

DEUTERON FORM FACTOR FROM t -CHANNEL ANALYTICITY

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Extended unitarity in the anomalous region is applied to the electromagnetic form factor of the scalar deuteron. A singular integral equation of the Omnes-Muskhelishvili type for the right-hand-cut part of the $d\bar{d} \rightarrow p\bar{p}$ partial-wave scattering amplitude is derived. If the partial waves of the $p\bar{p}$ elastic amplitude are known, the singular integral equation can be solved explicitly. Solutions are determined up to an N_B -order polynomial with arbitrary coefficients, where N_B is the number of bound states given by the Levinson theorem for the $p\bar{p}$ partial wave.

1. Introduction

The electromagnetic form factor of the deuteron is normally calculated from the deuteron wavefunction and the proton and neutron form factors. Precise measurements¹⁾ of form factors have made it possible to include also relativistic and mesonic effects in the calculation²⁾. Some of these effects are ambiguous because there is no reliable relativistic equation of motion for a two-nucleon system. The QCD structure of nucleons and pions, claimed in the literature, appears to be associated with considerable difficulties. We therefore turn our attention to methods based on unitarity and analyticity in the t -channel, which have been successfully applied to nucleon form factors³⁾.

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Owing to the weak deuteron binding, the nearest singularity is the anomalous threshold coming from the graph in Fig. 1. In the usual impulse approximation, this graph is taken into account and the spectator nucleon is placed on its mass shell. In our approach we use *extended unitarity* in the anomalous region:

(1.1)

Particle lines cut by the dashed line are on the mass shell. The off-shell parts follow from analyticity. At no point of the calculation is the spectator nucleon placed on its mass shell. On the contrary, (1.1) contains the full $d\bar{d} \rightarrow N\bar{N}$ amplitude. This amplitude has an imaginary part for $t = (D + \bar{D})^2 > 16MB$ (where B is the binding energy), in addition to the imaginary part arising from the $N\bar{N}$ cut, which leads to an anomalous threshold and to an integral equation of the Omnes-Muskhelishvili type:

(1.2)

Special care should be taken because the threshold is an anomalous one and the two $j = 1 N\bar{N}$ -states are involved simultaneously, which leads to a system of singular integral equations.

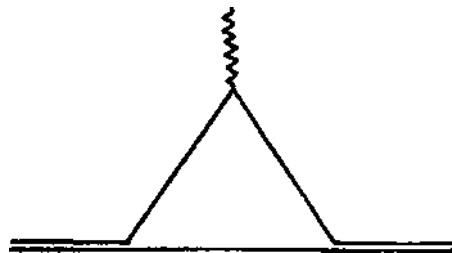


Fig. 1. Graph of a deuteron form factor exhibiting an anomalous threshold.

Indeed, a word of warning is needed here. According to a rule coming from particle physics, which is commonly accepted without questioning, the size of a nuclear system is considered to be inversely proportional to the distance from the

nearest t -channel singularity. At first sight, the closeness of the anomalous threshold to the physical region seems to be sufficient to explain the large size of the deuteron. However, as it has already been pointed out by Eden and Goldstone⁴⁾, the above rule cannot alone account for the large size of a nucleus because the position of the anomalous thresholds does not vary sufficiently with atomic weight A . Such an explanation is not unexpected because the shape of the discontinuity, and not its position, is responsible for the existence of a number of partial waves. That is, distant singularities with an appropriate oscillatory character might produce effects corresponding to large-size systems. This kind of item already appears in the simple two-pole approximation for the deuteron wavefunction.

It is therefore clear that it not possible to consider only the quasi-kinematical features of the theory (finding the positions of the various thresholds, etc.), but an attempt should be made to derive an integral equation involving analytic continuation and unitarity for the spectral function along the cuts. There are only a few papers dealing with these questions.

