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MULTIPOLE FIELD ELECTROMAGNETIC VELOMETERS

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Abstract: The aim of the paper is to demonstrate that the wall potentials on an insulating tube bounding a conductive fluid immersed in several multipole magnetic fields contain all information necessary to determine the flow velocity in all points of a cross-section, irrespective of the existence of axial symmetry in the flow.

1. Introduction

Electromagnetic velometry and electromagnetic measurement of flow are based on Faraday's law of induction. The electromotive forces induced in a moving conducting fluid subject to a magnetic field drive currents and produce a net distribution of electric potential in the body of the fluid, which can be sampled at the wall of the duct, thus conveying information on the flow without the introduction of obstacles. Electromagnetic flow measurement and velometry are reviewed in detail in a monograph by Shercliff[1].

The main problem in the theory of electromagnetic flowmeters and velometers is to relate the measured signal voltage and the nature of the flow as represented by various parameters which can be introduced to describe its properties. The aim of this paper is to show that the flow velocity can be established in any point of a nonconducting tube, provided the flow is submitted to several specially selected inhomogeneous fields and the variation of potential on the periphery of the duct recorded. The procedure permits to construct the distribution of flow velocity over the cross section of a cylindrical tube starting from data sampled on the tube periphery, without introducing any perturbing objects in the flow, and may prove to be of some value in the study of non-Newtonian fluids such as paper pulp and various slurries, where the optical Doppler shift velometer is not applicable due to lack of transparency and the Pitot tube fails by clogging of the orifice. In a round tube the method operates irrespective of the existence of symmetry of rotation in the flow.

2. *The weighted flow theorem*

We proceed by proving a theorem originally derived by Bevir[2)] in his study of ideal flowmeters. The proof is given here as it differs from the original derivation and the set of underlying assumptions is not exactly the same.

The theory of electromagnetic flowmeters and velometers rests on the generalized local form of Ohm's law

$$\frac{1}{\sigma}\vec{J} = \vec{E} + \vec{v} \cdot \vec{B}, \tag{1}$$

where we have denoted the vector current density by $\vec{J}$, the electric field by $\vec{E}$, the fluid velocity by $\vec{v}$, the magnetic induction vector by $\vec{B}$ and σ stands for the fluid conductivity, for which we assume scalar behaviour. Equ. 1 is correct for non-relativistic velocities. We shall further assume that the conductivity is everywhere the same within the fluid and above a certain value below which electromagnetic flowmeters do not operate. Taking the divergence of Equ. 1 then leads to the equation governing the behaviour of electromagnetic flowmeters and velometers

$$\nabla^2 V = \operatorname{div}(\vec{v} \cdot \vec{B}), \tag{2}$$

where V is the electrical potential defined by $\vec{E} \to -\nabla V$.

We now use a well known theorem of vector analysis, Green's second theorem,

$$\int_\tau (g\nabla^2 V - V\nabla^2 g)\,\mathrm{d}\tau = \int_S (g\nabla V - V\nabla g)\cdot \mathrm{d}\vec{S}, \tag{3}$$

where g is a function of position, not yet specified, which we require to satisfy $\nabla^2 g = 0$, i. e. g is a solution of the partial differential equation of Laplace, or a harmonic function, and τ stands for some volume in the fluid. Transforming one of the volume integrals and combining Equs. (2) and (3) we obtain

$$\int_\tau (\vec{v} \cdot \vec{B}) \cdot \nabla g\,\mathrm{d}\tau = \int_S V\nabla g \cdot \mathrm{d}\vec{S} + \int_S g(\vec{E} + \vec{v} \cdot \vec{B}) \cdot \mathrm{d}\vec{S}. \tag{4}$$

Assuming now that the volume τ is bounded by a nonconducting wall in the form of a cylinder of arbitrary cross-section and two planes normal to the axis of the cylinder, and that the current density vector is everywhere in planes normal to the tube axis, the second integral on the right hand side of Equ. (4) vanishes ($\vec{J} \cdot \mathrm{d}\vec{S} =$

