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LETTER TO THE EDITOR

ON THE SCHWINGER-DYSON EQUATION FOR THE DUFFIN-KEMMER PROPAGATOR IN SCALAR ELECTRODYNAMICS

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The first non-trivial approximation of the Schwinger-Dyson equation for the Duffin-Kemmer propagator in scalar electrodynamics is considered. To reach finite solution some rather difficult problems should be solved.

The spin-0 particles are usually described by the Klein-Gordon (*KG*) field but they can be also described by the Duffin-Kemmer (*DK*) field[1,2]. The DK fields for (pseudo)-scalar particles are 5-component fields and the *DK* equation of motion can be derived from the *KG* equation transforming it to the corresponding set of the 5 first-order differential equations. The *DK* and the *KG* formalisms are equivalent when they are used for description of the free particles[3], but, when the particles are in the interaction, some inequivalence may exist[4]. Scalar electrodynamics in one formalism is equivalent to itself in the other one in all orders of the usual perturbation expansion of the *S*-matrix. However, it is possible that this is not true for some modified (unusual) perturbation expansion. It is reasonable to expect that a non-trivial expansion of the Schwinger-Dyson (*SD*) equation for the scalar particle propagator in the *DK* formalism is not equivalent to the same procedure in the *KG* formalism. Namely, an infinite number of the Feynman diagrams contained in the first-order approximation of the *SD* equation in one formalism should not be equivalent to the Feynman diagrams contained in the first-order approximation of the other formalism. Such kind of inequivalence is a motivation for this work. We analyse the non-trivial first-order approximation of the *SD* equation for the spin-0 meson propagator in scalar electrodynamics of the *DK* formalism looking for its finiteness (without ultraviolet divergences before applying the renormalization procedure). The existence of finite scalar electrodynamics in the *KG* formalism is investigated by Fry[5]. He has made analysis along the lines of the Johnson-Baker-Willey[6] programme for finite spinor electrodynamics and his conclusion is that *a completely finite, closed theory of scalar electrodynamics is probably internally inconsistent.*

It is well known that many general expressions derived for spinor electrodynamics have the same form in scalar electrodynamics with the *DK* formalism[2]. One of such expressions is the general form of the *SD* equations. Hence the unrenormalized *SD* equation for the complete meson propagator $G(p)$ in the *DK* formulation of scalar electrodynamics is

$$G^{-1}(p) = G_0^{-1}(p) - \frac{ie_0^2}{(2\pi)^4} \int d^4 q \, \beta^\mu \, G(q) \, D_{\mu\nu}(p-q) \, \Gamma^\nu(p, q) \tag{1}$$

where $G_0^{-1}(p)$ is the inverse bare meson propagator, $D_{\mu\nu}(k)$ is the complete unrenormalized photon propagator, $\Gamma^\nu(p, q)$ is the complete unrenormalized vertex function and β^μ are 5×5 *DK* matrices.

The first non-trivial approximation of the *SD* equation (1) is

$$G^{-1}(p) = G_0^{-1}(p) - \frac{i\, e_0^2}{(2\pi)^4} \int d^4 q \, \beta^\mu \, G(q) \, D^0_{\mu\nu}(p-q) \, \beta^\nu \tag{2}$$

where $D^0_{\mu\nu}(k)$ is the bare photon propagator which is dependent of the longitudinal gauge parameter d_e:

$$D^0_{\mu\nu}(k) = -\frac{1}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) - \frac{d_e}{k^2}\frac{k_\mu k_\nu}{k^2}. \tag{3}$$

An equation in spinor electrodynamics which is analogous to Eq. (2) has been investigated in many papers[6]. An investigation of Eq. (2) in scalar electrodynamics is more difficult than in spinor electrodynamics because the algebra of the β^μ matrices is more complex than the γ^μ matrices. The matrices β^μ satisfy the *DK* algebra

$$\beta^\mu \beta^\nu \beta^\varrho + \beta^\varrho \beta^\nu \beta^\mu = g^{\mu\nu} \beta^\varrho + g^{\varrho\nu} \beta^\mu$$

where $g^{00} = -g^{11} = -g^{22} = -g^{33} = 1$, $g^{\mu\nu} = 0$ for $\mu \neq \nu$ and we shall use the following representation:

$$\beta^0 = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \quad \beta^1 = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -1 & \cdot & \cdot & \cdot \end{pmatrix} \quad \beta^2 = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & -1 & \cdot & \cdot \end{pmatrix}$$

$$\beta^3 = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & -1 & \cdot \end{pmatrix} \quad \beta = \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \quad \overline{\beta} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \end{pmatrix} \tag{4}$$

