

## USE OF SCHOTTKY'S DIFFUSION THEORY FOR NONSTATIONARY PLASMA COLUMN

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**Abstract:** The mechanism of ionization and diffusion in nonstationary plasma column is analyzed theoretically by the aid of Schottky's diffusion theory and application of single Langmuir probe theory.

Schottky's theory is shown to be a good tool for estimating rapid variation of gas pressure when the electron temperature is known.

The results are in good agreement with experimental measurements.

### *1. Introduction*

Schottky<sup>1)</sup> was the first to give the relationship between the electron temperature, pressure, and radius of the discharge tube. His theory was later extended by Tonks and Langmuir<sup>2)</sup>, and subsequently developed by R. Holm<sup>3)</sup>, Seeliger and Funk<sup>4)</sup> and A. v. Engle and Steenbeck<sup>5)</sup>.

By the aid of this theory, it is easy to calculate the value of electron temperature  $T_e$  and the radial and axial field strength in stationary plasma column.

The aim of the present work was to find out to what extent one can use Schottky's diffusion theory in the study of nonstationary plasma column, previously obtained<sup>6)</sup> in a pressure driven shock tube. A schematic diagram of the shock tube used is shown in Fig. 1. The tube has ring cathode and rod anode. The gases used are argon and nitrogen initially at pressure 1 Torr.

### *2. Theory*

The following analysis is proposed for nonstationary plasma column obtained in shock tube of radius  $R$ . We assume that the radial motion of charges in moving plasma is governed by ambipolar diffusion. For a steady state the electron

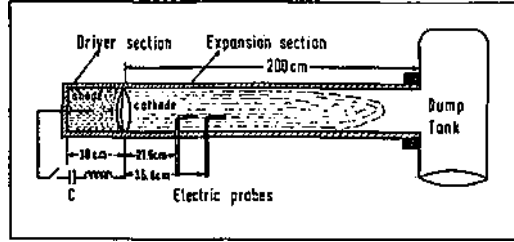


Fig. 1. Schematic diagram of the shock tube. The driver gases used are argon and nitrogen. The driver capacitor voltage  $C = 6$  kV.

concentration  $n_e$  and the positive ion  $n_i$  are each equal to  $n$ , and the loss rate must be equal to the production rate. Using Taylor's expansion one gets the number of particles leaving the elementary volume  $2\pi r dr$  (Fig. 2)

$$2\pi(r + dr)D_a \left( \frac{dn}{dr} \right)_{r+dr} - 2\pi r D_a \left( \frac{dn}{dr} \right)_r = 2\pi D_a \left( r \frac{d^2n}{dr^2} + \frac{dn}{dr} dr \right). \quad (1)$$

Since the production rate is  $Z_n(2\pi r dr)$ , then equating the two expressions one gets

$$\frac{d^2n}{dr^2} + \frac{1}{r} \left( \frac{dn}{dr} \right) + \frac{Z}{D_a} n = 0, \quad (2)$$

where  $D_a$  is the ambipolar diffusion coefficient.

We follow the same derivation of the classical Shottky's diffusion theory<sup>7)</sup>. We relate the electron temperature ( $T_e$ ) with the radius of the discharge tube ( $R$ ) and the pressure of the gas used ( $P$ ). We end with the following equation

$$\frac{e^{-eV_i/kT_e}}{\sqrt{eV_i/kT_e}} = 1.16 \cdot 10^7 \cdot C^2 \cdot P^2 \cdot R^2, \quad (3)$$

where  $C$  is a constant which depends on the gas used.

### 3. Results and discussion

*The electron temperature.* The Langmuir Mott-Smith single probe theory is used to calculate the electron temperature  $T_e$  of argon gas, as a driver gas in shock tube initially at 1 Torr pressure. The following results are obtained; the electron temperatures,  $T_e = 1.65 \cdot 10^4$  K and  $T_e = 2.34 \cdot 10^4$  K at distances 21.6 cm and 36.8 cm from cathode, respectively.

The success of this method in shock tube predicts that the electron energy distribution in the nonstationary plasma column is Maxwellian.

*Use of Schottky's diffusion theory.* We use Schottky's equation in calculating the pressure gradient along the expansion section of the shock tube of radius  $R = 2.5$  cm. The values of electron temperature ( $T_e$ ) are previously measured experimentally by the use of Langmuir Mott-Smith probe method. Knowing for argon gas the ionization potential  $V_i = 15.76$  V and the constant  $C = 5.3 \cdot 10^{-2}$ , from equation (3) the corresponding values of pressure  $P$  are calculated for each value of  $T_e$ .