$= 0$ on the whole bounding surface). Introducing the magnetic scalar potential by setting $\vec{B} = -\operatorname{grad} \Psi$ we have

$$\int_{\tau} \vec{v} \cdot (\nabla g \cdot \nabla \Psi) \, \mathrm{d}\tau = \int_{S} V \nabla g \cdot \vec{\mathrm{d}S}. \tag{5}$$

We can now consider some finite length of duct between two parallel planes normal to the tube axis and assume that both field and flow are axially persistent, so that the contributions to the surface integral on the left hand side of Equ. 5 from the two cylinder bases cancel, and we obtain

$$\int_{S_{\text{base}}} \vec{v} \cdot (\nabla g \cdot \nabla \Psi) \, \mathrm{d}S = \int_{(C)} V (\nabla g)_n \, \mathrm{d}s, \tag{6}$$

where ds is an element of arc on the contour (C), which is in fact the intersection of a plane normal to the axis and the internal face of the nonconducting wall.

The main assumption in the above derivation is that the problem is two--dimensional, with no axial variation in either flow field, or the function g. Regarding the nature of the fluid we have assumed electrical homogeneity and isotropy, expressed by the spatial constancy of the scalar conductivity, but we have not introduced any typically hydrodynamic conditions such as incompressibility or kinematic boundary conditions at the wall. We have also not assumed, at any stage of the deerivation, that the velocity in the transverse plane is nil. The theorem therefore holds in the presence of transverse or swirl flow.

3. *Multipole fields and the determination of velocity distributions*

The weighted flow theorem, expressed by Equ. (6), conveys the general idea that information on flow velocity in the volume of the fluid is present on the insulating wall of the duct. To make use of Equ. (6), we have to specify the magnetic field and the undetermined function g. The simplest choice would be to concentrate the field in some small volume, and determine local velocity, but this can not be done if the field is to be generated externally, outside the duct. In a round tube we choose a multipolar field defined by

$$\Psi = \frac{1}{l} A r^l \begin{matrix}\cos\\ \sin\end{matrix} l\Theta, \tag{7}$$

where l is an integer, the order of the multipole, A a constant and r and Θ are polar coordinates. For $l = 1$, the field is dipolar (and in fact homogeneous), $l = 2$ corresponds to a quadrupole field shown in Fig. 1 and the higher order multipoles are the hexapole, octupole etc. Details on multipole field magnets as used in accelerators can be found in Septier[3]. The field components are

$$B_r = -\frac{\partial \Psi}{\partial r} = A\, r^{l-1} \begin{matrix}\cos\\ \sin\end{matrix}\, l\Theta, \tag{8}$$

$$B_\Theta = -\frac{1}{r}\frac{\partial \Psi}{\partial \Theta} = A\, r^{l-1} \begin{matrix}\sin\\ \cos\end{matrix}\, l\Theta, \tag{9}$$

which makes the intensity of the vector of magnetic induction a function only of r,

$$B = A\, r^{l-1}, \tag{10}$$

and relates the constant A to the measurable field strength at a pole tip. The magnetic potential expressed by Equ. (7) is a permissible potential, as it satisfies Laplace's partial differential equation.

The second function in Equ. (6) is harmonic, but otherwise wholly arbitrary and we can choose it in a number of ways to achieve some useful results. Bevir[2] relates this function to the current flowing in a flowmeter without magnetic field but with an externally applied voltage on the electrodes, i. e. the function depends on the geometry of the duct and the position and shape of the electrodes. We can also relate the function g to the magnetic field itself, and also in a number of ways. For instance, by setting $g = \Psi$, we get

$$\int_{(C)} V\, B_n\, \mathrm{d}s = 0$$

for any two dimensional field configuration. We could also write $\nabla g = \vec{i}_3 \cdot \nabla \Psi$, where $\vec{i}_3$ is the unit vector of the tube axis, implying that the equiscalar surfaces of g are identical with the equiscalar surfaces of the magnetic potential but turned around for 90 degrees. The result is

