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ANALYSIS OF $^{12}C+^{12}C$ ELASTIC SCATTERING

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The optical potentials, generated using the Watanabe superposition, single-folding or double-folding approaches are used to calculate the differential cross sections for $^{12}C+^{12}C$ elastic scattering at 127 MeV incident energy. Four sets for the alpha-particle optical potential parameters are used to calculate the optical potentials for ^{12}C projectiles through the Watanabe procedure. One of these potential sets, namely the potential of the shallowest real depth and smallest volume integral J_R, fits the experimental data much better than the other three sets. The renormalized single- and double-folded optical potentials have produced good fits with the experimental data.

1. Introduction

Recently[1,2], we have obtained the optical potentials for ^{12}C-projectiles on a variety of targets and beam energies using the Watanabe superposition model. Taking advantage of the 3α-cluster structure of ^{12}C, the optical potentials for ^{12}C projectiles have been investigated in terms of the alpha-particle optical potentials, which were chosen phenomenologically from the literature. The elastic scattering cross-sections were calculated using these potentials and good fits to the experimental data were obtained without modifying any of the parameters.

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It should however be noticed that there are many ambiguities in the optical model potentials. These are familiar in phenomenological analyses where it is often found that several potentials fit the same data equally well. It is usually thought that one of these is the *physical* potential, namely one that is given by microscopic calculation. However the physical potential may have a form that is significantly different from any of the phenomenological potentials. Furthermore, a potential is of its nature a theoretical construction so care is necessary in describing it as *physical*. It is possible that even among microscopic potentials there are ambiguities in the sense that different types of calculations could conceivably give different potentials that nevertheless give equally good fits to the data[3].

In the present work, the effect of the folding approach on the discrete ambiguities of the phenomenological optical potential parameters for composite projectiles is investigated. Four sets for the alpha-particle optical potential parameters are used to generate the folded optical potentials for ^{12}C-projectiles on ^{12}C targets at 127 MeV bombarding energy. Also we use the alpha-nucleon (α-N) and nucleon-nucleon (N-N) interactions to calculate the folded optical potentials for ^{12}C projectiles through the single- and double-folded approaches, respectively. The elastic scattering differential cross sections are obtained using the resulting potentials and compared with the experimental data as well as with the phenomenological predictions.

2. *Formalism*

Consider the ^{12}C-nucleus composed of three rigid alpha-particles. According to the Watanabe superposition model, the optical potential of ^{12}C-projectiles can be expressed as the sum of the alpha-particle potentials averaged over the internal wavefunction of ^{12}C as follows[2]:

$$V_{^{12}C}(R) = \int |\Psi(r,\varrho)|^2 \left[V_{\alpha_1}\left(|\vec{R} + \frac{1}{3}\vec{\varrho} - \frac{1}{2}\vec{r}|\right) + \right.$$

$$\left. + V_{\alpha_2}\left(|\vec{R} + \frac{1}{3}\vec{\varrho} + \frac{1}{2}\vec{r}|\right) + V_{\alpha_3}\left(|\vec{R} - \frac{2}{3}\vec{\varrho}|\right)\right] \mathrm{d}\vec{r}\,\mathrm{d}\vec{\varrho} \ldots \qquad (1)$$

where $\Psi(r,\varrho)$ is the internal wavefunction of ^{12}C of the form:

$$\Psi(r,\varrho) = \left(\frac{2\sqrt{3}\gamma}{\pi}\right)^{3/2} \mathrm{e}^{-\gamma(2\varrho^2 + 3/2\, r^2)} \ldots \qquad (2)$$

with $\gamma = 0.04667\ \mathrm{fm}^{-2}$. Using the Taylor expansion of V_{α_1}, V_{α_2} and V_{α_3} up to the second order in $\vec{r}$ and $\vec{\varrho}$ we obtain:

$$V_{12C}(R) = 3V_\alpha(R) + \frac{1}{12\gamma}\left[\frac{\mathrm{d}^2 V_\alpha}{\mathrm{d}R^2} + \frac{2}{R}\frac{\mathrm{d}V_\alpha}{\mathrm{d}R}\right] \ldots \qquad (3)$$

Expression (3) is an approximate relation of $V_{12C}(R)$ in terms of the optical potential of the alpha-particle $V_\alpha(R)$.

