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VERTICAL SEA LEVEL MUON SPECTRA AT SUPER HIGH ENERGIES CALCULATED FROM THE RECENT PRIMARY COSMIC RAY SPECTRUM

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The vertical sea level muon spectrum has been calculated from the recent primary cosmic ray spectrum. On assuming the validity of accelerator data in the fragmentation region for $x > 0.1$ in the framework of Feynman scaling and using the meson atmospheric diffusion equation we have estimated the muon flux at first generation of parents from $pp \to \pi^{\pm} X$, $pp \to K^{\pm} X$, $\pi p \to \pi^{\pm} X$ inclusive reaction sources and second generation from $pp \to K^{\pm} X$ inclusive reaction increases with muon energy. The flux of muons for second generation and third generation for $pp \to \pi^{\pm} X$ inclusive reaction decreases with energy. The derived differential muon spectrum below 1 TeV is well in agreement with the direct magnetic spectrograph data of Holmes et al., Allkofer et al., Nandi and Sinha, Ayre et al. and Green et al. Above 1 TeV energy the calculated muon spectrum is in accord with the direct measurements of Thompson et al., Amineva et al. and Varkovitskaya et al. The derived differential muon spectrum in the range 0.5 — 30 TeV follows the form

$$\sum_i M_i (E_\mu) \, \mathrm{d}E_\mu = 9.5 \, E_\mu^{-3.6} \, \mathrm{d}E_\mu \, (\mathrm{cm}^2 \ \mathrm{s\ sr\ GeV})^{-1}$$

and the corresponding integral muon spectrum follows

$$\sum_i M_i (> E_\mu) = 3.65 \, E_\mu^{-2.6} \, (\mathrm{cm}^2 \ \mathrm{s\ sr})^{-1}.$$

This integral spectrum agrees the measured data of Chin et al., Akashi et al., Krishnaswamy et al. and Sheldon et al.

1. Introduction

The derivation of spectral shapes at different depths in the atmosphere from primary cosmic ray spectrum is necessary in cosmic ray phenomenology and for the tests of validity of accelerator data at super high energies.

In a recent investigation we (Bhattacharyya and Pal[1]) have calculated the sea level muon spectra up to 10 TeV energy from a predicted primary cosmic ray spectrum. In that study we used the Fermilab data of Johnson et al.[2] and Elbert et al.[3] along with the meson atmospheric diffusion equation originally developed by Zatsepin and Kuzmin[4] and later modified by Bugaev et al.[5]. In the present investigation we assumed that in the fragmentation region ($x > 0.1$) the spectrum of particles created in hadron-nucleus collisions coincides with the spectrum of particles in hadron-nucleon collision. In another study we have (Pal and Bhattacharyya[6]) derived the all particle primary spectrum from the recent balloon flight data[7,8] which has been found to follow a power law fit. Using this spectrum as hadron source the sea level muon spectra have been estimated for various generations of parents. The calculated differential spectrum have been compared with the magnetic spectrograph data of Holmes et al.[9], Allkofer et al.[10], Nandi and Sinha[11], Ayre et al.[12] and Green et al.[13]. Above 1 TeV energy the derived spectrum is in accord with the experimental data of Thompson et al.[14], Amineva et al.[15] and Varkovitskaya et al.[16]. The agreement of the derived integral muon spectrum with the directly measured data of Krishnaswamy et al.[17], Sheldon et al.[18] and with the indirect measurements of Chin et al.[19] and Akashi et al.[20] has been studied.

2. Nuclear physics and kinematics

A) Primary spectrum and calculation of pion and kaon spectra:

In a recent investigation we (Pal and Bhattacharyya[6]) have determined the all particle primary spectrum from the balloon flight proton and helium intensity data of Japanese American Cooperative Emulsion Chamber Experiments[7] and nuclei intensity data compilation of Abulova et al.[8]. On assuming that the nuclei break up into free nucleons near the top of the atmosphere, i. e. a heavy nucleus of mass A behaves as A independent nucleons in its interactions in the atmosphere the spectrum of primary partices follows the form:

$$N(E)\,\mathrm{d}E = 2.36\,E^{-2.7}\,\mathrm{d}E\,(\mathrm{cm}^2\ \mathrm{s\ sr\ GeV/nucleon})^{-1}. \qquad (1)$$