$$\int_S v_z\, B^2\, \mathrm{d}S = \int_{(C)} V\, B_T\, \mathrm{d}s, \tag{11}$$

where B_T is the component of magnetic induction tangential to the tube wall. This result was first established by Smyth[4] for a homogeneous field and led to a proposal to measure flow rates irrespective of profile by several pairs of electrodes on the tube periphery. In the context of the present paper, it is noteworthy to remark that with multipole fields Equ. (11) can be interpreted by saying that the l-th harmonic of the peripheral variation of electrical potential is proportional to some axial moment of the velocity distribution function. This is due to the fact that B dependes only on r, and $B_T = B_\Theta$, given by Equ. (9). The axial moment is a quantity similar to the moment of inertia of a distribution of density in a layered body of revolution, and is given by $\int_S r^{2l-1}\, v_z(r, \Theta)\, \mathrm{d}r\, \mathrm{d}\Theta$. The knowledge of several moments suffices to establish a velocity profile with some accuracy if the flow is axially symmetric, and the details of this procedure are given in Section 4.

In the absence of axial symmetry we define

$$g = r^k \begin{matrix}\cos\\ \sin\end{matrix}\, k\,\Theta, \tag{12}$$

and get from Equ. (6) either

$$a^k \int_{(C)} V(a, \Theta) \begin{matrix}\cos\\ \sin\end{matrix}\, k\,\Theta\, \mathrm{d}\Theta = A \int_S v_z(r, \Theta)\, r^{k+l-2} \sin(k - l)\,\Theta\, \mathrm{d}S,$$

$$\text{if} \qquad g = r^k \begin{matrix}\cos\\ \sin\end{matrix}\, k\,\Theta, \qquad \Psi = \frac{A}{l}\, r^l \begin{matrix}\cos\\ \sin\end{matrix}\, l\,\Theta, \tag{13}$$

or

$$a_k \int_{(C)} V(a, \Theta) \begin{matrix}\cos\\ \sin\end{matrix}\, k\,\Theta\, \mathrm{d}\Theta = \pm A \int v_z(r, \Theta)\, r^{k+l-2} \cos(k - l)\,\Theta\, \mathrm{d}S,$$

$$\text{if} \qquad g = r^k \begin{matrix}\cos\\ \sin\end{matrix}\, k\,\Theta, \qquad \Psi = \frac{A}{l}\, r^l \begin{matrix}\sin\\ \cos\end{matrix}\, l\,\Theta. \tag{14}$$

Let us now assume that we can sample the wall potential $V(a, \Theta)$ either by having a multitude of electrodes operated by an electronic switch, or by using a single rotating contact, referenced by an electrode at rest. (This second proposition is not quite impossible as there are many schemes developed to salvage weak signals developed in sensors rotating on shafts). The peripheral variation of potential is always a periodic function of azimuthal angle and we can write

$$V(a, \Theta) = \frac{1}{2} U_0 + \sum_{k=1}^{\infty} (U_k \cos k\,\Theta + V_k \sin k\,\Theta), \tag{15}$$

with

$$\frac{U_k}{V_k} = \frac{1}{\pi} \int_0^{2\pi} V(a, \Theta) \begin{matrix}\cos\\ \sin\end{matrix}\, k\,\Theta\, \mathrm{d}\Theta. \tag{16}$$

The left hand sides of Equ. 13 and 14 are therefore proportional to the k-th Fourier component of the variation of potential which arises on the wall when a multipole field of order l is applied. The two equations are in fact completely equivalent and we use either the one or the other depending on the orientation of the multipole structure relative to the arbitrary datum $\Theta = 0$ and the type of harmonic (cosine or sine) required.

The determination of the harmonics of $V(a, \Theta)$ either with a rotating contact or with a large number of contacts on the tube periphery is a matter of data handling and can be achieved in a number of ways already developed, such as analog and digital correlation techniques, FFT, or by the use of selective amplifiers followed by phase-sensitive detectors referenced by sine or cosine voltages.

Assuming that we have used several multipoles $l = 1, 2, 3, \ldots L$, and extracted several harmonics $k = 1, 2, \ldots K$, the question arises whether all these data are necessary. The quantities determined by the measurement of potential harmonics are the moments of the velocity distribution

$$I_{kl} = \int_S v_z(r, \Theta)\, r^{k+l-2} \cos(k-l)\,\Theta\, \mathrm{d}S, \tag{17}$$

and

$$\mathcal{J}_{kl} = \int_S v_z(r, \Theta)\, r^{k+l-2} \sin(k-l)\,\Theta\, \mathrm{d}S, \tag{18}$$

with $\mathrm{d}S = r\, \mathrm{d}\Theta\, \mathrm{d}r$.