In the following we shall try to solve singular integral equations connected with the anomalous threshold. The solution of such singular equations usually contains some free parameters that determine its multiplicity. It is important to give a full description of these parameters, as they might provide helpful tools in understanding experimental data.

Very important ingredients of deuteron electromagnetic form factors appear to be certain partial waves of the deuteron-antideuteron \rightarrow nucleon-antinucleon scattering amplitude in a deep unphysical region. These partial waves are related to the electromagnetic form factors of the deuteron by analytic continuation in the mass parameter of the unitarity relation

$$\text{Im} \left[\text{Diagram 1} \right] = \rho \left[\text{Diagram 2} \right] \cdot \left(\text{Diagram 3} \right)^* \tag{a}$$

This relation requires performing a simple integration once the deuteron-anti-deuteron \rightarrow nucleon-antinucleon anomalous amplitude is known. A more careful treatment should be used for the latter amplitude, since its knowledge is connected with the solution of a singular integral equation by the mass-analytic continuation of the Cauchy integral. This is related to the unitarity relation of the form

$$\text{Im} \left[\text{Diagram 4} \right] = \rho \left[\text{Diagram 5} \right] \cdot \left(\text{Diagram 6} \right)^* \tag{b}$$

To be more specific, we shall consider a partial-wave decomposition for both the form factor entering problem (a) and that entering problem (b).

When the full spin structure of the $D\bar{D} \rightarrow N\bar{N}$ process is taken into account, the corresponding two-particle unitarity equation (b) becomes a matrix equation.

There are two partial waves, the s wave and the d wave of the $N\bar{N}$ system, which are mixed. Thus, Eq. (b) is, in fact, a 2×2 matrix equation. In order to make the analytical problems associated with the anomalous threshold as transparent as possible, we shall consider a simplified case of the spinless deuteron. This follows if, for instance, the d wave is neglected.

2. The anomalous threshold

In perturbation theory, an equal-mass two-body scattering amplitude has cuts described by the usual unitarity relations. The scattering amplitude is a holomorphic function of mass-squared parameters in the positive imaginary half-plane. This provides us with a means of understanding how new cuts might emerge from the second Riemann sheet when masses are varied from equal masses to their actual physical values.

Mandelstam⁵⁾ was the first to publish a paper, now already classical, on the anomalous-threshold problem. He varied the Omnes form of the solution of the unitarity equation along the mass parameter M^2 . We prefer here to start with the conventional unitarity equation to make sure that none of the solutions are omitted. Indeed, the non-uniqueness of the final solution provides us with some free parameters that might be helpful in accommodating the physical data.

Let us suppose that we have a fictitious spinless deuteron of mass $M'_D = M$, where M is the nucleon mass. The partial-wave projection A_l of the deuteron-antideuteron proton-antiproton scattering amplitude obeying Eq. (b) satisfies the unitarity equation on the right-hand cut

$$\text{Im } A_l(t) = \rho(t) A_l(t) B_l^*(t), \quad t > 4M^2, \quad (2.1)$$

where $\rho(t) = (1 - 4M^2/t)^{1/2}$ and $B_l(t)$ is the partial-wave amplitude of the nucleon-antinucleon amplitude. Instead of $B_l^*(t + i\epsilon)$, we may use $B_l(t + i\epsilon)$, which is, in fact, B_l at $t + i\epsilon$ on the second Riemann sheet, i. e. $B_l^{II}(t + i\epsilon)$. Hence, we may write

$$\text{Im } A_l(t) = \omega_l(t), \quad t > 4M^2, \quad (2.2)$$

where $\omega_l(t)$ is now the boundary value of the holomorphic function

$$\omega_l(z) \equiv \rho(z) A_l(z) B_l^{II}(z), \quad (2.3)$$

which might also be continued outside the real-axis region.

