

LETTER TO THE EDITOR

ON SIX-DIMENSIONAL SPACE-TIME*)

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Attempts have been made to symmetrize formally the temporal and spatial parts of space-time. Demers¹⁾ considered a space-time with three temporal and three spatial dimensions. An event in it is specified by a six-vector $(t_1, t_2, t_3, x_1, x_2, x_3)$ (we have put $c = 1$). The scalar product of this vector with itself is defined as

$$-t_1^2 - t_2^2 - t_3^2 + x_1^2 + x_2^2 + x_3^2$$

and the squared interval is

$$ds_6^2 = dt_1^2 - dt_2^2 - dt_3^2 + dx_1^2 + dx_2^2 + dx_3^2.$$

To reconcile this six-dimensional space-time with the experimentally accessible four-dimensional space-time it is supposed that the three temporal components t_1, t_2, t_3 are not measurable separately but that a combination of them can be measured only. If this is taken to be the temporal part of the scalar product, i. e. $t^2 = t_1^2 + t_2^2 + t_3^2$, ds_6^2 coincides with the squared interval in ordinary flat four-dimensional space-time: $ds^2 = -dt^2 + dx_1^2 + dx_2^2 + dx_3^2$.

The advantages of this six-dimensional space-time in connection with generalized Lorentz transformations have been studied²⁾. A time-like state, described in a reference frame as a space-point extended in time, through a generalized Lorentz transformation can be transformed into a space-like state, described in another reference frame as a time-point extended in space. As for the first the

*) After the present paper was submitted for publication the author has become aware of the papers E. A. B. Cole, *Nuovo Cimento* 40A (1977) 171 and A. R. Lee and T. M. Kalotas, *Nuovo Cimento* 41B (1977) 365 which contain similar conclusions though on the basis of different arguments. In the first paper the transformation (1) is derived thoroughly. The author is indebted to E. A. B. Cole for sending a reprint of his paper.

four quantities $t^2 = t_1^2 + t_2^2 + t_3^2$ and x_1, x_2, x_3 can be measured, for the second the quantities t_1, t_2, t_3 and $x^2 = x_1^2 + x_2^2 + x_3^2$ should be measurable, i. e. x_1, x_2, x_3 should not be measurable separately. Thus, the generalization to six dimensions leads to a pleasant symmetry, avoiding imaginary quantities.

The described six-dimensional space-time, however, has properties which make it rather unsuitable. For it a general argument applies put forward against multidimensional spacetime by Dorling³⁾. In this space-time, as in all pseudo-euclidian spaces, three kinds of geodesics exist: time-like ($ds_6^2 < 0$), null ($ds_6^2 = 0$), and space-like ($ds_6^2 > 0$). In it a time-like geodesic between two fixed events can be bent either in a time-like or in a space-like direction getting thereby longer or shorter, respectively. Thus, in this symmetry case time-like geodesics are not maximal lines. In four-dimensional space-time, on the contrary, a time-like geodesic cannot be bent in a time-like direction, time being one-dimensional. So it cannot be made longer and is a maximal line. (Space-like geodesics are not maximal lines in both cases.)

Therefore, according to Dorling, in a six-dimensional space-time a particle can decay into particles with greater total rest mass. The additional temporal components take care of energy-momentum conservation. An electron then could decay, e. g. into a neutron, an antiproton, and an electron-neutrino. Even particles with zero rest mass could decay, e. g. a photon into an electron-positron pair. The supposition that not all three components of total energy W_1, W_2, W_3 are measurable but that the combination $W^2 = W_1^2 + W_2^2 + W_3^2$ could be measured only, does not rule out unobserved decays. A further supposition that particles decaying via both additional temporal components are in fact stable would effectively reestablish one-dimensional time.

A more specific evidence against six-dimensional space-time arises from the study of linear transformations. It is easy to construct a formally adequate six-dimensional Lorentz transformation

$$\begin{aligned} t_1' &= \gamma(t_1 - vx_1), & x_1' &= \gamma(x_1 - vt_1), \\ t_2' &= t_2, & x_2' &= x_2, \\ t_3' &= t_3, & x_3' &= x_3, \end{aligned} \quad (1)$$

where v is the parameter of the transformation and $\gamma = (1 - v^2)^{-1/2}$. This transformation reflects the symmetry of space and time. Under it, indeed, the scalar product of two six-vectors and the squared interval ds_6^2 remain invariant. It is represented by a 6×6 matrix obeying, as the usual Lorentz transformation, the relation $L^{-1}(z) = L(-v)$, in accord with the relativity principle.

The transformation (1), however, does not reproduce the relationship between coordinate time and proper time which in six dimensions should have the form

$$dt_1'^2 + dt_2'^2 + dt_3'^2 = (dt_1^2 + dt_2^2 + dt_3^2)/(1 - v^2).$$

Instead, it gives for $dx_1 = dx_2 = dx_3 = 0$,

$$dt_1'^2 + dt_2'^2 + dt_3'^2 = dt_1^2/(1 - v^2) + dt_2^2 + dt_3^2. \quad (2)$$

This is a very serious obstacle since the equation

$$dt'^2 = dt^2/(1 - v^2)$$

was proven with high accuracy comparing measured life-times of unstable particles at rest and in flight^{4,5)}.

It is not surprising that all three temporal components do not enter transformation (1) and equation (2) in a symmetric way. This indicates that for a time-like state the homogeneity and/or »isotropy« of time is affected. It is known that space-like states affect the homogeneity and/or isotropy of space. In schemes with imaginary superluminal transformations x_2 and x_3 become imaginary whereas x_1 does not⁶⁾ and in schemes with real superluminal transformations the element g_{11} of the metric tensor becomes negative whereas g_{22} and g_{33} do not⁷⁾.

A rather extensive search could not reveal a simple real linear six-dimensional transformation without the shortcomings of (1). So one cannot see how time homogeneity and space homogeneity and isotropy, which are well established on a macroscopic scale, could be warranted in six-dimensional space-time. All this leaves little hope that six-dimensional space-time may be of use in physics⁸⁾.

References

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*⁸⁾ In a broader context n-dimensional space-time with n-3 temporal dimensions was studied in detail by Kalitzin⁸⁾. His outlook, however, does not invalidate our argument, which is much more limited in scope.