The superposition model in the primary nucleus-air nucleus collisions is neglected here because in the recent charge transfer distribution after Breitenlohner[21] and Abdrakhmanov et al.[22] in hadron-hadron collisions it is found that irespective of the kind of projectile particles the quantum numbers were conserved in the target and projectile hemisphere in the centre of mass system. So one can safely replace hadron by the hadronic single matter and the projectile matter behaves according to hypothesis of limiting fragmentation which is nothing but Feynman[23] scaling in the forward region for $x > 0.1$. So $p - p$ collision data can safely be used in the present investigation due to the negligible contribution of intranuclear cascading in p-nucleus collisions.

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The pion and kaon production spectra in the atmosphere have been estimated using the conventional procedures described in Ref. 1 follows the forms:

$$\pi (E_\pi)\, dE_\pi = (Z_{p\pi^+} + Z_{p\pi^-})\, N (E_\pi)\, dE_\pi \cong I_\pi\, E_\pi^{-(\gamma+1)}\, dE_\pi \qquad (2)$$

$$K (E_k)\, dE_k = (Z_{pK^+} + Z_{pK^-})\, N (E_k)\, dE_k \cong I_k\, E_k^{-(\gamma+1)}\, dE_k \qquad (3)$$

where I_π and I_k are the spectral amplitudes of the pion and kaon spectra, respectively. The fractional hadronic energy moments of $pp \to \pi^+ X$, $pp \to K^\pm X$ and $\pi p \to \pi^\pm X$ inclusive reactions can be estimated by the expression:

$$Z_{ac} = \int_0^1 x^{\gamma-1} f_{ac} (x)\, dx \qquad (4)$$

where the scaling function for the inclusive reaction $a + b \to c + X$ defined as

$$f_{ac} (x) = \frac{\pi}{\sigma_{in}} \int_0^\infty f_{ac} (x, p_T)\, dp_T^2 \qquad (5)$$

for $a = \pi^\pm$ or p, $b = \pi^\pm$ or $K^\pm$.

B) The passage of pions and kaons through the atmosphere:

The kinetic equation for the passage of charged pions through the atmosphere has the form:

$$\frac{\partial \pi (E, Y)}{\partial Y} = \frac{\pi (E_\pi) \exp (-Y/\Lambda_p)}{\lambda_p} - \left(\frac{1}{\lambda_\pi} + \frac{H_\pi}{Y E_\pi}\right) \pi (E_\pi, Y) \qquad (6)$$

where λ_p and λ_π are the interaction mean free paths of nucleons and pions in the atmosphere, respectively, H_π is the critical energy for pion decay, Λ_p is the absorption mean free paths of nucleons in air. The solution of the above kinetic equation for pions generated via $pp \to \pi^\pm X$ inclusive reaction channel in the atmosphere has been considered for muon flux calculation at N generation of parents and the result follows after A. D. Erlykin (private communication) which account the muon flux arises from the pion source:

$$M_{p\pi}^N (E_\mu) = a^{(N-1)\gamma}\, D_{p\pi}^N (E_\mu) \qquad (7)$$

where a is the elasticity,

$$D_{p\pi}^N (E_\mu) = N (E_\mu)\, \langle Z_{p\pi} \rangle^N A_\pi (\lambda_\pi/\lambda_p)^N \frac{1}{(N-1)!}$$

$$\sum_{m=1}^{\infty} \frac{(1 - \lambda_\pi/\Lambda_p)^{m-1}\, m (m+1) \dots (m+N-2)}{1 + (m+N-1)\, R_\pi E_\mu/H_\pi}\, W (E_\mu, Y, Y_0), \qquad (8)$$