These data form two matrices

$$\begin{matrix} I_{11} & I_{12} & I_{13} \ldots \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \\ \ldots & & \end{matrix} \quad (19) \qquad \text{and} \qquad \begin{matrix} \mathcal{J}_{11} & \mathcal{J}_{12} & \mathcal{J}_{13} \ldots \\ \mathcal{J}_{21} & \mathcal{J}_{22} & \mathcal{J}_{23} \\ \mathcal{J}_{31} & \mathcal{J}_{32} & \mathcal{J}_{33} \\ \ldots & & \end{matrix} \quad (20)$$

the elements of which have the properties $I_{kl} = I_{lk}$, $\mathcal{J}_{kl} = -\mathcal{J}_{lk}$, i. e. the two mattrices are symmetric and antisymmetric, respectively. For instance, $I_{21} = I_{12}$ means that the same moment is obtained by either measuring the second harmonic of potential with a dipolar (homogeneous) field, or by measuring the first harmonic with a quadrupole field. In practice, these two measurements may prove to be accompained with stray signals, and it may prove useful to measure both I_{21} and I_{12} to check the operation of the system. From the point of view of theory, however, some of the measurements are redundant. If the number of multipoles used equals the number of harmonics determined ($L = K$) the matrices 19 and 20 are square and the total number of data to be determined is L^2, which is in fact the number of all diagonal terms in I and one half of all non-diagonal terms in both I and $\mathcal{J}$, as the diagonal terms in $\mathcal{J}$ are zero ($\mathcal{J}_{ll} = 0$).

The determination of the velocity distribution can now proceed in a number of ways, either analytical or purely numerical. Among the numerical methods, an obvious way is to set up a mesh in the plane r, Θ, consisting of L radii $\Theta =$ const. and L circles $r =$ const., yielding a total of L^2 knots at which L^2 values of $v_z(r, \Theta)$ can be determined from the resulting system of L^2 linear equations obtained by approximating the integrals in Equs. 17, 18, with the respective sums.

An analytic method to solve the problem of inversion is the Fourier-Laplace integral transform technique. For any distribution $v_z(r, \Theta)$ we can write

$$v_z(r, \Theta) = \frac{1}{2} u_0(r) + \sum_{n=1}^{\infty} (u_n(r) \cos n\,\Theta + v_n(r) \sin n\,\Theta), \tag{21}$$

with

$$\begin{matrix} u_n(r) \\ v_n(r) \end{matrix} = \frac{1}{\pi} \int_{-\pi}^{\pi} v_z(r, \Theta) \begin{matrix} \cos \\ \sin \end{matrix}\, n\,\Theta\, \mathrm{d}\Theta, \tag{22}$$

which results from the fact that $v_z(r, \Theta)$ for any $r = \text{const.}$ is a periodic function of period 2π, if the flow is stationary. We can now group together the diagonal elements in the matrices (19) and (20) in such a way that the index $n = k - l$ remains constant. From the definition of $I_{kl} = I_{n+l,l}$ (Equ. 17) we have

$$I_{n+l,l} = \int_0^a \int_{-\pi}^{\pi} v_z(r, \Theta)\, r^{n+2l-1} \cos n\,\Theta \, d\Theta \, dr,$$

$$I_{n+l,l} = \pi \int_0^a r^{n+2l-1}\, u_n(r)\, dr. \tag{23}$$

The latter equation is an integral equation in $u_n(r)$, which is easily reduced to the Laplace integral transform by writing $r = a\,e^{-t}$,

$$I_{n+l,l} = \pi\, a^{n+2l} \int_0^\infty u_n(t)\, e^{-(n+2l)t}\, dt, \tag{24}$$

the formal inversion of which is

$$u_n(t) = \frac{1}{\pi} \mathscr{L}^{-1} \frac{I_{n+l,l}}{a^{n+2l}}, \tag{25}$$

with $s = n + 2l$ as the symbolic variable and $t = \ln \frac{a}{r}$. A similar relation exists between $v_n(r)$ and $J_{n+l,l}$.

