89}\text{Sr}$ where the $3s_{1/2}$ single particle state of the ^{89}Sr nucleus is populated. However, the a_2 value in this case is almost model independent; $a_2 = -1$ (Ref.¹).

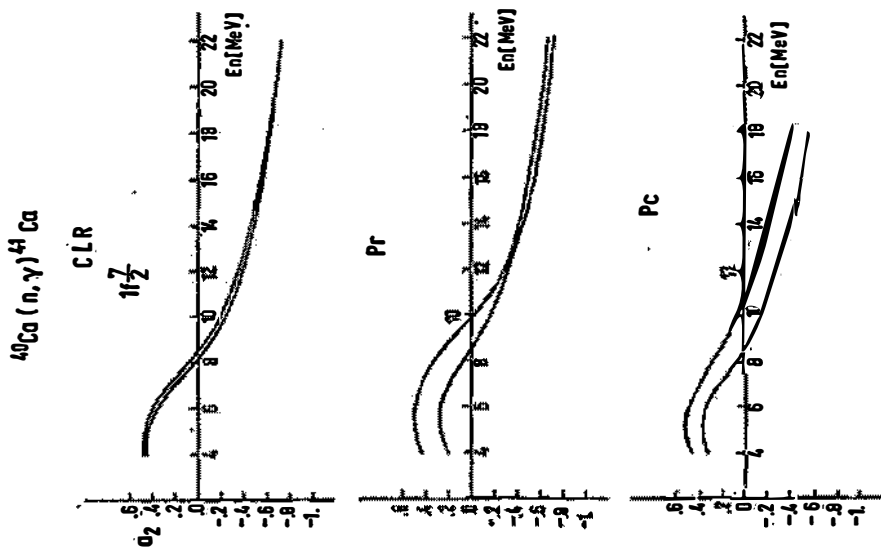


Fig. 3. Same as on Fig. 1 but for the reaction $^{40}\text{Ca}(n, \gamma)^{41}\text{Ca}$. The final single particle state of the captured neutron is $1f_{7/2}$.

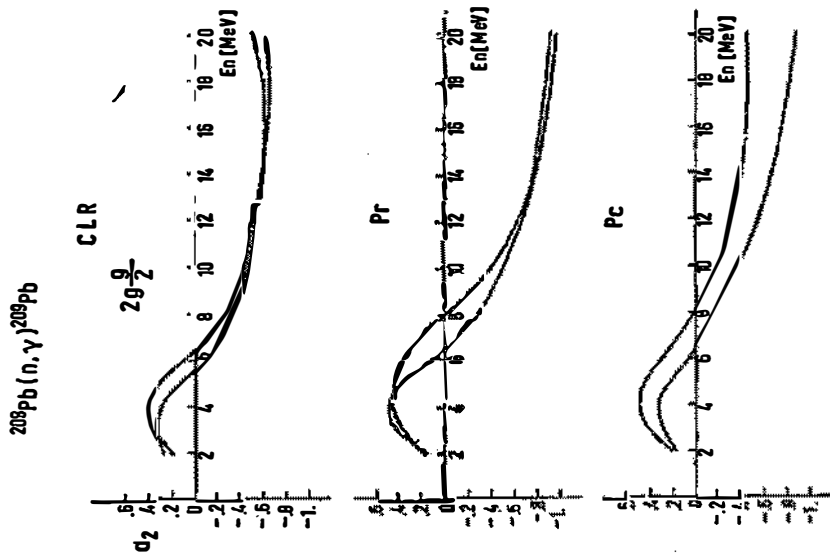


Fig. 4. Same as on Fig. 1 but for the reaction $^{208}\text{Pb}(n, \gamma)^{209}\text{Pb}$. The final single particle state of the captured neutron is $2g_{9/2}$.