The general form of the inverse propagator $G^{-1}(p)$ can be done by

$$G^{-1}(p) = \sum_{i=1}^{5} G_i(-p^2)\, g_i(p) \tag{5}$$

where $G_i(-p^2)$ are unknown scalar functions and $g_i(p)$ are linearly independent and complete set of the following matrices:

$$g_1(p) = I,\ g_2(p) = p_\mu \beta^\mu = \hat{p},\ g_3(p) = \beta^\mu \beta_\mu,\ g_4(p) = \hat{p}\hat{p},\ g_5(p) = \hat{p}\, \beta^\mu \beta_\mu. \tag{6}$$

The propagator $G(p)$ can also be expanded in terms of the same matrices $g_i(p)$

$$G(p) = \sum_{i=1}^{5} H_i(-p^2)\, g_i(p) \tag{7}$$

where $H_i(-p^2)$ are some other scalar functions. Starting from the condition

$$G^{-1}(p)\, G(p) = I \tag{8}$$

we obtain relations connecting functions $H_i(-p^2)$ and $G_j(-p^2)$. Note that the following relations

$$\begin{gathered} \hat{p}\hat{p}\hat{p} = p^2 \hat{p},\ \beta^\mu \beta_\mu = \beta + 4\overline{\beta},\ \beta + \overline{\beta} = I,\ \beta\overline{\beta} = \overline{\beta}\beta = 0, \\ \beta\hat{p}\hat{p} = p^2 \beta,\ \overline{\beta}\beta^\mu = \beta^\mu \beta,\ \beta\beta^\mu = \beta^\mu \overline{\beta} \end{gathered} \tag{9}$$

are very useful in many calculations. Substituting the expressions (3), (5), (7) and

$$G_0^{-1}(p) = m_0 - \hat{p} \tag{10}$$

in Eq. (2) we get one matrix equation for unknown functions $G_i(-p^2)$ (or $H_i(-$ $-p^2)$). Multiplying such equation successively by matrices $g_i(p)$ ($i = 1, 2, 3, 4, 5$) and applying traces technique we obtain the 5 coupled non-linear integral equations for the 5 scalar functions $G_i(-p^2)$. Going to Euclidean momentum space by the Wick rotation ($p_0 \to ip_4$, $q_0 \to iq_4$) we can perform the integration over spherical angles. Since the above calculations are rather long we shall give only the final result:

$$1.\quad G_1(x) = m_0 + \frac{\varepsilon}{x} \int_0^x y\, dy \left\{ 8H_3 - 2y\, H_4 + (d_e - 1)\left(2H_3 - \frac{y}{2} H_4\right)\right\} +$$

$$+ \varepsilon \int_x^\infty y\, dy \left\{ \frac{8}{y} H_3 - 2H_4 + (d_e - 1)\left(\frac{2}{y} H_3 - \frac{x}{2y} H_4\right)\right\},$$

$$2.\quad G_2(x) = -1 + \frac{\varepsilon}{2x^2}\int_0^x y^2\,\mathrm{d}y\,\{2H_2 + 10H_5 - (d_e - 1)(H_2 + 5H_5) +$$

$$+ \frac{\varepsilon}{2}\int_x^\infty \mathrm{d}y\,\{2H_2 + 10H_5 - (d_e - 1)(H_2 + 5H_5)\}, \qquad (11)$$

$$3.\quad G_3(x) = \frac{\varepsilon}{x}\int_0^x y\,\mathrm{d}y\left\{2H_1 + (d_e - 1)\left[\frac{y}{2x}H_1 + 2\frac{y-x}{x}H_3 + \frac{y(x-y)}{2x}H_4\right]\right\} +$$

$$+ \frac{\varepsilon}{x}\int_x^\infty y\,\mathrm{d}y\left\{\frac{2x}{y}H_1 + (d_e - 1)\left[\frac{x}{2y}H_1 + \frac{x(x-y)}{2y}H_4\right]\right\},$$

$$4.\quad G_4(x) = \frac{2\varepsilon(d_e - 1)}{x^3}\int_0^x y\,\mathrm{d}y\,(y - x)\,(H_1 + 4H_3 - y\,H_4)$$

$$5.\quad G_5(x) = \frac{\varepsilon}{x^2}\int_0^x y\,\mathrm{d}y\left\{-y\,H_5 + (d_e - 1)\frac{y}{2}H_5\right\} + \varepsilon\int_x^\infty \mathrm{d}y\left(-H_5 + \frac{d_e - 1}{2}H_5\right)$$

where $x = -p^2 = \vec{p}^{\,2} + p_4^2$ and $\varepsilon = e_0^2/32\,\pi^2 = \alpha_0/8\pi$.