$W(E_\mu, Y, Y_0)$ represents the survival probability that a muon produced at an atmospheric depth Y should be observed at a certain depth Y_0 g · cm^{-2}, N is the number of generation of parents, $N(E_\mu)$ is the differential primary spectrum of energy E_μ,

$$A_i = \frac{1 - r_i^{2(\gamma+1)}}{(1 - r_i^2)(\gamma + 1)}, \tag{9}$$

$$R_i = \frac{(\gamma + 2)(1 - r_i^{2(\gamma+1)})}{(\gamma + 1)(1 - r_i^{2(\gamma+2)})}, \tag{10}$$

$$r_i = \frac{(m_i^2 + m_\mu^2)}{2m_i^2}, \tag{11}$$

$$H_i = m_i\, c^2\, H/(c\, \tau_i) \tag{12}$$

is the critical energy for meson decay with scale height of the atmosphere H, m_i and τ_i are the mass and life time of i meson at rest, $i = \pi$ or K. In a similar manner one can estimate the muon flux at different generation N from $pp \to K^\pm X$ inclusive reactions follows:

$$M_{pk}^N(E_\mu) = a^{(N-1)\gamma}\, D_{pk}^N(E_\mu) \tag{13}$$

where a is the total elasticity,

$$D_{pk}^N(E_\mu) = b_{k\mu}\, N(E_\mu)\, \langle Z_{pK}\rangle^N A_k\, (\lambda_k/\lambda_p)\, \frac{1}{(N-1)!}$$

$$\sum_{m=1}^{\infty} \frac{(1 - \lambda_k/\Lambda_p)^{m-1}\, m(m+1)\dots(m+N-2)}{1 + (m+N-1)\, R_k\, E_\mu/H_k}\, W(E_\mu, Y, Y_0), \tag{14}$$

$b_{k\mu}$ is the branching ratio for $K \to \mu_2$ decay.

The muon flux generated by secondary pions at first generation of parents from $\pi p \to p^\pm X$ inclusive reactions calculated by the following expression:

$$M_{\pi\pi}^1(E_\mu) = A_\pi \frac{\lambda_\pi}{\lambda_p} \frac{Z_{p\pi}\, Z_{\pi\pi}\, N(E_\mu)}{[1 + 2R_\pi E/H_\pi]} \left| 1 - b_{\pi\pi} \left[\frac{H_\pi}{r_\pi E_\mu}\right]^{a_{\pi\pi}} \right| W(E_\mu, Y, Y_0). \tag{15}$$

Differential muon spectrum at an atmospheric depth Y_0 from the decays of pions and kaons can be estimated by the following expression:

$$\sum_{i=p\pi,\, pK,\, \pi\pi} M_i(E_\mu, Y_0) = \sum_{N=1}^{3} M_{p\pi}^N(E_\mu, Y_0) + \sum_{N=1}^{3} M_{pk}^N(E_\mu, Y_0) + M_{\pi\pi}^1(E_\mu, Y_0). \tag{16}$$

3. *Results and discussion*

The all particle primary spectrum[6] is estimated from the recent direct measurement of proton and helium intensity data of Japanese American Cooperative Emulsion Experiment[7] and nuclei intensity data compilation of Abulova et al.[8] at balloon altitudes. It is assumed that the primary cosmic ray nuclei split up near the top of the atmosphere into its constituents like nucleons. The superposition method has been used for the estimation of total nucleon flux from the spectra of different nuclei and the differential nucleon spectrum follows the form:

$$N(E)\,\mathrm{d}E = 2.36\;10^4\,E^{-2.7}\,\mathrm{d}E\,(\mathrm{m}^2\,\mathrm{s\;sr\;GeV/nucleon})^{-1} \tag{17}$$

in the spectral range $10^3 - 10^6$ GeV. We used this spectrum as hadron source.

TABLE 1.

Secondary particle	A_h	$\bar{n}_h$	m_h^2
π^+	0.72	2.86	0.66
π^-	0!55	3.52	0.74
K^+	0.15	3.20	0.64
K^-	0.09	4.89	0.90

Fitting parameters of the Lorentz invariant cross section estimated from the Fermilab data[2].