In practical applications the method may require a large number of multipoles, and the functional dependence $I_{n+l,l}(l)$ may prove difficult to establish due to the fact that the symbolic variable s is a continuous variable and l changes in steps of 1. The fact that the method is formally operative and correct was checked with various simple distributions. It is important to realize that in Equ. (25) n should be kept constant, and l is eliminated by setting $l = (s - n)/2$, after which the standrad Laplace transform technique is applied.

4. *Axially symmetric flow*

The problem is greatly simplified if the flow is axially symmetric. Only the diagonal elements I_{nn} of I are non-zero, all other elements of both I and J vanish. In addition, it is not necessary to measure the entire potential distribution at the wall, $V(a, \Theta)$. It can be proved by separation of variables that the wall potential with axially symmetric flow is of the general type,

$$V(a, \Theta) = U_n \sin n\,\Theta, \tag{26}$$

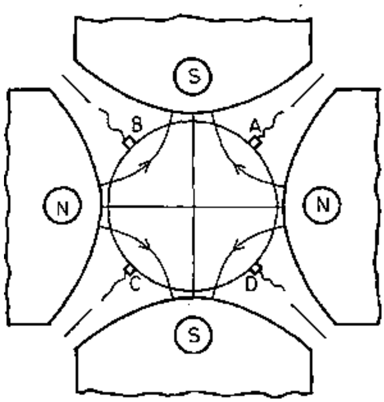

Fig. 1. A quadrupole magnet and a quadrupole field. The pole profile is a hyperbola. The conducting fluid flows normal to the plane of the paper in the round tube. A, B, C and D are possible electrode positions.

where U_n is the voltage between two points such as A and B in Fig. 1. These voltages are directly proportional to the axial moments I_{nn}; combining equations (9), (10), (11) and (26) we get

$$I_{nn} = 2\pi \int_0^a r^{2n-1} v_z(r)\, dr = -\frac{\pi a^n}{A} U_n. \tag{27}$$

The velocity profile can be obtained from the measured voltages and moments by writing $r = a\, e^{-t}$ and inverting the resulting Laplace transform,

$$v_z(t) = \mathscr{L}^{-1} \frac{I_{nn}}{2\pi a^{2n}}, \tag{28}$$

with the symbolic variable $s = 2n$.

5. *Conclusions*

The use of specially selected inhomogeneous magnetic field can convert the standard technique of electromagnetic inducive flowmeters into a method to determine the spatial variation of flow velocity in a round duct, without the introduction of probes in the flow. The quantities that are determined by measurement are the harmonics of the peripheral variation of electrical potential. Theory then relates these observables with the flow velocity distribution in the plane normal to the duct axis. Although the paper is written as if the various fields (dipole, quadrupole, etc.) are applied one by one, it is also possible to generate these by a single set of coils, switched in the same way as this is done with electrical machines of variable number of poles.

With axially symmetric flows it is not necessary to record the entire potential distribution at the wall, and it suffices to measure the voltage at two suitably positionned electrodes within each multipole structure.

The method can be useful with various non-Newtonian conductive fluids and slurries where probles and optical techniques are not applicable. The conductivity should be essentially above 1 mS/m and well below the conductivity of liquid metals. This last condition is essential with AC fields, which are normally applied in electromagnetic flowmeters to avoid electrode polarization effects. With liquid metals a DC field variant of the method is feasable, provided the magnetic Reynolds number can be kept sufficiently low. A D. C. version with the standard multipole magnets used in accelerators is also possible with electrolytic fluids, but only for the study of the fluctuating velocity components in a turbulent flow.

The method is further limited to two-dimensional flows and fields and electrically homogeneous fluids.

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ELEKTROMAGNETSKI BRZINOMJERI SA VIŠEPOLNIM POLJEM

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Sadržaj

U provodnom fluidu koji struji kroz nekoliko nehomogenih multipolnih magnetskih polja, indukuju se električni potencijali čije vrednosti na zidu izolacione cevi u potpunosti određuju brzinu strujanja u svakoj tački poprečnog preseka cevi, bez obzira na to da li je strujanje osno simetrično ili ne.