The set of Eqs. (11) can be simplified by taking an appropriate value of the longitudinal gauge parameter d_e. There are two possibilities which lead to the considerable simplifications: 1) $d_e = 1$ (the Feynman gauge) leading to $G_4(x) = 0$ and 2) $d_e = 3$ (the Yennie gauge) leading to $G_2(x) = -1$ and $G_5(x) = 0$. Hence, in the Yennie gauge we have only 3 integral equations and they can be written as follows:

$$G_1(x) = m_0 + I_1(\varepsilon, m_0) - x\,I_2(\varepsilon, m_0) + 12\,\varepsilon\int_0^x \left(\frac{y}{x} - 1\right) H_3\,\mathrm{d}y +$$

$$+ \varepsilon\int_0^x y\left(2 - \frac{3y}{x} + \frac{x}{y}\right) H_4\,\mathrm{d}y,$$

$$G_3(x) = x\,I_2(\varepsilon, m_0) + I_3(\varepsilon, m_0) + \varepsilon\int_0^x \left(\frac{y^2}{x^2} + \frac{2y}{x} - 3\right) H_1\,\mathrm{d}y +$$

$$+ 4\varepsilon\int_0^x \left(\frac{y^2}{x^2} - \frac{y}{x}\right) H_3\,\mathrm{d}y + \varepsilon\int_0^x \left(\frac{y^2}{x} - \frac{y^3}{x^2} + y - x\right) H_4\,\mathrm{d}y \qquad (12)$$

$$G_4(x) = \frac{4\varepsilon}{x^3}\int_0^x y\,(y - x)\,(H_1 + 4H_3 - y\,H_4)\,\mathrm{d}y$$

where

$$I_1(\varepsilon, m_0) = \varepsilon \int_0^\infty (12 H_3 - 2y H_4)\, dy, \quad I_2(\varepsilon, m_0) = \varepsilon \int_0^\infty H_4\, dy,$$

$$I_3(\varepsilon, m_0) = \varepsilon \int_0^\infty (3 H_1 - y H_4)\, dy \tag{13}$$

are divergent integrals.

To get finite (without *UV* divergences) solutions for the equations given in (12), the integrals $I_1(\varepsilon, m_0)$, $I_2(\varepsilon, m_0)$ and $I_3(\varepsilon, m_0)$ should be finite. In principle, they can be made finite by choosing appropriate values of their parameters. Since we have two parameters (ε and m_0), but three integrals (I_1, I_2 and I_3), it is probably impossible to reach finiteness of all these quantities consistently. Moreover, one can start with the interaction Lagrangian

$$\mathscr{L}_t(x) = \mathrm{e}_0\, \bar{\chi}(x)\, \beta_\mu\, \chi(x)\, A_\mu(x) + \lambda\, (\bar{\chi}(x)\, \bar{\beta}\, \chi(x))^2 \tag{14}$$

which constains an additional interaction proportional to coupling constant λ and which gives a renormalizable scalar electrodynamics. The corresponding *SD* equations for scalar particle would have an additional parameter λ but also at least one additional integral $I_4(\varepsilon, \lambda, m_0)$ and hence the problem of finiteness still remains unsolved. For the definite answer to this problem a more complete analysis should be done. We may now conclude that scalar electrodynamics in the *DK* formalism, like scalar electrodynamics in the *KG* formalism[5], probably is not finite, at least in the first non-trivial approximation with the 4 space-time dimensions.

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O SCHWINGER-DYSONOVOJ JEDNAČINI ZA DUFFIN-KEMMEROV PROPAGATOR U SKALARNOJ ELEKTRODINAMICI

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Razmatrana je prva netrivijalna aproksimacija Schwinger-Dysonove jednačine za Duffin-Kemmerov propagator u skalarnoj elektrodinamici. Da bi se postiglo konačno rešenje potrebno je rešiti još neke prilično teške probleme.