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INFLUENCE OF THE ENERGY CONTINUUM OF THE NN SUBSYSTEM ON THE π d ELASTIC SCATTERING LENGHT

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The Schwinger functional type equation for the pion-nucleus elastic scattering amplitude is derived. The iteration procedure leads to the Watson series. A truncated scheme for the Green function is studied. The pion-deuteron scattering length is calculated with the first two terms of the series. The results obtained are compared with those for which other methods are used.

1. Introduction

Within the framework of the non-relativistic quantum mechanics the pion-deuteron scattering belongs to those rare cases which can be solved exactly, if the pion absorption is neglected and the interaction between the particles is described by the-two body potentials. Then, using the Faddeev equations the problem of the π d scattering can be solved exactly[1]. The low-energy π d scattering has been studied by a method of summing the multiscattering series[2,3], by equations obtained in the coupled channels procedure[4], by summing the non-relativistic Feynman diagrams[5] and by the method of evolution with respect to the coupling constant[6]. Account of the relativistic effects has been considered in Ref. 7. Estimate of the contribution of the absorption channel into the elastic scattering is given in Refs. 8,9, in which it is shown to be small. The calculation of the $\pi\, d$ scattering length can also be found in Ref. 10.

We shall derive an approximate equation for the pion-nucleus scattering based on the Schwinger variational functional. Along with specifying an elastic--scattering amplitude we shall also give the relevant formulae for the approximate equation for the pion-nucleus scattering, the elastic scattering and the truncated Green functions.

The equations will be written for the $\pi\, d$ elastic scattering case, in particular, for the $\pi\, d$ scattering length (Section 3). Finally, the results of the calculated $\pi\, d$ scattering length are shown and discussed. The comparison with other published results is given in Section 4.

2. *An approximate equation for the pion-nucleus scattering, the elastic scattering and the truncated Green functions*

Let us consider a collision process of a particle by a nucleus. The interaction potential V can be written as a sum of the interaction potentials between the incoming particle and each constituent particle of the target, i. e. $V = \sum_{\alpha=1}^{A} V_\alpha$. In the same spirit the wave function of the whole system is written as follows: $|\psi \pm\rangle = |\varphi\rangle + \frac{1}{E - H_0 - h \pm i\varepsilon} \sum_{\alpha=1}^{A} \tau_\alpha\, |\psi_\alpha^\pm\rangle$. Then, the equation of the multiple scattering theory is obtained:

$$|\psi_\alpha^\pm\rangle = |\varphi\rangle + \frac{1}{E - H_0 - h \pm i\varepsilon} \sum_{\beta \neq \alpha = 1}^{A} \tau_\beta\, |\psi_\beta^\pm\rangle \tag{1}$$

where τ_α is a scattering operator for the incoming particle with a nucleon inside the nuclear matter, satisfying the equation:

$$\tau_\alpha + t_\alpha + t_\alpha (G - G_0)\, \tau_\alpha \tag{2}$$

$$G = \frac{1}{E - H_0 - h \pm i\varepsilon}; \quad G_0 = \frac{1}{E - E_0 - H_0 \pm i\varepsilon}$$

with H_0 being the kinetic energy operator of the incoming particle and h is the target Hamiltonian, E_0 is the energy parameter. The free-nucleon collision operator t_α satisfies the integral equation:

$$t_\alpha = V_\alpha + V_\alpha\, G_0\, t_\alpha. \tag{3}$$

The variational functional for Eq. (1) takes the form[11]:

$$T = \sum_{\alpha=1}^{A} [\langle\psi^- |\tau_\alpha|\varphi\rangle + \langle\varphi|\tau_\alpha|\psi_\alpha^+\rangle - \langle\psi_\alpha^- |\tau_\alpha|\psi_\alpha^+\rangle + \sum_{\gamma\neq\alpha} \langle\psi_\alpha^-|\tau_\alpha G \tau_\gamma|\psi_\gamma^+\rangle]. \quad (4)$$

By writing the initial state $|\varphi\rangle$ as a product of the free particle state $|\vec{K}\rangle$ and the target state, e. q. $|\varphi\rangle = |\vec{K}\rangle|1\rangle$, and the trial functions $|\psi_\alpha^\pm\rangle$ as another product $|\psi_\alpha^\pm\rangle = |F_\alpha^\pm\rangle\,|\,1\rangle$, and varying it with respect to $|F_\alpha^\pm\rangle$, one obtains:

$$\tau_{11}|F_\alpha\rangle = \tau_{11}|\vec{K}\rangle + \sum_{\gamma\neq\alpha} W_{\gamma\alpha}|F_\gamma\rangle \quad (5)$$

where:

$$W_{\alpha\gamma} = \langle 1|\tau_\alpha G \tau_\gamma|1\rangle$$

$$\tau_{11} = \langle 1|\tau_\alpha|1\rangle.$$

For the A-nucleon system Eq. (5) reads:

$$\tau_{11}|F\rangle = \tau_{11}|\vec{K}\rangle + (A-1)\,W_{12}|F\rangle$$

$$w_{12} = \langle 1|\tau_1 G \tau_2|1\rangle \quad (6)$$

$$|F\rangle = |F_\alpha\rangle.$$

Here the rearrangement channels are excluded due to the factorization of the state function. The amplitude for the elastic scattering f, is given by

$$f = -\frac{\mu}{2\pi} A \langle\vec{K}'|\tau_{11}|F\rangle \quad (7)$$

where μ is a reduced mass of the πA system. It should be noted that the solution of Eq. (5), obtained by iteration procedure, reproduces the Watson series, which, up to the second term, is:

$$f = -\frac{\mu}{2\pi} A \{\langle\vec{K}'|\tau_{11}|\vec{K}\rangle + (A-1)\langle\vec{K}'|W_{12}|K\rangle\}. \quad (8)$$

It is tremendously difficult to find the solution of Eq. (6) for the general case due the complex nature of the Green function.

However, the situation appears to be promissing for the πd scattering with the aid of some reasonable approximation. At the same time it has been claimed in Ref. 12 that for energies below the threshold the target Hamiltonian can be truncated, i. e. one can put:

$$h \to h^N = \sum_{n=1}^{N} E_n|n\rangle\langle n| \quad (9)$$

where $|n\rangle$ forms a complete set of the wave functions satisfying:

$$h|n\rangle = E_n|n\rangle. \tag{10}$$

In this case the expression for the Green function is simplified. One has:

$$G \to G^N = G_0(E)(1 - \sum_{n=1}^{N} |n\rangle \langle n|) + \sum_{n=1}^{N} G_0(E - E_n)|n\rangle \langle n|. \tag{11}$$

The equation (6) is reduced to:

$$\tau_{11}|F\rangle = \tau_{11}|\vec{K}\rangle + (A-1)\sum_{n=1}^{N} \tau_{1n} G_0(E - E_n)|F\rangle +$$

$$+ [\langle 1|\tau_1 G_0(E)\tau_2|1\rangle - \sum_{n=1}^{N} \tau_{1n} G_0(E)\tau_{n1}]|F\rangle. \tag{12}$$

3. *The πd elastic scattering and the πd scattering length*

Here the three-particle πd system will be descried by Jacobi coordinates where indices 2 and 3 are used for nucleons and 1 for the incoming pion. The relative momentum of nucleons will be denoted by $\vec{K}_{23}$, and the momentum of a pion with respect to the center of mass of deuteron by $\vec{p}_1$.

We shall assume that the two-body interaction is described by the separable potential:

$$\langle \vec{K}|V^T|\vec{K}'\rangle = -\frac{2\pi}{\mu}\lambda_T g_T(\vec{K}) g_T(\vec{K}')$$

$$g_T(\vec{K}) = \frac{1}{K^2 + \beta_T^2} \tag{13}$$

where μ is a reduced mass of the interacting particles, λ_T is a constant, β_T is a parameter and a superscript T denoted the total isotropic spin index of the potential V according to the decompozition:

$$V_\alpha = V_\alpha^{3/2} P_\alpha^{3/2} + V_\alpha^{1/2} P_\alpha^{1/2}$$

where P^T is the projection operator to the isospin state T. The Coulomb interaction will be neglected. Since we are interested in the πd scattering length the incoming pion energy tends to zero, $\frac{m_\pi}{n_n} \ll 1$. Finally, we shall use the s. c. adiabatic hypothesis and replace τ_t by t_t. Then Eq. (6) becomes:

$$\int t_{11}(\vec{p}_1, \vec{p}_1') F(\vec{p}_1) \frac{d\vec{p}_1'}{(2\pi)^3} = t_{11}(\vec{p}_1, 0) + \int w(\vec{p}_1, \vec{p}_1') F(\vec{p}_1') \frac{d\vec{p}'}{(2\pi)^3}, \tag{14}$$

$$w(\vec{p}_1, \vec{p}_1') = \frac{2}{g} \int [2t^{1/2}(\vec{p}_1, \vec{p}_1'') t^{3/2}(\vec{p}_1'', \vec{p}_1') +$$

$$+ 2t^{3/2}(\vec{p}_1, \vec{p}_1'') t^{1/2}(\vec{p}_1'', \vec{p}_1') + t^{1/2}(\vec{p}_1, \vec{p}_1'') t^{1/2}(\vec{p}_1'' \vec{p}_1')] \times$$

$$\times \langle \vec{K}_{23}' | G\left(\varepsilon_d - \frac{\vec{p}_1''^2}{2\mu_{\pi d}}\right) | - \vec{K}_{23}'' \rangle \varphi_d^* (\vec{K}_{23}) \varphi_d(\vec{q}_{23}) \times \frac{d\vec{K}_{23}}{(2\pi)^3} \frac{d\vec{q}_{23}}{(2\pi)^3} \frac{d\vec{p}_1''}{(2\pi)^3},$$

$$\vec{K}_{23}' = \vec{K}_{23} - \frac{1}{2}(\vec{p}_1'' - \vec{p}_1), \quad \vec{K}_{23}'' = q_{23} - \frac{1}{2}(\vec{p}_1'' - \vec{p}_1').$$