Neglecting subtraction problems and omitting the index l for simplicity, we can write

$$A(t) = A_L(t) + A_R(t), \quad (2.4)$$

where

$$A_R(t) = \frac{1}{\pi} \int_{4M^2}^{\infty} \frac{\text{Im } A(t' + i\epsilon)}{t' - t} dt' = \frac{1}{\pi} \int_{C'} \frac{\omega(z')}{z' - z} dz' \quad (2.5)$$

and A_L is the left-hand-cut contribution corresponding to the partial wave due to the integration of the $M_D'^2$ pole. The integration region is shown in Fig. 2 as double-arrow segments. It is obvious that this integral vanishes for t above t_1 because the delta function corresponding to the pole term of Fig. 3 cuts the integration region. Hence $t_1(M_D'^2)$ is the rightmost point of the left-hand cut.

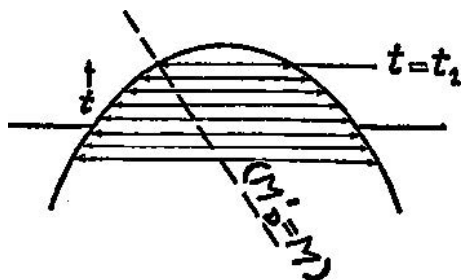


Fig. 2. The position of the pole of the scattering amplitude of antinucleons on a fictitious deuteron of mass $M_D' = M$. The double arrows represent the cosine integration range of the 3rd channel.

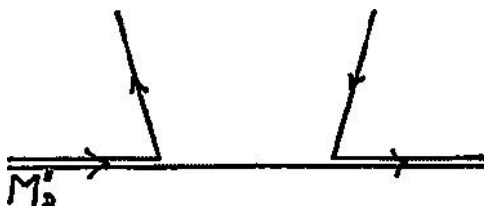


Fig. 3. Graph of a one-nucleon pole.

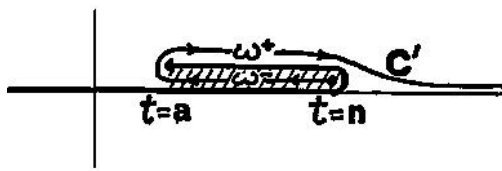


Fig. 4. Hooking of the integration contour C' by the left-hand cut. The values of the function ω in the shaded area correspond to a second Riemann sheet defined by the left cut.

Now, if one varies the mass $M_D'^2$ of the hypothetical deuteron along the upper lip of the real-axis up to the actual value $M_D'^2 = M_D^2$, the pole moves rightwards. The rightmost point of the left-hand cut t_1 increases and *hooks* (see Fig. 4) the integral path corresponding to the right-hand cut before reaching its final position at $t = a$, as shown in Fig. 5 (a being the anomalous threshold). On the other hand, since the function $\omega_l(t)$ from the right-hand-cut dispersion integral (2.5) is itself holomorphic in t , the integration path of (2.5) may be displaced before M_D' is varied. Hence, if this path is contorted enough, no collision with the left-hand

cut will occur. Here we have neglected the possibility of meeting a pole of B'' on the real axis corresponding to an antibound state of the nucleon-antinucleon partial wave.

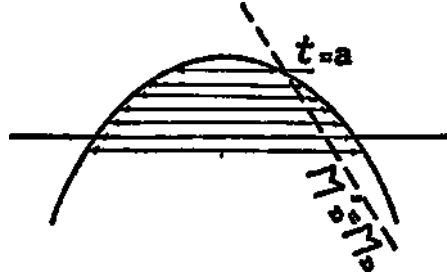


Fig. 5. The final position ($t = a$) of the left-hand cut branching point corresponding to the actual value of the deuteron mass $M'_D \equiv M_D$.

