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INFLUENCE OF THREE-BODY CORRELATIONS IN THE SPECTRUM OF ELEMENTARY EXCITATIONS IN LIQUID ^{4}He

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Using the variational ansatz we obtained an appropriate expression for energy of elementary excitations. Reduction to the Jackson-Feenberg form and the numerical computation pointed out the necessity for precision in the perturbation calculation. We also analysed the influence of three-body correlation on the Jackson-Feenberg spectrum in convolution approximation. Consequently we used and tested in this way Feenberg as well as Woo form of three-body correlation part of the wave function. It was demonstrated that the influence of Feenberg's three-body correlations on convolution approximation is actually insignificant. The same is correct for Woo correlation in phonon region, but in maxon and roton region the influence is about 16% and 5%, respectively. The discrepancy with experimental data of all approximants in maxon and especially in roton region shows that the basis with one and two-phonon funtions describes well only phonons in liquid ^{4}He.

1. Introduction

The energy spectrum of elementary excitations in liquid ^{4}He has been theoretically studied by a great number of researchers. However, no satisfactory agreement between theoretical data and experimental results has been achieved so far in the whole region of the spectrum (particularly in the roton part).

Brillouin-Wigner (*BW*) perturbation calculation is one of the most useful procedure when dealing with this problem. Jackson and Feenberg (*JF*) were the first to use *BW* perturbation procedure when dealing with elementary excitations

in liquid ^{4}He[1,2]. The energy calculation was performed up to the second order. The whole formulation was expressed by a two-body structure factor which was later on obtained theoretically as well[3]. Lee and Lee[4] expanded the perturbation calculation on this basic introducing members up to the fourth order. Their results are, when compared with others, in better agreement with experimental results.

Considering it rigorously $\mathcal{J}F$ expression for energy contains an additional approximation with respect to the energy expression up to the second order[5] (for the non-orthogonalized basis of the perturbation calculation). In the numerator of the $\mathcal{J}F$ correction of the second order we have Bijl-Feynman (BF) energy ε_0 instead of complete energy ε. The same remark refers to the previously mentioned Lee and Lee work.

In this paper we first present additional analytical forms for the energy of elementary excitation of the boson system, state the matrix elements and then study the influence of three-body correlations on the $\mathcal{J}F$ spectrum using therein a special form of three-body structure factor[6,7].

2. *Basic relations*

Let us consider N identical bosons in volume Ω interacting with a two-body potential

$$V = \frac{1}{2} \sum_{ij} V(r_{ij}). \tag{1}$$

To obtain the spectrum of elementary excitation we can start from the variational ansatz for energy

$$E < \frac{\langle \psi_k | H | \psi_k \rangle}{\langle \psi_k | \psi_k \rangle}, \tag{2}$$

where the Hamiltonian of the system

$$H = -\frac{\hbar^2}{2m} \sum_i \Delta_i + V \tag{3}$$

and ψ_k wave function describing the system with excitation momentum $\hbar\vec{k}$. As an essential assumption Jackson an Feenberg assumed to know the solution of the problem of the ground state

$$H\psi_0 = E_0 \psi_0, \tag{4}$$

here E_0 is the energy of the ground state and ψ_0 normalized wave function of the ground state. Excitation energy is defined by the difference

$$\varepsilon = E - E_0. \tag{5}$$

For the basis of the perturbation procedure $\mathcal{F}F$ took one and two-phonon functions describing states with the same momentum $\hbar\vec{k}$:

$$\{ | \vec{k} >, | \vec{k} - \vec{l}, \vec{l} >, \text{ all } \vec{l} \}, \tag{6}$$

where

$$| \vec{k} > = \frac{1}{\sqrt{N S_k}} \varrho_{\vec{k}} \psi_0, \tag{7}$$

$$| \vec{k} - \vec{l}, \vec{l} > = \frac{1}{\sqrt{N^2 S_{k-l} S_l}} \varrho_{\vec{k}-\vec{l}} \varrho_{\vec{l}} \psi_0. \tag{8}$$

$\varrho_{\vec{k}}$ is the Fourier component of the density operator and S_k two-body structure factor defined by the relation

$$S_k = \frac{1}{N} \langle \psi_0 | \varrho_{-\vec{k}} \varrho_{\vec{k}} | \psi_0 \rangle, \ \vec{k} \neq 0. \tag{9}$$

Basis (6) is normalized but not orthogonalized.