The fractional hadronic energy moments for $pp \to \pi^\pm X$ and $pp \to K^\pm X$ inclusive reactions have been estimated from the approximate scaling distribution on the inclusive charged hadron production Fermilab data in 100 — 400 GeV $p - p$ collision of Johnson et al.[2] which shows

$$\frac{1}{\sigma_{in}} E\,(\mathrm{d}^3\sigma/\mathrm{d}^3p) \sim \frac{A_h\,(1-x)^{\bar{n}_h}}{(1+p_T^2/m_h^2)^4} \tag{18}$$

where the values of the parameters A_h, $\bar{n}_h$ and m_h^2 have been presented in Table 1 for different meson production. The scaling function $f_{p\pi}(x)$ follows:

$$f_{p\pi}(x) = \frac{\pi}{\sigma_{in}} \int E(\mathrm{d}^3\sigma/\mathrm{d}^3p)\,\mathrm{d}p_T^2 = \frac{\pi A_n\,(1-x)^{\bar{n}_h}\,m_h^2}{3} \tag{19}$$

and hadronic energy moments $Z_{p\pi}$ follows the form

$$Z_{p\pi} = \int_0^1 x^{\gamma-1} f_{p\pi}(x)\,\mathrm{d}x = \frac{\pi A_h\,m_h^2}{3} \frac{\Gamma(\gamma)\,\Gamma(\bar{n}_h+1)}{\Gamma(\gamma)\,\Gamma(\gamma+\bar{n}_h+1)}. \tag{20}$$

TABLE 2.

$Z_{p\pi^+}$	$Z_{p\pi^-}$	Z_{pK^+}	Z_{pK^-}
0.039565	0.0264	0.0070	0.00344

Fractional hadronic energy moments calculated from the Fermilab data of Johnson et al.[2].

Table 2 shows the calculated fractional hadronic energy moments estimated from the Fermilab accelerator data after Johnson et al.[2] using the relation (20). The fractional hadronic energy moments for meson-proton collisions have been calculated after Elbert et al.[3] and Beaupre et al.[24]. These energy moments have been used to estimate the muons produced by pions arising from the interactions of secondary pions only and have been presented in Table 3.

TABLE 3.

γ	$Z_{\pi^+\pi^+}$	$Z_{\pi^+\pi^-}$	$Z_{\pi^-\pi^+}$	$Z_{\pi^-\pi^-}$
1.70	0.18	0.052	0.0413	0.168

The fractional energy moments for $\pi p \to p \pm X$ inclusive reactions after Elbert et al.[3] and Beaupre et al.[24].

The spectra of pions and kaons in the atmosphere using relations (2) and (3) follow the forms:

$$\pi (E_\pi)\, dE_\pi = 0.1557\, E_\pi^{-2.7}\, dE_\pi\, dE_\pi\, (\text{cm}^2\ \text{s sr GeV})^{-1} \tag{21}$$

and

$$K (E_k)\, dE_k = 0.02464\, E_k^{-2.7}\, dE_k\, (\text{cm}^2\ \text{s sr GeV})^{-1}. \tag{22}$$

The values of I_π and I_k are 0.1557 and 0.02464, respectively. We have used the meson atmospheric diffusion equations after Bugaev et al.[5] viz., the relations (7), (13) and (15) along with the following parametric values:

λ_p = interaction mean free path of nucleons in air = 85 g · cm^{-2};
Λ_p = absorption mean free path of nucleons in air = 110 g · cm^{-2};
λ_π = interaction mean free path of pions in air = 120 g · cm^{-2};
λ_k = interaction mean free path of kaons in air = 150 g · cm^{-2};
H_π = critical energy for pion decay = 121 GeV;
H_k = critical energy for kaon decay = 897 GeV;
A_π = kinetic constant depending on $\pi - \mu$ decay = 0.6963;
A_k = kinetic constant depending on $K - \mu_2$ decay = 0.4928;

r_π = energy degradation factor for $\pi - \mu$ decay = 0.78;
r_k = energy degradation factor for $K - \mu_2$ decay = 6.52;

$R_\pi = 1.2157$; $R_k = 1.3409$; $a_{\pi\pi} = 0.421$;
$b_{\pi\pi} = 0.384$; $a_{\pi\bar{\pi}} = 0.579$; $b_{\pi\bar{\pi}} = 0.271$;
$b_{K\mu}$ = branching ratio for $K \to \mu_2$ decay = 0.63;