In fact, it is not necessary to take such a complicated integration contour, but we may simply write

$$A_L(t) = \frac{1}{\pi} \int_{-\infty}^a \frac{\Delta_L^*(t')}{t' - t} dt'; \quad \Delta_L(t') \equiv \text{Im } A(t' + i\epsilon), \quad t' < a. \quad (2.6)$$

However, we have to keep in mind that $A_L(t)$ enters $\omega(t)$ via Eq. (2.3), and hence its values are also needed in the shaded area of the contour C' (see Fig. 4). Note that $\omega(t)$ corresponds to the values of A_L on its second Riemann sheet

$$A_L^{II} = A_L^I + 2i\Delta_L, \quad (2.7)$$

where $\Delta_L(t)$ is an analytic continuation of the imaginary part of A_L for $t < a$. There are no technical difficulties in writing the explicit form of Δ_L , at least in the one-pole integral approximation of A_L , where A_L is a well-defined logarithmic-type function. Both A_L and Δ_L are considered to be input functions for the integral equations to be discussed below.

Let us define

$$\delta(t) = \omega(t + i\epsilon) - \omega(t - i\epsilon) \text{ for } a < t < n. \quad (2.8)$$

Then the integral (2.5) along the curve C' (Fig. 4) may be written as

$$A_R(t) = \frac{1}{\pi} \int_a^n \frac{\delta(t')}{t' - t} dt' + \frac{1}{\pi} \int_n^\infty \frac{\omega(t')}{t' - t} dt', \quad (2.9)$$

where, combining (2.8) with (2.3) (here $A^{I,II} \equiv A_L^{I,II} + A_R^{I,II}$), one has

$$\delta(t) = \varrho(t)(A_L^I - A_L^{II})B^{II} + \varrho(A_R(t_+) - A_R(t_-))B^{II}, \tag{2.10}$$

$$a < t < n, t_{\pm} = t \pm i\epsilon.$$

Now, it follows from the integral (2.9) that for the values of t between a and n the difference $A_R(t_+) - A_R(t_-)$ is just $2i\delta(t)$. On the other hand, $A_L^I(t) - A_L^{II}(t)$ is the (known!) function $\Delta_L(t)$. (In the one-pole approximation (Fig. 3), both A_L and its jump $\Delta_L(t)$ are easily computable analytic functions.) Thus, we are left with a simple first-degree algebraic equation for the discontinuity across the anomalous part of the cut:

$$\delta = \varrho\Delta_L B^{II} + 2i\varrho\delta, a < t < n. \tag{2.11}$$

If necessary, the solution of Eq. (2.11) may then be rewritten in terms of the value of the known partial wave of the nucleon-antinucleon amplitude B^I only on the first Riemann sheet, namely

$$\delta = \varrho\Delta_L(t) \frac{B^{II}}{1 - 2i\varrho B^{II}} \equiv \varrho\Delta_L B^I, \quad a < t < n. \tag{2.12}$$

3. Singular integral equations and their solutions

The explicit forms (2.12) and (2.3) for the discontinuities $\delta(t)$ and $\omega(t)$ across the *anomalous* cut and the *normal* cut, respectively, allow us to write a singular integral equation for the right-hand part A_R of the deuteron-antideuteron proton-antiproton partial-wave amplitude $A = A_L + A_R$. Here the functions $A_L, \Delta_L, \varrho, B^I$ and B^{II} are the inputs of the problem and are supposed to be known. Hence the integral equation for A_R reads

$$A_R = \frac{1}{\pi} \int_a^n \frac{\varrho\Delta_L B^I}{t' - t} dt' + \frac{1}{\pi} \int_n^\infty \frac{\varrho(A_L + A_R)B^{II}}{t' - t} dt'. \tag{3.1}$$

Furthermore, it is convenient to modify Eq. (3.1) and write

$$A_R(t) = A_{R1}(t) + \Phi(t), \tag{3.2}$$

where

$$A_{R1}(t) \equiv \frac{1}{\pi} \int_a^n \frac{\varrho\Delta_L B^I}{t' - t} dt' + \frac{1}{\pi} \int_n^\infty \frac{\varrho A_L B^{II}}{t' - t} dt'. \tag{3.3}$$

is assumed to be known, and

$$\Phi(t) = \frac{1}{\pi} \int_n^\infty \frac{\rho A_{R1} B^{II}}{t' - t} dt' + \frac{1}{\pi} \int_n^\infty \frac{\rho \Phi B^{II}}{t' - t} dt'. \quad (3.4)$$

Now both the function Φ and its inhomogeneous term

$$\frac{1}{\pi} \int_n^\infty \frac{\rho A_{R1}(t') B^{II}(t')}{t' - t} dt' \quad (3.5)$$

have only a right-hand cut from n to ∞ .