Variational ansatz (2) can be considered quite generally and we can take for a trial function the expression of the *first order* in a nonorthogonal (but normalized) basis[5]

$$| \psi > = | m > = a \sum_{n(\neq m)} \frac{\langle n | H - E | m \rangle}{E - H_{nn}} | n \rangle, \tag{10}$$

where a is the variational parameter. Substituting (10) into (2) we obtain:

$$E = H_{mm} + 2a \sum_{n(\neq m)} \frac{| \langle m | H - E | n \rangle |^2}{E - H_{nn}} +$$

$$+ a^2 \sum_{\substack{n \\ (\neq m)}} \sum_{n'} \frac{\langle m | H - E | n \rangle \langle n | H - E | n' \rangle \langle n' | H - E | m \rangle}{(E - H_{nn})(E - H_{n'n'})}. \tag{11}$$

Minimizing this expression with respect to a, we have

$$a = - \sum_{n(\neq m)} \frac{| \langle m | H - E | n \rangle |^2}{E - H_{nn}} \times$$

$$\times \left\{ \sum_{\substack{n \\ (\neq m)}} \sum_{n'} \frac{\langle m | H - E | n \rangle \langle n | H - E | n' \rangle \langle n' | H | m \rangle}{(E - H_{nn})(E - H_{n'n'})} \right\}^{-1}, \tag{12}$$

i. e. for energy

$$E = H_{mm} - \left\{ \sum_{n(\neq m)} \frac{|\langle m | H - E | n \rangle|^2}{E - H_{nn}} \right\} \times$$

$$\times \left\{ \sum_{\substack{n \\ (\neq m)}} \sum_{n'} \frac{\langle m | H - E | n \rangle \langle n | H - E | n' \rangle \langle n' | H - E | m \rangle}{(E - H_{mm})(E - H_{n'n'})} \right\}^{-1}. \tag{13}$$

This relation is one of thegenerali zed form of Goldhamer-Feenberg perturbation--iterative formula[8,9]. Let us state that relation (13) is also correct without regard to statistics. The study of excitations in neutron matter on the basis of relation (13) is still in progress (J. W. Clark and M. Flynn).

We analyse relation (13) in the case of liquid ^{4}He. In the basis (6) after substitution

$$|m> \to |\vec{k}>$$

$$|n> \to |\vec{k} - \vec{l}, \vec{l}>$$

$$\sum_n \to \sum_{\vec{l}}{}',$$

$$E = E_0 + \varepsilon$$

(the prime indicates that all the wave vectors are $\neq 0$), equation (13) has the following form

$$\varepsilon = \varepsilon_0 - \left\{ \frac{1}{2} \sum_{\vec{l}}{}' \frac{|\langle \vec{k} | W | \vec{k} - \vec{l}, \vec{l} \rangle|^2}{\varepsilon - \varepsilon_0(l) - \varepsilon_0(|\vec{k} - \vec{l}|)} \right\}^2 \times$$

$$\times \left\{ \frac{1}{4} \sum_{\vec{l}} \sum_{\vec{l}'} \frac{\langle \vec{k} | W | \vec{k} - \vec{l}, \vec{l} \rangle \langle \vec{k} - \vec{l}, \vec{l} | W | \vec{k} - \vec{l}', \vec{l}' \rangle}{\varepsilon - \varepsilon_0(l) - \varepsilon_0(|\vec{k} - \vec{l}|)} \times \right.$$

$$\left. \times \frac{\langle \vec{k} - \vec{l}', \vec{l}' | W | \vec{k} \rangle}{\varepsilon - \varepsilon_0(l') - \varepsilon_0(|\vec{k} - \vec{l}'|)} \right\}^{-1}, \tag{14}$$

where we introduced $W = H - E_0 - \varepsilon$ and used relations for BF energies:

$$\langle \vec{k} | H - E_0 | \vec{k} \rangle = \frac{\hbar^2 k^2}{2m} \frac{1}{S_k} \equiv \varepsilon_0(k) \equiv \varepsilon_0$$

$$\langle \vec{k} - \vec{l}, \vec{l} | H - E_0 | \vec{k} - \vec{l}, \vec{l} \rangle = \varepsilon_0(|\vec{k} - \vec{l}|) + \varepsilon_0(l).$$

$\mathcal{J}F$ approximation of relation (14) reads

$$\varepsilon = \varepsilon_0 + \frac{1}{2}\sum_{\vec{l}}{}' \frac{|\langle \vec{k}|H - E_0 - \varepsilon_0|\vec{k}-\vec{l},\vec{l}\rangle|^2}{\varepsilon - \varepsilon_0(l) - \varepsilon_0(|\vec{k}-\vec{l}|)}. \tag{15}$$

In the numerator we have ε_0 instead of ε.