Since the internal wavefunction of ^{12}C, $\Psi(r, \varrho)$ depends explicitly on the spatial coordinates r and ϱ then $\Psi(r, \varrho)$ is symmetric with respect to the exchange of the spatial coordinates of the alpha clusters constituting ^{12}C. This allows Eq. (1) to be written as

$$V_{12C}(R) = 3 \int |\Psi(r, \varrho)|^2 \, V_\alpha \left(\left|\vec{R} - \frac{2}{3}\vec{\varrho}\right|\right) \mathrm{d}\vec{r}\, \mathrm{d}\vec{\varrho} \ldots \tag{4}$$

On the other hand, the real part of the optical potential for ^{12}C ions is obtained by folding model approaches. In the single-folding approach, the alpha-nucleus potential is calculated by folding an effective alpha-nucleus interaction $V_{\alpha-N}$ into the target density $\varrho_T(r)$[4], i. e.

$$V_{\alpha(S.F.)}(R) = \int \varrho_T(r) \, V_{\alpha-N}(|\vec{R} - \vec{r}|) \, \mathrm{d}\vec{r} \ldots \tag{5}$$

In the present work $V_{\alpha-n}$ is chosen of the Gaussian form[5]:

$$V_{\alpha-N}(S) = -V_0 \, \mathrm{e}^{-KS^2} \tag{6}$$

with $V_0 = 37$ MeV and $K = 0.25\ \mathrm{fm}^{-2}$. The nuclear matter distribution used has the form[6]:

$$\varrho(r) = \varrho_0 (1 + \alpha r^2) \, \mathrm{e}^{-\beta r^2} \tag{7}$$

with $\alpha = 0.49877\ \mathrm{fm}^{-2}$ and $\beta = 0.37408\ \mathrm{fm}^{-2}$. ϱ_0 can be evaluated from the normalization condition:

$$\int \varrho_{12C}(r) \, \mathrm{d}\vec{r} = 12.$$

Thus using Eqs. (5—7) we get:

$$V_{\alpha(S.F.)}(R) = \left(\frac{\pi}{\lambda}\right)^{3/2} V_0 \left(1 + \frac{3\alpha}{2\lambda} + \frac{K^2R^2}{2}\right) \mathrm{e}^{-\eta R^2} \ldots \tag{8}$$

where $\lambda = K + \beta$ and $\eta = K - \dfrac{K^2}{\lambda}$.

Substituting Eq. (8) for Eq. (4) we obtain the single-folded potential for ^{12}C--projectile as

$$V_{12C(S.F.)}(R) = C_1 (C_2 + C_3 R^2) \, \mathrm{e}^{-C_4 R^2} \ldots \tag{9}$$

where

$$C_1 = -24 \, V_0 \, \varrho_0 \left(\frac{\pi\gamma}{\lambda h}\right)^{3/2}.$$

$$C_2 = 1 + \frac{3\alpha}{2\lambda} + \frac{2\alpha K^2}{3\lambda^2 h},$$

$$C_3 = \frac{\alpha K^2}{\lambda^2}\left(1 + \frac{16\,\eta^2}{81\,h^2} - \frac{8\eta}{9h}\right),$$

$$C_4 = \eta - \frac{4\,\eta^2}{9h} \text{ and } h = \frac{4\eta}{9} + 4\gamma.$$

In the double folding approach, a nucleon-nucleon interaction $V_{N-N}(r)$ is folded into the densities of both the projectile and target nuclei[4]:

$$V_{^{12}C(D.F.)}(R) = \int \varrho_1(r_1)\,\varrho_2(r_2)\,V_{N-N}(|\vec{R} - \vec{r}_1 + \vec{r}_2|)\,\mathrm{d}\vec{r}_1\,\mathrm{d}\vec{r}_2 \ldots \qquad (10)$$

Two choices of $V_{N-N}(r)$ have been tried. The first is of the form[7]:

$$V_{N-N}(r) = -\,V_1\,\mathrm{e}^{-K_1 r^2} \ldots \qquad (11A)$$

with $V_1 = 22.332$ MeV and $K_1 = 0.46\ \mathrm{fm}^{-2}$, and the second has the form[8]:

$$V_{N-N}(r) = -\,V_2\,\mathrm{e}^{-K_2 r^2} - V_3\,\mathrm{e}^{-K_3 r^2} \qquad (11B)$$

with $V_2 = 5.447$ MeV, $K_2 = 0.292\ \mathrm{fm}^{-2}$, $V_3 = 12.448$ MeV and $K_3 = 0.415\ \mathrm{fm}^{-2}$. Using expression (7), (10) and (11A) or (11B) yields the double folding expression for the real part of $^{12}C+^{12}C$ potential.