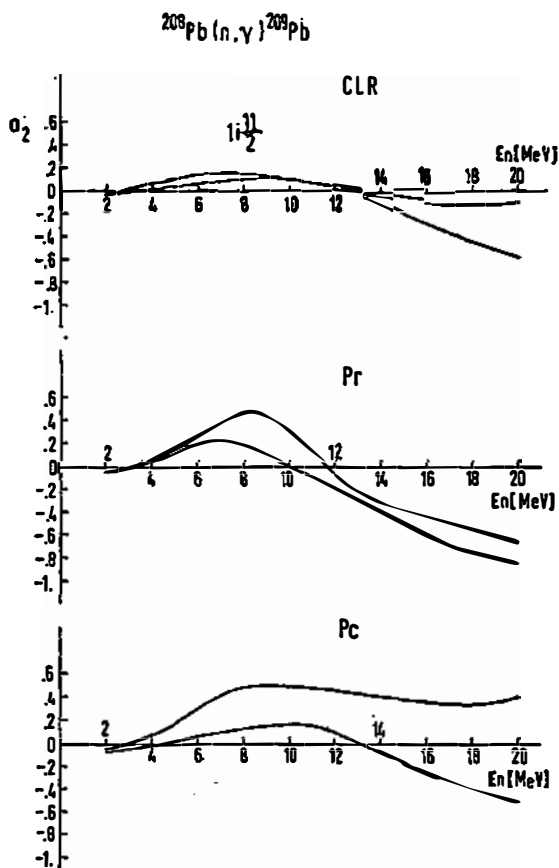


Fig 5. Same as on Fig. 1 but for the reaction $^{208}\text{Pb}(n,\gamma_0)^{209}\text{Pb}$. The final single particle state of the captured neutron is $1i_{11/2}$.

From the inspection of the figures and keeping in mind the results for the reaction $^{88}\text{Sr}(n,\gamma_1)^{89}\text{Sr}$ the following general conclusions about the sensitivity of the a_2 coefficient can be obtained:

- the approach of Clement, Lane and Rook⁴⁾ to the form factor leads in all the cases considered to very insensitive angular distributions, although the intervals of the variation of the free parameters were the widest just here;
- the use of the complex form factors yields a_2 coefficients most sensitive to the variation of the free parameters. This, of course, should be expected since two parameters were varied independently during the search instead of one as in the other cases; and
- there is a tendency that the capture to the single particle states of the target nuclei with higher values of the orbital angular momentum gi-

ves more sensitive results than the capture to single particle states with low values of l ($l \leq 3$). This tendency is the strongest when the complex interaction function is taken into account.

Generally speaking the sensitivity of the a_2 coefficient on the free parameters V_1 and W_1 of the DSD model is rather weak. It must be kept in mind that particularly severe variations of the free parameters have been taken to see the effect clearly. Thus, once the free parameters are obtained from the best fit of the excitation functions, the calculated angular distributions are accurately determined regardless of the poor quality of experimental data or the poor quality of the obtained fits. One must notice that the uncertainty of the free parameters obtained by the best fit procedure really exceeds $\pm 30\%$. Within this limit of the free parameters the uncertainty of the a_2 Legendre coefficient is always smaller than the typical error of the angular distribution experiments^{6,7}; its value is -0.1 . On the other hand the different form factors predict measurably different a_2 coefficients when high l -values of final single particle states are considered. In this respect the measurements of the angular distributions of γ -rays following the capture of fast neutrons to find single particle states with high values of orbital angular momentum would answer the question of how adequate the actual form factors are for the description of the capture process.

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OBČUTLJIVOST KOTNIH PORAZDELITEV ŽARKOV GAMA NA PARAMETRE DSD MÔDELA ZA ZAJETJE NEVTRONOV

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Vsebina

Izračunali smo kotne porazdelitve žarkov gama iz sevalnega zajetja hitrih nevtronov z uporabo različnih pristopov k direktno-semidirektnemu modelu za zajetje. Študirali smo občutljivost izračunanih kotnih porazdelitev na glavne parametre modela.

Pokazali smo, da so Legendrovi koeficienti pri zajetju v endoelčna stanja končnega jedra z nizko orbitalno vrtilno količino l praktično neovisni od parametrov sklopitve delca z dipolnim veleresonančnim stanjem. Občutljivost se poveča, če je l končnega enodelčnega stanja večji.

Primerjava računanih in eksperimentalnih kotnih porazdelitev omogoča izbiro primerne pristopa k DSD modelu.