The results are shown in Table 1 and the variation of  $\ln T_e/V_i$  against  $\ln CRP$  are plotted in Fig. 3. Results indicate the pressure gradient along the shock tube. The pressure is high in the driver section and low in the expansion section. The ring cathode separates the two regions and the pressure decreases along the expansion section.

From Table 1 one finds that at pressure  $P = 1$  Torr the corresponding electron temperature  $T_e$  is  $1.3 \cdot 10^4$  K. This result is in good agreement with the experimental one obtained in Ref.<sup>3)</sup>, in which the same type of shock tube was used. The theoretical data in Table 1 show that the electron temperature in-



Fig. 2. Section of the positive column of glow discharge.

Table 1

## Results for Argon

Ionization potential  $V_t = 15.76$  V  
 Radius of the tube  $R = 2.5$  cm  
 Constant  $C = 2.3 \cdot 10^{-2}$

$P$ (Torr)	$T_c$ (K)	$RP$ (cm. Torr)	$\ln T_c/V_t$	$\ln CRP$
1.4	$1.1 \cdot 10^4$	3.5	6.43	1.12
1.0	$1.3 \cdot 10^4$	2.5	6.60	1.45
0.55	$1.5 \cdot 10^4$	1.38	6.74	2.08
0.34	$1.64 \cdot 10^4$	0.85	6.78	2.36
0.30	$1.7 \cdot 10^4$	0.75	6.84	2.65
0.14	$2.0 \cdot 10^4$	0.33	6.98	3.42
0.10	$2.1 \cdot 10^4$	0.25	7.08	3.76
0.064	$2.34 \cdot 10^4$	0.22	7.17	4.26
0.051	$2.5 \cdot 10^4$	0.13	7.20	4.56
0.041	$2.7 \cdot 10^4$	0.10	7.32	4.67
0.032	$2.9 \cdot 10^4$	0.08	7.42	4.84
0.021	$3.1 \cdot 10^4$	0.053	7.65	5.20
0.012	$3.3 \cdot 10^4$	0.030	7.84	5.32

Table 2

## Results for Nitrogen

Ionization potential  $V_t = 14.54$  V  
 Radius of the tube  $R = 2.5$  cm  
 Constant  $C = 3.5 \cdot 10^{-2}$

$P$ (Torr)	$T_c$ (K)	$RP$ (cm. Torr)	$\ln T_c/T_t$	$\ln CRP$
1.99	$1.2 \cdot 10^4$	4.98	6.4	1.77
1.00	$1.33 \cdot 10^4$	2.50	4.48	2.2
0.71	$1.4 \cdot 10^4$	1.78	6.58	2.8
0.65	$1.6 \cdot 10^4$	1.63	6.62	2.96
0.21	$1.8 \cdot 10^4$	0.53	6.74	3.56
0.14	$2.0 \cdot 10^4$	0.35	6.88	4.56
0.092	$2.2 \cdot 10^4$	0.23	6.94	4.88
0.064	$2.4 \cdot 10^4$	0.16	7.82	5.32
0.054	$2.6 \cdot 10^4$	0.14	7.06	5.48
0.042	$2.8 \cdot 10^4$	0.11	7.1	5.64
0.023	$3.0 \cdot 10^4$	0.065	7.29	6.36
0.010	$3.2 \cdot 10^4$	0.025	7.38	7.06
0.005	$3.4 \cdot 10^4$	0.015	7.45	7.20
0.0042	$3.6 \cdot 10^4$	0.0105	7.54	7.52
0.0031	$3.8 \cdot 10^4$	0.00775	7.68	7.62
0.0015	$4.0 \cdot 10^4$	0.00375	7.89	7.82

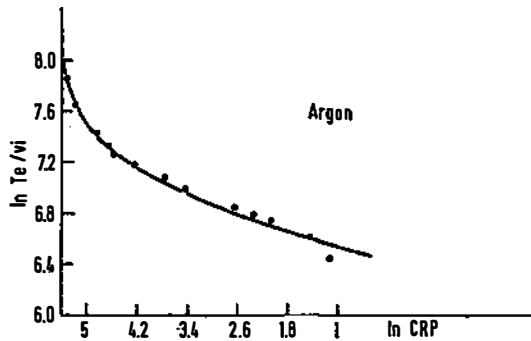


Fig. 3. Verification of Schottky's relation in nonstationary plasma using argon gas.