Let us assume that in the energy range considered there are N_B resonances contributing to the nucleon-antinucleon partial wave $B(t)$. In fact, if we express the nucleon-antinucleon amplitude as

$$B = |B| \exp [i\delta_B(t)],$$

then we have

$$\delta_B(0) = 0, \quad \delta_B(\infty) = N_B \pi, \quad (3.6a)$$

$$|B(\infty)| = 0, \quad (3.6b)$$

where N_B is a known integer. Of course, the singular integral equation (3.4) may be written in its equivalent form ($\Phi^\pm \equiv \Phi(t \pm i\epsilon)$)

$$\Phi^+ - \Phi^- = 2i\rho A_{R1} B^{II} + 2i\rho \Phi^+ B^{II},$$

which might be solved with respect to Φ^+ to yield

$$\Phi^+ = \frac{1}{1 - 2i\rho B^{II}} \Phi^- + 2i\rho A_{R1} \frac{B^{II}}{1 - 2i\rho B^{II}}, \quad (3.7)$$

or simply

$$\Phi^+(t) = G(t) \Phi^-(t) + g(t), \quad n < t < \infty. \quad (3.8)$$

In the above expressions we have used the usual mathematical notation for the Riemann problem, but the reader will notice that $G(t)$ is here the partial-wave component of the S matrix for the nucleon-antinucleon system

$$G = \frac{B^{II}}{1 - 2i\rho B^{II}} \cdot \frac{1}{B^{II}} \equiv \frac{B^I}{B^{I*}} \equiv \exp \{2i\delta_B\}, \quad (3.9a)$$

while the inhomogeneous term g is given by

$$g \equiv 2iq A_{R1} B^l. \tag{3.9b}$$

Eq. (3.8) is an inhomogeneous Riemann problem and its solution is to find a holomorphic function $\Phi(z)$ which has a finite order at infinity and satisfies the condition (3.8) on the boundary line L . In our case, this boundary line is $n < t < \infty$, where $G(t)$ and $g(t)$ are defined; furthermore, $G(t) \neq 0$ everywhere on L .

The usual way of solving such a singular integral equation⁶⁾ is to express the function $G(t)$ as a ratio $X^+(t)/X^-(t)$, (t being on the cut), while $g(t)/X^+(t)$ is $\Psi^+(t) - \Psi^-(t)$, where the functions X and Ψ are both well-defined analytic functions of t , with a cut between $t = n$ and $t = \infty$. In this way, Eq. (3.8) reads as follows:

$$\frac{\Phi^+(t)}{X^+(t)} - \Psi^+(t) = \frac{\Phi^-(t)}{X^-(t)} - \Psi^-(t), \quad n < t < \infty, \tag{3.10a}$$

where $X^+(t)/X^-(t) \equiv G(t)$ and

$$\Psi^+(t) - \Psi^-(t) \equiv g(t)/X^+(t), \quad n < t < \infty. \tag{3.10b}$$

Eq. (3.10a) simply states that the function $\Phi(z)/X(z) - \Psi(z)$ has no cut at all; it is a polynomial in z .

As it is known from the theory of singular integral equations for open contours, special care should be taken at $t = n$ and $t = \infty$ in order avoid spurious singularities. These conditions are counterparts of the index condition appearing for close contours. This strictly limits and determines the multiplicity of the solutions.