As far as we know there are in the same basis two more analytical forms for the energy of elementary excitation in liquid ^{4}He:

1. C. E. Campbell[10]

$$\varepsilon = \left\{\varepsilon_0 + \sum_{\vec{l}}{}' \frac{|g_{kl}|^2}{\varepsilon - \varepsilon_0(l) - \varepsilon_0(|\vec{k}-\vec{l}|)} + \varepsilon \sum_{\vec{l}}{}' \frac{|g_{kl}|^2}{[\varepsilon - \varepsilon_0(l) - \varepsilon_0(|\vec{k}-\vec{l}|)]^2}\right\} \times$$
$$\times \left\{1 + \sum_{\vec{l}}{}' \frac{|g_{kl}|^2}{[\varepsilon - \varepsilon_0(l) - \varepsilon_0(|\vec{k}-\vec{l}|)]^2}\right\}^{-1}, \tag{16}$$

where the function g_{kl} is defined in the Hamiltonian

$$H = \sum_{\vec{k}} \varepsilon_0(k)\, a^+_{\vec{k}}\, a_{\vec{k}} + \sum_{\vec{k}}\sum_{\vec{l}} g_{lk}\,[a^+_{\vec{l}}\, a_{\vec{l}-\vec{k}}\, a_{\vec{k}} + a^+_{\vec{k}}\, a^+_{\vec{l}-\vec{k}}\, a_{\vec{l}}]$$

and

2. Lin-Liu and Woo[11]

$$\varepsilon = \varepsilon_0 + \frac{1}{2}\sum_{\vec{l}}{}' \frac{|\langle \vec{k}|W|\vec{k}-\vec{l},\vec{l}\rangle|^2}{\varepsilon - \varepsilon_0(l) - \varepsilon_0(|\vec{k}-\vec{l}|)} +$$
$$+ \frac{1}{4}\sum_{\vec{l}}{}'\sum_{\vec{l}'}{}' \frac{\langle \vec{k}|W|\vec{k}-\vec{l},\vec{l}\rangle\langle \vec{k}-\vec{l},\vec{l}|W|\vec{k}-\vec{l}',\vec{l}',\rangle\langle \vec{k}-\vec{l}',\vec{l}'|W|\vec{k}\rangle}{[\varepsilon - \varepsilon_0(l) - \varepsilon_0(|\vec{k}-\vec{l}|)]\,[\varepsilon - \varepsilon_0(l') - \varepsilon_0(|\vec{k}-\vec{l}'|)]} \tag{17}$$

3. *Matrix elements*

Most matrix elements contained in relation (14) have been computed earlier[1,2,4,12]:

$$1.\ \langle \vec{k}|H - E_0|\vec{k}-\vec{l},\vec{l}\rangle = \frac{1}{\sqrt{N S_k S_l S_{k-l}}} \frac{\hbar^2}{2m}\{k^2 +$$
$$+ \vec{k}(\vec{k}-\vec{l})(S_l - 1) + \vec{k}\cdot\vec{l}(S_{k-l} - 1). \tag{18}$$

$$2.\ \langle \vec{k}|1|\vec{k}-\vec{l},\vec{l}\rangle = \frac{1}{\sqrt{N S_k S_l S_{k-l}}} S_3(\vec{l}, -\vec{k}, \vec{k}-\vec{l}), \tag{19}$$

where the three-body structure factor is defined by the relation

$$S_3(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{1}{N} \langle \psi_0 \mid \varrho_{\vec{k}_1} \varrho_{\vec{k}_2} \varrho_{\vec{k}_3} \mid \psi_0 \rangle, \sum_i \vec{k}_i = 0, \ \vec{k}_i \neq 0. \tag{20}$$

$$3.\ \langle \vec{k} - \vec{l}, l \mid H - E_0 \mid \vec{k} - \vec{l}', \vec{l}' \rangle = \frac{1}{N\sqrt{S_k S_{l'} S_{k-l} S_{k-l'}}} \times \frac{\hbar^2}{2m} \times$$