3. *Results and discussion*

Four phenomenological optical potential parameter sets for the alpha-particles scattered from ^{12}C at 56 MeV are used with expression (3) to calculate the optical potentials for ^{12}C at 127 MeV incident energy. These potential parameter sets are given in Table 1 as well as the volume integrals per interacting nucleon pair, $\mathcal{J}_R$, for the real parts of the alpha-particle potentials.

TABLE 1.

Set	V_0	r_0	a_0	W_v	r_I	a_I	$\mathcal{J}_R$
A	40.4	1.75	0.495	16.3	1.75	0.615	260.9
B	115.8	1.5	0.555	24.0	1.5	0.4	514.8
C	151.9	1.24	0.665	28.05	1.24	0.64	467.5
D	216.8	1.3	0.58	28.05	1.5	0.32	685.7

The optical potential parameters for the alpha-particle projection in ^{12}C target at energy 56 GeV taken from Ref. 10. These potentials are parametrized in Woods-Saxon form.

All lengths are in fm, depths in MeV and volume integrals in MeV · fm^3, where 1 fm = 10^{-15} m and 1 MeV = 1.602×10^{-13} J.

The elastic scattering differential cross sections are calculated using the Watanabe potentials obtained by expression (3). The results are shown in Fig. 1 compared with the experimental data as well as the phenomenological Wood-Saxon potential prediction[9]. Coulomb potential used is the potential due to uniform spherical distribution of charges of radius $R_C = 0.8\,(A_P^{1/3} + A_T^{1/3})$ fm where A_P and A_T are the mass number of the projectile and target, respectively. From

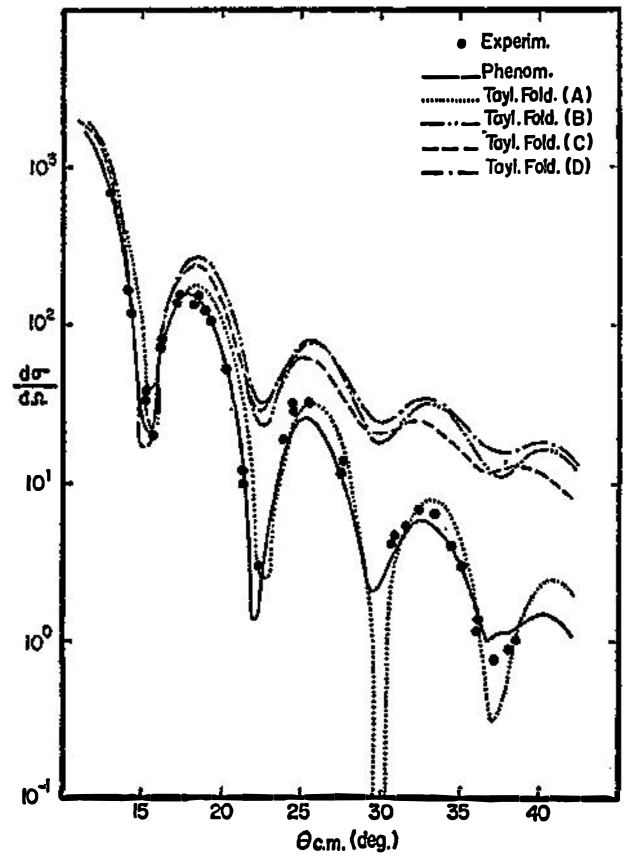

Fig. 1. The differential cross-sections in m^{-31} sr^{-1} for the elastic scattering of ^{12}C-projectiles on ^{12}C-targets at 127 MeV using four sets of the α-particle optical potential parameters (A, B, C and D) given in Table 1. The phenomenological optical potential parameters for ^{12}C-projectiles are[9]: $V_0 = 44.2$ MeV, $N_v = 28.2$ MeV, $R_0 = R_I = 2.1 \times 12^{1/3}$ fm and $a_0 = a_I = 0.683$ fm, where 1 MeV = 1.602×10^{-13} J, 1 fm = 10^{-15} m.