It is easy to find a function $X'(z)$ satisfying the condition $X'(t + i\varepsilon)/X'(t - i\varepsilon) = G(t)$ on the cut. Indeed, since $e^{2i\pi\nu} \equiv 1$ only for the integer ν , it is sufficient to have

$$\ln X'(t + i\varepsilon) - \ln X'(t - i\varepsilon) = \ln G(t) - 2\pi i\nu. \tag{3.11}$$

A suitable candidate for such a quantity $X'(t)$ might be found by taking $\nu = N_B$, since in this case

$$\lim_{t' \rightarrow \infty} (\ln G(t') - 2i\pi\nu) = 0$$

and hence the integral

$$\Gamma(t) \equiv \frac{1}{2\pi i} \int_n^\infty \frac{(\ln G(t') - 2\pi i N_B)}{t' - t} dt' \equiv \frac{1}{i\pi} \int_n^\infty \frac{(\delta_B(t') - \pi i N_B)}{t' - t} dt' \tag{3.12}$$

is convergent for any t in the cut complex plane. Moreover,

$$X'(t) \equiv \exp \{ \Gamma(t) \}, \tag{3.13}$$

where $\Gamma(t)$ is finite (or zero) at $t = \infty$. The function $\Gamma(t)$ has a logarithmic singularity at $t = n$, but its sign is such that it produces zero of order N_B in $X'(t) = \exp(\Gamma(t))$. (For example, $A(t) = \int_n^\infty a(t') dt'$ behaves as $\text{const} \cdot \ln(t - t_n)$ for $t \rightarrow n$ when $a(t \rightarrow n) = \text{const} (\neq 0)$.) Thus we have

$$X'(t \rightarrow \infty) = 1, \tag{3.14}$$

$$X'(t \rightarrow n) \approx (t - n)^{N_B} \times (\text{non-zero factors}).$$

(N_B is the number of $N\bar{N}$ resonances and is therefore a nonnegative integer!). This N_B -fold zero at the threshold may cause difficulties in the construction of the function $\Psi(t)$. Indeed, $g(t)/X^+(t)$ for $n < t < \infty$ means that the function $\Psi(t)$ may be expressed as

$$\Psi(t) = \frac{1}{2\pi i} \int_n^\infty dt' \frac{g(t')}{X^+(t')} \cdot \frac{1}{t' - t} = \frac{1}{\pi} \int_n^\infty \frac{\varrho(t') A_{R1}(t') B(t')}{X^{'+}(t')(t' - t)} dt', \tag{3.15}$$

but the factor $1/X^{'+}(t')$, as it stands, has a pole of order N_B at $t' = n$. Therefore, one has to change the definition of X' by moving the zero out of the integration range and write

$$X(t) \equiv \frac{t^{N_B+1}}{(t - n)^{N_B}} X'(t) \equiv \frac{t^{N_B+1}}{(t - n)^{N_B}} e^{\Gamma(t)} \tag{3.16}$$

where $X(t)$ is chosen such

$$X(t + i\epsilon)/X(t' + i\epsilon) = G(t) \text{ (on the cut)} \tag{3.17}$$

that no more poles are present at $t = n$ or elsewhere in the integration region $n < t' < \infty$. The factor t^{N_B+1} has been introduced to produce $1/t^{N_B-1}$ in the integrand $g(t)/X^+(t)$ and to ensure the convergence of the integral in Eq. (3.15) for $t' \rightarrow \infty$. The discontinuity condition (3.9) may now be written in the form

$$\frac{\Phi^+(t)}{X^+(t)} - \Psi^+(t) = \frac{\Phi^-(t)}{X^-(t)} - \Psi^-(t), \tag{3.18}$$

where $X(t)$ is given by Eq. (3.17) and $\Psi(t)$ by Eq. (3.18). Eq. (3.18) simply states that the composite function