$$\times \{\vec{l} \cdot \vec{l}' \cdot S_3(\vec{k} - \vec{l}, -\vec{k} + \vec{l}', \vec{l} - \vec{l}') +$$

$$+ \vec{l} \cdot (\vec{k} - \vec{l}) \cdot S_3(\vec{k} - \vec{l}, -\vec{l}', -\vec{k} + \vec{l} + \vec{l}') + \tag{21}$$

$$+ \vec{l}' \cdot (\vec{k} - \vec{l}) \cdot S_3(-\vec{k} + \vec{l}', \vec{l}, \vec{k} - \vec{l} - \vec{l}') +$$

$$+ (\vec{k} - \vec{l}) \cdot (\vec{k} - \vec{l}') S_3(\vec{l}, -\vec{l}', -\vec{l} + \vec{l}')\}.$$

$$4.\ \langle \vec{k} - \vec{l}, \vec{l} \mid 1 \mid \vec{k} - \vec{l}', \vec{l}' \rangle = \frac{1}{N\sqrt{S_l S_{l'} S_{k-l} S_{k-l'}}} \times$$

$$\times S_4(\vec{l}, \vec{k} - \vec{l}, -\vec{l}', -\vec{k} + \vec{l}'), \tag{22}$$

where the four-body structure factor is defined by the relation

$$S_4(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) = \frac{1}{N} \langle \psi_0 \mid \varrho_{\vec{k}_1} \varrho_{\vec{k}_2} \varrho^{\vec{k}_3} \varrho_{\vec{k}_4} \mid \psi_0 \rangle, \sum_i \vec{k}_i = 0, \ \vec{k}_i \neq 0. \tag{23}$$

There are, in literature, several forms[3,4,13-15] for the three-body structure factor. *JF* introduced convolution approximation

$$S_3^c(\vec{k}_1, \vec{k}_2, \vec{k}_3) = S_{k_1} S_{k_2} S_{k_3}, \tag{24}$$

which was found to be satisfactory and which is a special case of the form[6,7]

$$S_3(\vec{k}_1, \vec{k}_2, \vec{k}_3) = S_3^c [1 + \varrho^2 u_3(\vec{k}_1, \vec{k}_2, \vec{k}_3)]. \tag{25}$$

Function $u_3(\vec{k}_1, \vec{k}_2, \vec{k}_3)$ is the Fourier transform of the three-body function $u_3(r_1, r_2, r_3)$ which describes three-body correlations, beyond the Jastrow, and appears in Feenberg function[17]

$$\psi_0(r_1, r_2, \ldots, r_N) = e^{\frac{1}{2} \bar{u}(1,2,\ldots N)} \tag{26}$$

with

$$\bar{u}(1, 2, \ldots N) = \sum_{i<j} u_2(r_1, r_2) + \sum_{i<j<k} u_3(r_1, r_2, r_3) + \ldots \tag{27}$$

Woo[18] and Feenberg[17] found the following relations for $u_3(\vec{k}_1, \vec{k}_2, \vec{k}_3)$

$$\varrho^2 u_3(\vec{k}_1,\vec{k}_2,\vec{k}_3) = -\frac{\vec{k}_1 \vec{k}_2 \varrho u_2(k_1) \varrho u_2(k_2) + (2,3) + (3,1)}{k_1^2 + k_2^2 + k_3^2}, \tag{28}$$

$$\varrho^2 u_3(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \frac{\vec{k}_1 \cdot \vec{k}_2 \left(1 - \frac{1}{S_{k_1}}\right)\left(1 - \frac{1}{S_{k_2}}\right) + (2,3) + (3,1)}{\frac{k_1^2}{S_{k_1}} + \frac{k_2^2}{S_{k_2}} + \frac{k_3^2}{S_{k_3}}}. \tag{29}$$

The same result as Woo was derived by C. E. Campbell[10].

Using Lee and Lee result[4] for *overlap* integral between two-phonon function we can write for a four-body structure function, according to (23), in the convolution approximation

$$S_4(\vec{l}, \vec{k} - \vec{l}, -\vec{l}', -\vec{k} + \vec{l}') = S_l S_{l'} S_{k-l} S_{k-l'} \times [-2 + S_k + S_{l-l'} + \\ + S_{k-l-l'}]. \tag{30}$$

This result is implicite included in Campbell work as well[12].