Fig. 1 it is clear that the alpha-particle optical potential with the shallowest real depth and smallest J_R fits the experimental data for $^{12}C + ^{12}C$ elastic scattering much better than the other three, i. e. although different potentials give the same elastic scattering cross-sections for the alpha-particle projectiles, their use in the folded potentials for ^{12}C projectiles leads to different results.

Fig. 2 shows the calculated differential cross-sections of $^{12}C + ^{12}C$ elastic scattering at 127 MeV using the single or double folding real potentials with an imaginary potential taken as a phenomenological fitted Woods-Saxon one from Ref. 9. In the analysis, the folded real potentials were allowed an overall adjustable normalization factor N_R. The fits shown in Fig. 2 require $N_R = 0.7$ and 0.6 for the double folded real potentials with the interaction (11A) and (11B), respectively, and $N_R = 0.7$ for the single folded real potentials of expression (9). Recently, the double folding method has been applied for $^{12}C + ^{12}C$ system with $M3Y$ interaction in the energy range 70 — 126 MeV, the fits required $N_R = 1.01 -$ $- 1.13$[11] and with a δ-function for 300 MeV incident energy $N_R = 0.50$[12].

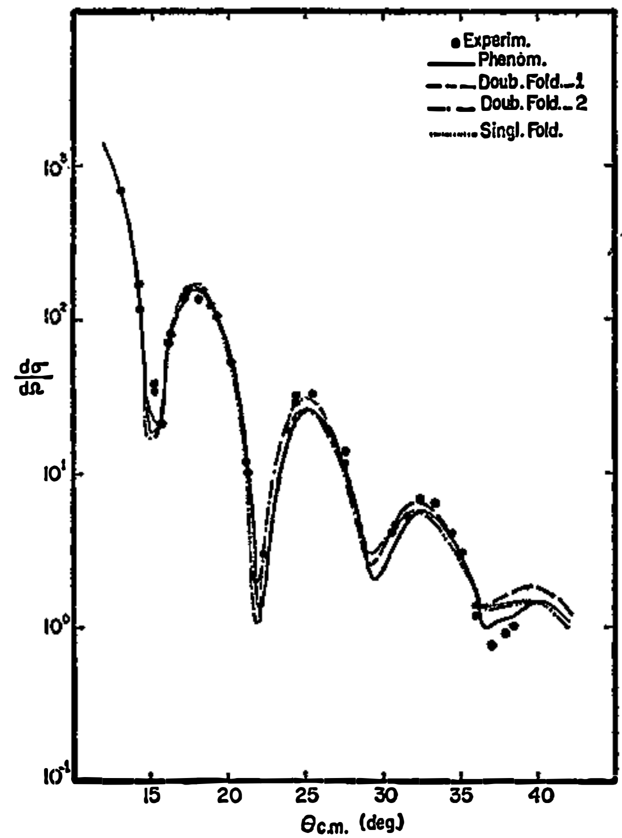

Fig. 2. The differential cross-sections in m^{-31} sr^{-1} for the elastic scattering of ^{12}C-projectiles on ^{12}C-targets at 127 MeV where the dashed curve represents the double-folding results using the interaction (11 A) with $N_R = 0.8$, the dot-dashed curve represents the double-folding result using the interaction (11 B) with $N_R = 0.6$ and the dotted curve represents the single folding result with $N_R = 0.7$.

Fig. 3 shows the real part of the potentials calculated by the Watanabe, single and double folding approaches compared with the phenomenological real potentials. Around the strong absorption radius R_S which is typically about $1.5\,(A_P^{1/3} + A_T^{1/3})$ fm[4], the Watanabe potential is smaller in magnitude than the other potentials.