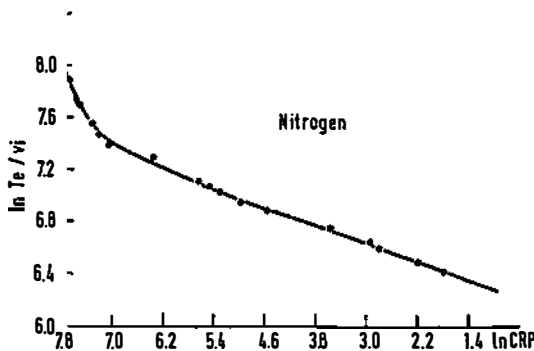


Fig. 4. Verification of Schottky's relation using nitrogen gas.

creases with decreasing pressure. For example, at pressure  $P = 0.34$  Torr the electron temperature  $T_e = 1.63 \cdot 10^4$  K and at  $P = 0.064$  Torr,  $T_e = 2.34 \cdot 10^4$  K.

These results too are in good agreement with the experimental results, obtained in Ref.<sup>3)</sup>.

We also applied Schottky's theory to calculate the electron temperature  $T_e$  for nitrogen as the driver gas in the same shock tube. The results are given in Table 2. and shown in Fig. 4.

*The pressure of nonstationary plasma column.* Schottky's diffusion theory is thus a good tool to measure the gas pressure when the electron temperature is known without need to use any pressure measuring instruments, such as McLeod gauge or ionization gauge.

In the type of shock tube used in Ref.<sup>5)</sup>, where the gas pressure rapidly varies along the expansion section, it is very difficult to measure this variation experimentally. By the aid of Langmuir probes and Schottky's theory, one can

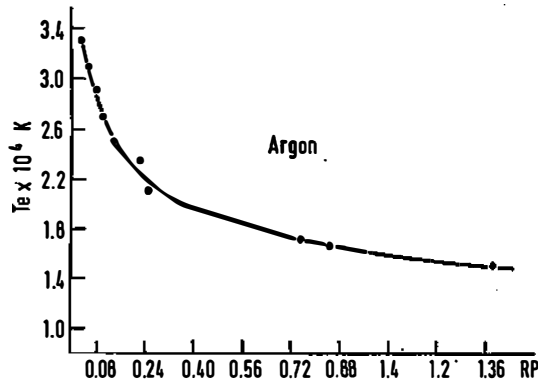


Fig. 5. Calibration curve showing variation of electron temperature with gas pressure for argon.

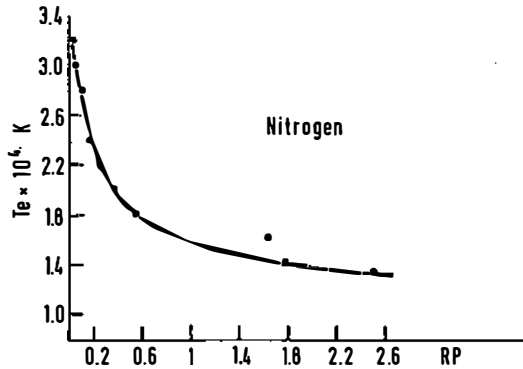


Fig. 6. Calibration curve showing variation of electron temperature with gas pressure for nitrogen.

get the driver gas pressure in this nonstationary plasma column. Electrical probe can be introduced in place where pressure measurement is needed.

The two calibration curves for nonstationary plasma column of shock tube of radius 2.5 cm, using argon and nitrogen gas as driver gases, are shown in Figs. 5 and 6.

From these two curves the pressure  $P$ , at any value of the electron temperature  $T_e$  can be deduced.

#### References

- 1) W. Schottky, *Z. Physik* **25** (1925) 342, 635;
- 2) L. Tonsk and I. Langmuir, *Phys. Rev.* **33** (1929) 195; **34** (1929) 876;
- 3) R. Holm, *Z. Physik* **75** (1932) 171;
- 4) R. Seeliger and Funk, *Z. Physik* **110** (1928) 271;
- 5) A. von Engel and M. Steenbeck, *Elektrische Gasentladungen*, Berlin, Spinger (1934);
- 6) T. Basha, Thesis for the M. Sc. degree, Assiut University, Egypt (1971).