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THE ROLE OF POLARITONS IN SUPERCONDUCTIVITY PHENOMENON

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Analysing the influence of electromagnetic waves on the magnitude of the superconductivity energy gap it was shown, that some increase can be reached if there is a sandwich between a semi-infinite metal and a semi-infinite dielectric with optical characteristics. It was predicted theoretically that the superconductive motion of electons is possible at the temperatures of about 100-200 K. This effect can be expected only for those electrons which are very close to dielectric part of the sandwich.

1. Introduction

In this work we are interested in exciton as well as in polariton mechanism of superconductivity which can be realized in metal-dielectric contacts. Recently, two ways of increasing of the critical temperature by exciton mechanism have been considered. The first one, suggested by Ginzburg[1], is linked with the penetration of polarized waves from dielectric into the metal. In the second variant, suggested in Ref. 2, conductive electrons tunnel from the metal into the dielectric. In both cases Cooper's pairs are produced by virtual exchange of excitons in a limited layer of metal or dielectric only. Our intention is to consider the polariton mechanism in the metal-dielectric contact. The analysis of the influence of polariton mechanism to the phenomenon of superconductivity is a more realistic approach to the problem of formation of a superconductive state by electromagnetic waves, because the polaritons are real excitations in dielectrics.

2. The effective Hamiltonian

The system which we consider is a metal-dielectric contact without chemical influence. It is supposed that polaritons from dielectric penetrate into the metal and interact with electrons. The Hamiltonian of this system can be written as

$$H = H_e + H_p + H_{int} \tag{1.1}$$

where

$$H_e = \sum_{\vec{q}} E_e(\vec{q})\, a^{+}_{\vec{q}}\, a_{\vec{q}}$$

is the Hamiltonian of the electrons, $a^{+}_{\vec{q}}$, $a_{\vec{q}}$ are Fermi-operators which create (annihilate) the electrons,

$$H_p = \sum_{\vec{k}} \sum_{\varrho=1}^{3} E_\varrho(\vec{k})\, b^{+}_{\vec{k}\varrho}\, b_{\vec{k}\varrho}$$

is the Hamiltonian of polaritons, $b^{+}_{\vec{k}\varrho}$, $b_{\vec{k}\varrho}$ are Bose-operators which create (annihilate) the polaritons and H_{int} is the Hamiltonian of electron-polariton interaction.

Assuming that polaritons penetrate into the metal we consider the electron-polariton interaction as the interaction of electrons and electromagnetic radiation. The interaction of the electron current with the external field can be written in the form

$$H_{int}(t) = -\frac{1}{c}\int \vec{A}(\vec{r}, t)\,\vec{j}(\vec{r}, t)\, \mathrm{d}\vec{r}. \tag{1.2}$$

Now, if the operator of the vector potential $\vec{A}$[3,4] is expressed over polariton operators and the operator of the electron current is expressed over Fermi operators creating and annihilating electrons, we obtain the final expression describing the electron-polariton interaction:

$$H_{int} = \sum_{\vec{k},\varrho} \sum_{\vec{p},\vec{q}} \varphi(\vec{p}, \vec{q}, \vec{k}) \left[F_\varrho(\vec{p}, \vec{q}, \vec{k})\, a^{+}_{\vec{q}}\, a_{\vec{p}}\, b_{\vec{k}\varrho} + F^{*}_\varrho(\vec{p}, \vec{q}, -\vec{k})\, a^{+}_{\vec{\varrho}}\, a_{\vec{p}}\, b_{-\vec{k}\varrho}\right] \tag{1.3}$$

where:

$$\varphi(\vec{p}, \vec{q}, \vec{k}) = \int \mathrm{d}\vec{r}\, e^{i\vec{r}(\vec{k}+\vec{p}-\vec{q})} \tag{1.4}$$

$$F_\varrho(\vec{p}, \vec{q}, \vec{k}) = -\frac{1}{c}\sum_{j=1}^{2} \left(\frac{2\Pi\hbar c}{kV}\right)^{1/2} \frac{\hbar e}{2mV}\, \vec{l}_{\vec{k}j}(\vec{p}+\vec{q})\,[u_{j\varrho}(\vec{k}) + v_{j\varrho}(\vec{k})]. \tag{1.5}$$

The expressions for $u_{j\varrho}$ and $v_{j\varrho}$ can be found in Ref. 5.