4. *Numerical computation and discussion*

Numerical computation of the complete equation (14), could not be carried out with necessary accuracy, although matrix elements were known, as the denominator contains six-dimensional integral. It is possible to start from relation (14) and get a relation which by its form is similar to Goldhamer-Feenberg

$$\varepsilon = \varepsilon_0 + \frac{\varepsilon_2}{1 - \frac{\varepsilon_3}{\varepsilon_2}}, \tag{31}$$

if we note that in the double sum for $n' = n$

$$\langle \vec{k} - \vec{l}, \vec{l} \mid H - E_0 - \varepsilon \mid k - l, l \rangle = \{\varepsilon - \varepsilon_0(l) - \varepsilon_0(|\vec{k} - \vec{l}|)\} + O\left(\frac{1}{N}\right). \tag{32}$$

The following notations are introduced into relation (31)

$$\varepsilon_2 = \frac{1}{2} \sum_{\vec{l}}' \frac{|\langle \vec{k} | W | \vec{k} - \vec{l}, \vec{l} \rangle|^2}{\varepsilon - \varepsilon_0(l) - \varepsilon_0(|\vec{k} - \vec{l}|)} \tag{33}$$

$$\varepsilon_3 = \frac{1}{4} \sum_{\substack{\vec{l} \\ \vec{l} \neq \vec{l}'}}' \sum_{\vec{l}}' \frac{\langle \vec{k} | W | \vec{k} - \vec{l}, \vec{l} \rangle \langle \vec{k} - \vec{l}, \vec{l} | W | \vec{k} - \vec{l}', \vec{l}' \rangle \langle \vec{k} - \vec{l}', \vec{l}' | W | k \rangle}{[\varepsilon - \varepsilon_0(l) - \varepsilon_0(|\vec{k} - \vec{l}|)][\varepsilon - \varepsilon_0(l') - \varepsilon_0(|\vec{k} - \vec{l}|)]}. \tag{34}$$

Leaving out the term ε_3 we have $\varepsilon = \varepsilon_0 + \varepsilon_2$ or in the form prepared for the numerical calculation

$$\varepsilon = \varepsilon_0(k) + \frac{1}{8\pi^2 \varrho} \frac{1}{k} \int_0^\infty \mathrm{d}l \cdot l \int_{|k-l|}^{k+l} \mathrm{d}p \cdot p \frac{1}{S_k S_l S_p} \frac{1}{\varepsilon - \varepsilon_0(l) - \varepsilon_0(p)} \times$$

$$\times \left\{ \frac{\hbar^2}{2m} \left[\left(k^2 + \frac{1}{2}(p^2 - k^2 - l^2) \right)(S_l - 1) - \frac{1}{2}(p^2 - k^2 - l^2)(S_p - 1) \right] + \right.$$

$$\left. + \frac{\hbar^2 k^2}{2m} - \varepsilon S_3(p, l, -k) \right\}. \tag{35}$$

This relation is transformed into $\mathcal{J}F$ if we substitute ε with ε_0 in the numerator.

In the numerical solution of the equation (35) we used the iterative procedure. For each iteration we used 32-point Gauss integration formula with double precision. Relative error was 0.1%. First we took $\mathcal{J}F$ form of relation (35), i. e. $\varepsilon \to \varepsilon_0$ and for S_3 a) convolution approximation (24) and b) approximation (25) in Woo (28) and Feenberg approximation (29). Let us say again that only approximation (25) includes three-body correlations beyond Jastrow's. The obtained results ε_{CA}, ε_W, and ε_{Fee} are presented in Table in the third, fourth and fifth column. The second column of the Table presents BF energy ε_0, the sixth Chang-Campbell theoretically derived structure factor[19] which has been used in this calculation and the seventh column the experimental results[20].

In the phonon region of the spectrum ε_{Fee} coincides with ε_{CA} and they slightly differ from ε_W. In the maxon region, neighbourhood of $k = 1.1 \times 10^{10}\ \mathrm{m}^{-1}$, ε_{CA} and ε_{Fee} follow experimental data better than ε_W. In the roton region of the spectrum, neighbourhood of $k = 1.9 \times 10^{10}\ \mathrm{m}^{-1}$, all the three approximations show significant deviations and ε_W is slightly closer to experimental data.