$$R(t) \equiv \Phi(t)/X(t) - \Psi(t) \tag{3.19}$$

has no cuts at all and hence has to be some rational function of t . The actual form of $R(t)$ is important since the solution $\Phi(t)$ of the integral equation is expressed in terms of it, namely

$$\Phi(t) = X(t)(R(t) + \Psi(t)). \quad (3.20)$$

(In fact, $R(t)$ determines the multiplicity of the solution $\Phi(t)$; $X(t)$ and $\Psi(t)$ are fixed functions given by Eqs. (3.16) and (3.15), respectively.) If we look for a solution $\Phi(t)$ which is finite for $t \rightarrow \infty$, then it follows from Eq. (3.19) and the fact that both $1/X(t)$ and $\Psi(t)$ vanish at $t \rightarrow \infty$, that $R(t)$ also vanishes. Hence, the unique poles of the rational function $R(t)$ might be connected (see Eq. (3.18) with the $(N_B + 1)$ th-order zero of $X(t)$ at $t = 0$ and we might write

$$\Phi(t) = \frac{e^{\Gamma(t)}}{(t-n)^{N_B}} (\Psi(t) + a_{N_B} t^{N_B} + a_{N_B-1} t^{N_B-1} + \dots a_0), \quad (3.21)$$

where the factor $(t-n)^{-N_B}$ is cancelled by zero of order N_B of $\exp \Gamma(t)$ (see Eqs. (3.12) and (3.18) for the definition of $\Gamma(t)$ and $\Psi(t)$), while $a_{N_B}, \dots a_0$ are arbitrary real constants and represent free parameters of the solution.

4. Conclusion

To summarize, once the nucleon-antinucleon elastic partial wave $B(t)$ is known (and hence the number N_B of its resonances supposed to be finite), the integrals (3.12) and (3.18) defining the functions Γ and Ψ might be evaluated explicitly. Hence the corresponding deuteron-antideuteron partial-wave amplitude $A(t)$ is found as

$$A(t) = A_L(t) + A_{R1}(t) + \Phi(t), \quad (4.1)$$

where $A_L(t)$ is the left-hand-cut contribution (Eq. (2.6)), and A_{R1} is a kind of once iterated Born term containing the *anomalous part* of the cut. $\Delta(t)$ appearing in the first integral of Eq. (3.3) is (see Eq. (2.7)) the value of the difference of A_L on its first and second Riemann sheets, as well as a part of the discontinuity across the normal cut (see Eq. (3.3)). Furthermore, $\Phi(t)$ is the solution of the singular equation and contains $N_B + 1$ free parameters.

As we have stated in the *Introduction*, we have treated here only a simplified spinless example which nevertheless exhibits the main features of the anomalous-threshold problem of the deuteron electromagnetic form factor and of the solutions of the corresponding singular integral equations.

Acknowledgment

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References

- 1) G. G. Simon, *Dissertation*, Mainz (1978);
- 2) J. L. Friar, in *Mesons in Nuclei*, eds. M. Rho and D. Wilkinson, North Holland (1979);
- 3) G. Höhler, E. Pietarinen, I. Sabba-Stefanescu, F. Borkowski, G. G. Simon, V. H. Walther and R. D. Wendling, *Nucl. Phys. B* **114** (1976) 505;
- 4) R. J. Eden and J. Goldstone, *Nucl. Phys.* **49** (1963) 33;
- 5) S. Mandelstam, *Phys. Rev.* **115** (1959) 1741;
- 6) N. I. Muskhelishvili, *Singular Integral Equations*, P. Noord N. V., Groningen, Holland (1946).

FORM FAKTOR DEUTERONA IZ ANALITIČNOSTI t -KANALA

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Proširena unitarnost u anomalnom području primijenjena je na elektromagnetski form faktor skalarnog deuteronu. Izvedena je i riješena singularna integralna jednadžba tipa Omnes-Muskhelishvili za desni rez u amplitudi parcijalnog vala za raspršenje $d\bar{d} \rightarrow p\bar{p}$.