In order to derive the definite conclusions about the possibilities and efficiency of the polariton mechanism in increasing of superconductive critical temperature, we shall apply the standard Fröhlich's procedure[6] to the electron-phonon Hamiltonian:

$$H_{eq} = e^{-S} H e^{S} \approx H - [S, H] + \frac{1}{2} [S, [S, H]] \tag{1.6}$$

where S is the antihermitian operator $S = S_1 - S_1^+$

$$S_1 = \sum_{\vec{p}, \vec{q}, \vec{k}} \sum_{\varrho=1} X_\varrho(\vec{p}, \vec{q}, \vec{k})\, a^+_{\vec{q}}\, a_{\vec{p}}\, b_{\vec{k}\varrho}. \tag{1.7}$$

After elimination of the linear terms from electron-polariton interaction and after averaging over the polariton vacuum we finally obtain the effective Hamiltonian in the form:

$$H_{eff} = \langle 0 \mid H_{eq} \mid 0 \rangle = \sum E_e(\vec{q})\, a^+_{\vec{q}}\, a_{\vec{q}} - \sum_{\vec{p}, \vec{q}} \sum_{\vec{\alpha}, \vec{\beta}} \sum_{\vec{k}, \varrho} \varphi(\vec{\alpha}, \vec{\beta}, -\vec{k})$$

$$\varphi(\vec{p}, \vec{q}, \vec{k})\, F_\varrho(\vec{\alpha}, \vec{\beta}, -\vec{k})\, F^*_\varrho(\vec{p}, \vec{q}, -\vec{k}) \left[\frac{1}{E_e(\vec{\alpha}) - E_e(\vec{\beta}) + E_\varrho(\vec{k})} + \right.$$

$$\left. + \frac{1}{E_e(\vec{q}) - E_e(\vec{p}) + E_\varrho(\vec{k})}\right] a^+_{\vec{\beta}}\, a_{\vec{\alpha}}\, a^+_{\vec{q}}\, a_{\vec{p}}\,. \tag{1.8}$$

From (1.8) we can see that the interaction among electrons and polaritons leads to the effective electron Hamiltonian which is formally similar to that which arises due to virtual exchange of excitons.

3. *The analysis of the elementary excitations spectrum*

From Hamiltonian (1.8) we shall separate terms $\vec{q} = -\vec{\beta}$ and $\vec{p} = -\vec{\alpha}$. Then we have:

$$H_{eff} = \sum_{\vec{q}} E_e(\vec{q})\, a^+_{\vec{q}}\, a_{\vec{q}} - \frac{1}{2} \sum_{\vec{p}, \vec{q}} W(\vec{p}, \vec{q})\, a^+_{\vec{q}}\, a^+_{\vec{p}}\, a_{\vec{p}}\, a_{\vec{q}} \tag{2.1}$$

where

$$W(\vec{p}, \vec{q}) = \sum_{\vec{k}, \varrho} \frac{2E_{\varrho(\vec{k})} \mid F_\varrho(\vec{p}, -\vec{q}, \vec{k}) \mid^2 \ \mid \varphi(\vec{p}, -\vec{q}, \vec{k}) \mid^2}{E^2_\varrho(\vec{k}) - [E_e(\vec{p}) - E_e(\vec{q})]^2}. \tag{2.2}$$

In this way we get the Hamiltonian which describes the system of interacting electrons with opposite momenta. The function $W(\vec{p}, \vec{q})$ is the Fourier component of interaction energy of two electrons, caused by virtual exchange of polaritons.

Now we are going to take into account the spin of the electrons. In the first term (2.1) we summ over both spins, while in the second term the summation ranges over the opposite spins:

$$H_{eff}\left\{\vec{q}\left(\frac{1}{2}\right);\ -\vec{q}\left(-\frac{1}{2}\right);\ \vec{p}\left(\frac{1}{2}\right);\ -\vec{p}\left(-\frac{1}{2}\right)\right\} \equiv \mathscr{H}. \tag{2.3}$$