It can also be seen that Feenberg three-body effects (in $\mathcal{J}F$ approximation) on the convolution approximation are almost insignificant. This conclusion is correct for Woo correlations in phonon region. But in maxon and roton region Woo correlations influence about 16% and 5%, respectively. Campbell determined these effects to be about 10% in the ground state. Later this result was confirmed by Schmidt et al.[21].

TABLE

k (10^{10} m^{-1})	ε_0	ε_{CA}	ε_W	ε_{Fee}	S_k	ε_{exp}	ε
0.1	2.263	2.248	2.232	2.249	0.039		2.248
0.115						2.899	
0.2	4.925	4.803	4.723	4.804	0.072	5.107	4.805
0.3	8.102	7.665	7.499	7.671	0.101	7.743	7.669
0.4	11.648	10.522	10.344	10.531	0.126	10.214	10.523
0.5	15.389	13.096	13.117	13.124	0.150	12.629	12.870
0.6	19.075	15.168	15.658	15.264	0.174	14.838	
0.7	22.478	16.667	17.875	16.819		16.218	
0.8	25.884	17.905	20.021	18.120	0.227	17.508	
0.9	28.138	18.385	21.289	18.643		18.150	
1.0	30.443	18.925	22.541	19.225	0.295	18.702	
1.1	31.362	18.921	22.753	19.230		19.047	
1.2	32.494	19.220	23.109	19.532	0.388	18.978	
1.3	31.865	18.919	22.228	19.194		18.633	
1.4	31.738	19.035	21.764	19.274	0.523	17.874	
1.5	30.018	18.527	20.294	18.699		16.839	
1.6	29.040	18.465	19.465	18.578	0.718	15.459	
1.7	27.215	17.980	18.180	18.026		14.148	
1.8	26.081	17.847	17.465	17.836	0.974	12.767	
1.9	25.397	17.925	17.114	17.868	1.103	11.957	
2.0	25.700	18.520	17.420	18.427	1.204	12.353	
2.1	27.388	19.939	18.655	19.822	1.256	13.803	
2.2	30.466	22.170	20.790	22.039	1.258	16.080	
2.3	34.844	25.066	23.711	24.934	1.225	18.702	
2.4	40.185	28.219	27.041	28.104	1.176	21.394	
2.5	46.019	31.317	30.459	31.236		22.705	
2.6	52.694	34.437	34.036	34.403	1.077	23.464	
2.7	59.081	37.156	37.185	37.164		23.878	
2.8	66.164	39.793	40.260	39.847	1.005	24.154	
2.9	72.510	42.025				24.431	
3.0	79.312	44.542			0.963	24.638	
3.2	91.589				0.946		

All energy ε-s are in 10^{-23} J; ε_o — Bijl-Feynman, ε_{CA} — Jackson-Feenberg, ε_W — Woo, ε_{Fee} — Feenberg, ε_{exp} — experimental and ε — without approximation $\varepsilon \to \varepsilon_0$, energy; S_k — Chang-Campbell structure factor.

If the relation (35) is numerically computed, leaving ε in the numerator, we derive the solution only for $k < 6 \times 10^{10}$ cm^{-1} (eight column of the Table). For greater k the process does not converge. This result and relation (32) show how important it is to distinguish ε and ε_0 in the numerator of (35). Namely the *third* order contains also elements of the *second* order. The substitution of ε with ε_0 somehow includes the change of approximation order of *BW* perturbation procedure. Summarising all this we come to the conclusion that the basis (6), formed by one and two-phonon functions, (including three-body correlations) can describe well only phonon excitations in liquid ^{4}He.

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Koristeći varijacioni princip dobijen je analitički izraz za energiju elementarnih pobuđenja. Restrikcija na Jackson-Feenbergovu formu spekta elementarnih pobuđenja ukazala je na nužnost preciznosti u računu smetnje. Utjecaj tročestičnih korelacija na spektar elementarnih pobuđenja određen u konvolucionoj aproksimaciji također je analiziran. Pokazano je da je ovaj utjecaj neznatan u domeni fononskog i maksonskog dijela spektra. Za rotonski dio spektra, gdje je podudaranje s eksperimentom i dalje loše, utjecaj je nešto izraženiji. Kako je valna funkcija izražena preko jedno- i dvo- fononskih stanja, to je pokazano da i sa uključenim tročestičnim korelacijama ovaj tip funkcije ne opisuje kvantitativno dobro rotonska stanja spektra.