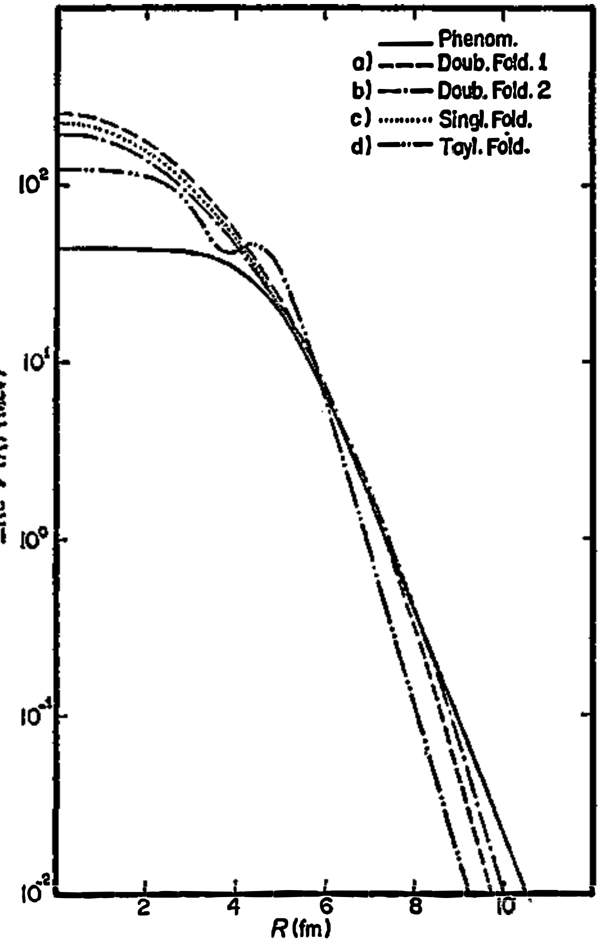

Fig. 3. The real part of the ^{12}C optical potential. The dashed curve is the double folded potential using the interaction (11 A) with $N_R = 0.8$, the dot-dashed curve is the double folded potential using the interaction (11 B) with $N_R = 0.6$, the dotted curve is the single-folded potential with $N_R = 0.7$ and the double dot-dashed curve is the folded potential using Watanabe procedure using the expression (3).

On the other hand, Figs. 1,2 show that the calculated differential cross-sections using the renormalized single or double folded real potentials are in better agreement with the experimental data than in the case of the Watanabe potential. This may be due to the fact that heavy ion scattering data is sensitive only to the tail of the nucleus-nucleus potential, in the vicinity of some strong absorption radius[4].

Finally, the following conclusions may be drawn:

a) The Watanabe superposition procedure describes well the optical potentials of ^{12}C-ions in terms of the alpha-particle optical potentials (where the calculated differential cross-sections agree well with the experimental data).

b) The Taylor expansion used is a good approximation to calculate the optical potentials. This simplification reduces the potential computing time enormously.

c) It seems that the family of the α-particle optical potential parameters with the shallowest real depth and the smallest volume integral J_R reproduces well the experimental data, i. e. the discrete ambiguities of the α-particle potentials disappear when we introduce these potentials in the folding calculations. More detailed conclusions must await studies of other cases.

d) The alpha-nucleon effective interaction has been used successfully through the combination between the Watanabe and single-folding approaches but after normalizing the real calculated potential by a factor 0.7.

e) The two Gaussian effective nucleon-nucleon interactions have been used successfully to reproduce the elastic scattering data through the double--folding approach with real normalization factors of about 0.8 and 0.6. It may be concluded that the factor N_R depends on the used potential.

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ANALIZA ELASTIČNOG RASPRŠENJA $^{12}C+^{12}C$

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Diferencijalni udarni presjeci elastičnog raspršenja $^{12}C+^{12}C$ pri upadnoj energiji od 127 MeV-a analizirani su pomoću optičkog potencijala dobivenog (a) Watanabeovom superpozicijom te (b) metodom jedno- i dvostrukog preklapanja (folding). Za analizu Watanabeovom superpozicijom korištena su 4 skupa parametara alfa-čestičnih optičkih potencijala. Najbolje slaganje s eksperimentom postiže se primjenom skupa parametara kod kojeg je realni potencijal najplići i volumni integral J_R najmanji. Dobro slaganje s eksperimentom postiže se nakon renormalizacije, i s potencijalima dobivenim metodom jedno- i dvostrukog preklapanja.