Following the ideas of Bogolyubov[7,8] successfully used in the theory of superfluidity and superconductivity, in Hamiltonian (2.3) we go over to the new Fermi operators $A_{\vec{q}1}$ and $A_{\vec{q}2}$ by the canonical transformation

$$a_{\vec{q}}\left(\frac{1}{2}\right) = u_{\vec{q}} A_{\vec{q}_1} + v_{\vec{q}} A_{-\vec{q}_2};\ u^2_{\vec{q}} + v^2_{\vec{q}} = 1. \tag{2.4}$$

For the final expression we obtain:

$$\mathscr{H}^{(2)} = \mathscr{H}_0 + \mathscr{H}_2, \tag{2.5}$$

$$\mathscr{H}_0 = -\frac{1}{2}\sum_{\vec{q}} \frac{[E_e(\vec{q}) - \varepsilon(\vec{q})]^2}{\varepsilon(\vec{q})}. \tag{2.6}$$

$$\mathscr{H}_2 = \sum_{\vec{p}} \varepsilon(\vec{q})\,[A^+_{\vec{q}_1} A_{\vec{q}_1} + A^+_{-\vec{q}_2} A_{-\vec{q}_2}], \tag{2.7}$$

where

$$\varepsilon(q) = [E^2_e(\vec{q}) + \Delta^2(\vec{q})]^{1/2};\ E_e(\vec{q}) = \frac{\hbar^2 k^2}{2m} - \mu \tag{2.8}$$

and

$$\Delta(\vec{q}) = \sum_{\vec{p}} \frac{W(\vec{p}, \vec{q})\,\Delta_{\vec{p}}}{[E^2_e(\vec{p}) + \Delta^2(\vec{p})]^{1/2}}. \tag{2.9}$$

This means that the Hamiltonian $\mathscr{H}_2$ (2.7) describes the system of quasi-particles with the energy gap, defined by the equation (2.9).

4. *The critical temperature*

It is obvious that further analysis requires certain approximations. Assuming that the excitons interact only with one photon branch[3] and using standard approximations of the *BCS* theory we find:

$$\Delta = 2\frac{\hbar^2}{m} p_F\, p_g\, e^{-\frac{2\Pi^2 \hbar^2}{V\langle W\rangle p_F m}} \tag{3.1}$$

$$\langle W\rangle = \frac{2\Pi\hbar^2}{V}\left(\frac{\hbar e}{2mV}\right)^2 \langle\sum_{\vec{k}} \frac{|\vec{p}-\vec{q}|^2 |\varphi(\vec{p}, -\vec{q}, \vec{k})|^2}{E_{ft}^2(\vec{k})}\rangle \tag{3.2}$$

where p_g is determined by the relation $\hbar v_F p_g = \hbar c k_0$ taking $\varepsilon_k \approx E_{ft}(k)$; ($E_{ex_{min}} \sim 10^{-19}$ J, $E_{ft_{min}} \sim 10^{-20}$ J).

The $\hbar c k_0$ energy is the optimum (E_{ft}) energy exchanged by electrons. In order to calculate the critical temperature we use the relation[9]:

$$T_c = 1.14 \frac{\hbar c k_0}{k_B} e^{-\frac{2\Pi^2 \hbar^2}{\langle W\rangle V p_F m}}. \tag{3.3}$$

We shall calculate the critical temperature for two cases. In the first case we assume that polaritons penetrate into the entire volume of the metal and in the second one that they behave as vacuum photons i. e. that they penetrate into the metal layer of thickness λ, which is of the order 10^{-8} m.

In the first case the function (1.4) is:

$$\varphi(\vec{p}, -\vec{q}, \vec{k}) = \int e^{i\vec{r}(\vec{p}+\vec{q}+\vec{k})} \, d\vec{r} = V\delta(\vec{p}+\vec{q}+\vec{k}) \tag{3.4}$$

so that $\langle W\rangle$ becomes

$$\langle W\rangle = \frac{2\Pi\hbar^2}{V}\left(\frac{\hbar e}{2m}\right)^2 \langle \frac{|\vec{p}-\vec{q}|^2}{\hbar^2\omega_0^2 + \hbar^2 c^2 |\vec{p}+\vec{q}|^2}\rangle = \frac{1}{V}\left(\frac{\hbar e}{2m}\right)^2 \frac{a^6}{(2\Pi)^3} \frac{p_F^6}{5c^2}. \tag{3.5}$$

For $p_F = 10^{10}\,\mathrm{m}^{-1}$ and $a = 3.8 \times 10^{-10}$ we get

$$T_c \sim e^{-5.243\times 10^{8}} \approx 0. \tag{3.6}$$

This result shows that in the case when polaritons penetrate into the whole volume of the metal, the dielectric-metal system does not exibit the superconducting properties.