The symbols used are: ε_d is the deuteron binding energy, N is a normalization constant, μ_d is the two nucleon reduced mass, $\varphi_d(\vec{q}) = N \frac{g_0(\vec{q})}{q^2 + a^2}$ is the deuteron wave function and $a^2 = 2\mu d |\varepsilon_d|$.

The Green function expressed through t-matrix is:

$$\langle \vec{K} | G(E) | \vec{K}' \rangle = \frac{(2\pi)^3 \delta(\vec{K} - \vec{K}')}{E - \frac{K^2}{2\mu_d}} + \frac{\langle \vec{K} | t_0(E) | \vec{K}' \rangle}{\left(E - \frac{K^2}{2\mu_d}\right)\left(E - \frac{K'^2}{2\mu_d}\right)} \tag{15}$$

with

$$\langle \vec{K} | t_0(E) | \vec{K}' \rangle = -\frac{2\pi}{\mu_d} \lambda_0 \frac{g_0(\vec{K}) g_0(\vec{K}')}{D(E)}. \tag{16}$$

and

$$D(E) = 1 + \frac{2\pi}{\mu_d} \lambda_0 \int \frac{d\vec{q}}{(2\pi)^3} \frac{g_0^2(\vec{q})}{E - \frac{q^2}{2\mu_d}}. \tag{17}$$

The solution of Eq. (14) after the first iteration, for the πd scattering length, gives:

$$A_{\pi d}[W] = A_{\pi d}^{(1)}[W] + A_{\pi d}^{(2)}[W] \tag{18}$$

$$A_{\pi d}^{(1)}[W] = \frac{\mu_{\pi d}}{\pi} t_{11}(0,0)$$

$$A_{\pi d}^{(2)}[W] = \frac{\mu_{\pi d}}{\pi} W(0,0) \tag{19}$$

where $A_{\pi d}^{(1)}$ is the impulse-approximation term, while in $A_{\pi d}^{(2)}$ the double-scattering contribution is taken into account.

The expression (18) is written in a convenient form which can be compared with the calculation of the $\pi\, d$ scattering length using the Born series expansion. Here the first two terms are:

$$A_{\pi d}\,[\text{Born}] = A^{(1)}_{\pi d}\,[\text{Born}] + A^{(2)}_{\pi d}\,[\text{Born}) \tag{20}$$

with

$$A^{(1)}_{\pi d}\,[\text{Born}] = \frac{\mu_{\pi d}}{2\pi} \langle 1|V|1\rangle \tag{21}$$

$$A^{(2)}_{\pi d}\,[\text{Born}] = \frac{\mu_{\pi d}}{2\pi} \langle 1|VG(\varepsilon_d)\, V|1\rangle.$$

4. Results and discussion

In calculating the $\pi\, d$ scattering length by the approximate scheme which yields expressions (18) and (20), we have used the following set of parameters. All the quantities which enter the nucleon-nucleon potential are consistent with the deuteron binding energy $\varepsilon_d = -2.225$ MeV and with the nucleon-nucleon scattering length $a^t = 5.378$ fm. $\lambda_{T=1/2,3/2}$ is taken from Ref. 1 and $\beta_{T=1/2,3/2} = 3 \cdot 5\ \text{fm}^{-1}$. For the fixed values of the pion-nucleon scattering lengths[14] $a^{1/2}_{\pi N} = -0.257$ fm and $a^{3/2}_{\pi N} = 0.154$ fm. The results of the calculation are listed in Table 1. For the $\pi\, d$-scattering length we used the same values of parameters[13, 18] as in solution of the Faddeev equations. Along with the detailed calculation of the expressions (19) and (21) the validity of the truncated Green function (11) has been tested.

TABLE 1.

References	$A^{(1)}_{\pi d}$ (fm)	$A^{(2)}_{\pi d}$ (fm)
The value obtained from the formula (18) with $E_0 = 0$	0.053	0.042
The value obtained from the formula (18) with use of the approximation (11) with $E_0 = 0$	0.053	0.016
The value obtained from the formula (18) with $E_0 = \varepsilon_d$	0.036	0.044
The value obtained from the formula (20)	0.174	−0.117
The value obtained from the formula (20) with use of the approximation (11)	0.174	−0.139
The non-relativistic Feyman diagram[5]	0.036	0.037
The multi-scattering theory[6]	0.034	0.045
The method of the evaluation of the coupling constant[6]	0.037	0.036

Comparison of the values for $A^{(1)}_{\pi d}$ and $A^{(2)}_{\pi d}$ caulculated in this work with the results obtained by other methods.