Now we are going to analyse the second mentioned case. Assuming that the z-axis is orthogonal to the metal surface we obtain:

$$\left|\int e^{i\vec{r}(\vec{p}+\vec{q}+\vec{k})}\right|^2 = V^2 \delta_{k_x+p_x+q_x}\,\delta_{k_y+p_y+q_y} \frac{\sin^2\frac{\lambda}{2}(p_z+q_z+k_z)}{\frac{\lambda^2}{4}(p_z+q_z+k_z)}. \tag{3.7}$$

For the Fourier component of interaction energy of two electrons we get:

$$\langle W^{\lambda}\rangle = \frac{2\Pi\hbar^2}{V}\left(\frac{\hbar e}{2m}\right)^2 \frac{L}{2\Pi} \times$$

$$\times \int \frac{\mathrm{d}k_z\,|\vec{p}-\vec{q}\,|^2}{\hbar^2\omega_0^2 + \hbar^2 c^2\,|(p_x+q_x)^2+(p_y+q_y)^2+k_z^2|} \cdot \frac{\sin^2\frac{\lambda}{2}(p_z+q_z+k_z)}{\frac{\lambda^2}{4}(p_z+q_z+k_z)} \approx$$

$$\approx \frac{2\pi\hbar^2}{V}\left(\frac{\hbar e}{2m}\right)^2 \left\langle \frac{|\vec{p}-\vec{q}\,|^2}{\hbar^2\omega_0^2+\hbar^2c^2\,|p+q\,|^2}\right\rangle \frac{2L}{\Pi\lambda}\int_0^{x_m\approx 10^3} \frac{\sin^2 x}{x^2}\,\mathrm{d}x, \tag{3.8}$$

Consequently:

$$\langle W^{\lambda}\rangle = \langle W\rangle \frac{L}{\lambda}. \tag{3.9}$$

where $\langle W\rangle$ is given by (3.5).

If we take that $L = 10^{-2}$ m and $\lambda = 10^{-8}$ m the critical temperature becomes:

$$T_c = 115.5 - 216.6\,\mathrm{K}. \tag{3.10}$$

These values correspond to the polariton wave vector of the order $0.8373\times10^7 - 1.57\times10^7\,\mathrm{m}^{-1}$ (visible light).

5. *Conclusions*

We can make two conclusion from our observations.

The superconductive motion of electrons at relatively high temperatures in a semi-infinite metal can be obtained only in the layer which is very close to dielectric.

In the sandwich consisting of a metal film (of thickness λ) and massive dielectric with optical characteristics the appearance of high temperature superconductivity is impossible. Such a conclusion is in accordance with all experimental data until now, and with theoretical considerations[10] in which typical values are used in calculations (Fermi wave vector, depth of penetration etc.). However, in the sandwich consisting of semi-infinite metal and the semi-infinite dielectric, the polariton mechanism leads to relatively high critical temperatures. The phonon and polariton mechanisms of superconductivity are acting one against the other. So, our conclusions are valid only if the metal is a good conductor for high temperatures. Besides, the result (3.10) is obtained for $p_q \sim 10^9\,\mathrm{m}^{-1}$ which means that metal with large Fermi-sphere momenta are more convenient for construction of high temperature superconductors. The less approximative approach requires inclusion of electron-phonon interaction as well as the estimation of the infuence of surface polariton and electron states. The more exact solution of Eq. (2.9) is also necessary. This will be the object of our further investigations.

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ULOGA POLARITONA U FENOMENU SUPERPROVODNOSTI

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Razmatran je uticaj elektromagnetskog zračenja na povišenje superprovodnog praga energije. Teorijski je dobijeno da se u spojevima polubeskonačnog metala i polubeskonačnog dielektrika sa optičkim svojstvima, ali samo u onom dijelu metala koji se nalazi neposredno uz dielektrik, može dobiti superprovodno kretanje elektrona na temperaturama od 100 do 200 K.