For comparison the already published results obtained by other methods are also given.

Note the sensitivity of our results with respect to the off-mass shell contribution. If the amplitude is taken over the mass shell, the difference between our results and those obtained by the use of other theoretical schemes is negligible. In addition, it should be pointed out that the double-scattering contribution is as important as an impulse-approximation term is.

In Table 2 the dependence of $A^{(1)}_{\pi d}[W]$ and $A^{(2)}_{\pi d}[W]$ as well as of $A_{\pi d}[W]$ on the different values of πN scattering lengthes proposed in literature are presented.

TABLE 2.

πN scattering length (fm)	πd scattering length (fm)				
References	$a^{1/2}_{\pi d}$	$a^{3/2}_{\pi d}$	$A^{(1)}_{\pi d}$	$A^{(2)}_{\pi d}$	$A_{\pi d}$
13	−0.242	0.124	0.017	0.033	0.051
14	−0.257	0.154	0.053	0.042	0.094
15	−0.262	0.145	0.036	0.041	0.077
16	−0.244	0.126	0.019	0.034	0.053
17	−0.257	0.126	0.011	0.036	0.047
18	−0.240	0.130	0.028	0.034	0.062

The calculated values for $A^{(1)}_{\pi d}$, $A^{(2)}_{\pi d}$ and $A_{\pi d}$ obtained by changing the values of $a^{1/2}_{\pi d}$ and $a^{3/2}_{\pi d}$.

It follows from Table 2 that the values of the $A^{(1)}_{\pi d}[W]$ and $A^{(2)}_{\pi d}[W]$ are comparable with each other. Similar results have been obtained by other authors[5,6]. This accounts for the fact that only the isoscalar part of the πN scattering amplitude, which is close to zero, contributes to the πd scattering length for the single scattering. For the double scattering one obtains a noticeable contribution due to the isovectorial component of the elementary amplitude. The scattering of the higher parity orders gives essentially a negligible contribution[5].

It is seen that the $A^{(2)}_{\pi d}[W]$ term is less sensitive to the change of the πN scattering length parameter than the $A^{(1)}_{\pi d}[W]$. This is in agreement with the conclusion derived in Ref.6.

The complete results of the calculated $A_{\pi d}[W]$ and $A_{\pi d}$[Born] together with those obtained by using the approximation (11), which we shall denote in terms of $A_{\pi d}$[W] and $A_{\pi d}$[Born] for different values of πN-scattering lengthes, are presented in Table 3. Here the results obtained by Faddeev method[1] are also reported.

It is seen from Table 3 that the values obtained by the iteration method (Eq. (14)) are similar to those of the Faddeev equations provided that the energy parameter in t-matrix of the πN interaction, E_0, is assumed to be equal to ε_d. We shall denote this quantity by $A^{m}_{\pi d}[W]$.

TABLE 3.

πN scattering length (fm)	The πd scattering length (fm)					
References	$A_{\pi d}^{(1)}$ [Born]	$A_{\pi d}^{(2)}$ [Born]	$A_{\pi d}^{(1)}$ [W]	$A_{\pi d}^{(2)}$ [W]	$A_{\pi d}^{m}$ [W]	$A_{\pi d}^{(1)}$
13	0.032	0.016	0.051	0.030	0.039	0.037
14	0.057	0.035	0.094	0.069	0.080	0.075
15	0.046	0.027	0.077	0.052	0.063	0.058
16	0.034	0.017	0.053	0.032	0.041	0.038
17	0.028	0.010	0.047	0.025	0.035	0.031
18	0.040	0.023	0.062	0.041	0.050	0.046

The πd scattering length obtained with the formulae (18) and (20).

One concludes, that the approximation (11) diminishes the $A_{\pi d}$ values, but the important simplification achieved should be kept in mind.

Finally, it should be mentioned that the experimentally found πd scattering length is given with a large error $(0.074 \pm {}^{0.031}_{0.024})$ fm which makes the theoretical argument inconclusive.

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O UTJECAJU ENERGETSKOG KONTINUUMA *NN* PODSISTEMA NA πd

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Izvedene su funkcionalne jednačine Schwinger-ovog tipa za elastično rasejanje π-mezona na atomskim jezgrama. Pokazano je da se iteracionom procedurom dobija Watson-ov red. Proučena je Greenova funkcija konačnog ranga. Dužina rasejanja za slučaj rasejanja π-mezona na deutronu je izračunata za prva dva člana reda. Dobijeni rezultat je uporeden sa vrednostima ove veličine dobijene drugim metodama.