$W(E_\mu, Y, Y_0)$ is the survival probability of muons which allow for muon decay energy loss and follows the form after Maurayama et al.[25]. The probability $W(E_\mu, Y, Y_0)$ of the surviving muon of energy E_μ to travel from Y to Y_0 is obtained from the equation:

$$\frac{\mathrm{d}W_\mu(x')}{\mathrm{d}x'} = -\frac{W_\mu(x')\, m_\mu\, c}{E(x')\, p(x')\, \tau_\mu} \tag{23}$$

where $W_\mu(x')$ is the survival probability of muon from Y to x', m_μ is the rest mass and τ_μ is the proper life time of muons. The solution of the above equation leads to the following:

$$W(E_\mu, Y, Y_0) = \left[\frac{Y}{Y_0}\frac{E_\mu}{E_\mu + \beta(Y_0 - Y)\, E_\mu}\right]^{B_\mu/(E_\mu + \beta Y_0)} \tag{24}$$

where $B_\mu = m_\mu c^2 H'/(c\,\tau_\mu)$, $H' = RT_2/Mg$, T_2 is the temperature of the lower atmosphere where muons being produced $\cong 256$ K. The change of energy of cosmic ray muon along its penetration path in the atmosphere can, therefore, be calculated from the following semi-empirical formula after Maeda[26]

$$\beta = -\frac{\mathrm{d}E}{\mathrm{d}Y} = a_{ion} + bE_\mu \tag{25}$$

where a_{ion} is the energy loss by collision in the atmosphere $\approx 2.5 \cdot 10^{-3}$ GeV/$g \cdot \mathrm{cm}^{-2}$ and b is the energy loss by radiation, electron pair production and nuclear interaction in the atmosphere $\approx 2.78 \cdot 10^{-6}/g \cdot \mathrm{cm}^{-2}$. We found $B_\mu \approx 1.25$ GeV and

$$W(E_\mu, y, y_0) \approx \left[\frac{0.0968\, E_\mu}{1.002594\, E_\mu + 2.3325\ \mathrm{GeV}}\right]^{\frac{1.25\ \mathrm{GeV}}{1.00287\, E_\mu + 2.5825\ \mathrm{GeV}}} \tag{26}$$

where E_μ is the sea level muon energy expressed in GeV units. Fig. 1 shows the derived survival probability for muons produced at mean atmospheric depth $Y = 100\ \mathrm{g\cdot cm^{-2}}$ reaching sea level atmospheric depth $Y_0 = 1033\ \mathrm{g\cdot cm^{-2}}$. The relative muon flux from different parents generation N via different channels viz. $pp \to \pi^\pm X$ inclusive reaction yields muon flux at generation N by the term $M^N_{p\pi}(E_\mu, y_0)$; for $pp \to K^\pm X$ channel yields $M^N_{pK}(E_\mu, y_0)$ and $\pi p \to \pi^\pm X$ channel gives the flux $M^1_{\pi\pi}(E_\mu, y_0)$. In these calculations we have calculated the total elasticity by the expression

$$a = 1 - K_T = 1 - [1 - (1 - \lambda_p/\Lambda_p)^{1/\gamma}] \tag{27}$$
$$\cong 0.4414.$$

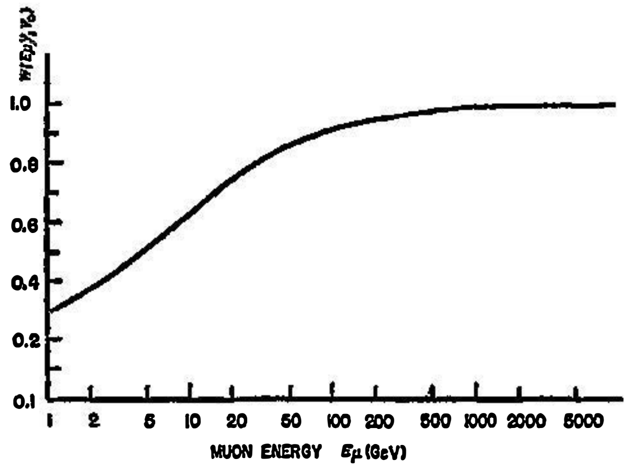

Fig. 1. Survival probability of muons $W(E\mu, Y, Y_0)$ produced at median atmospheric depth $Y = 100\,\text{g-cm}^{-2}$ reaching sea level depth $Y_0 = 1033\,\text{g} \cdot \text{cm}^{-2}$.

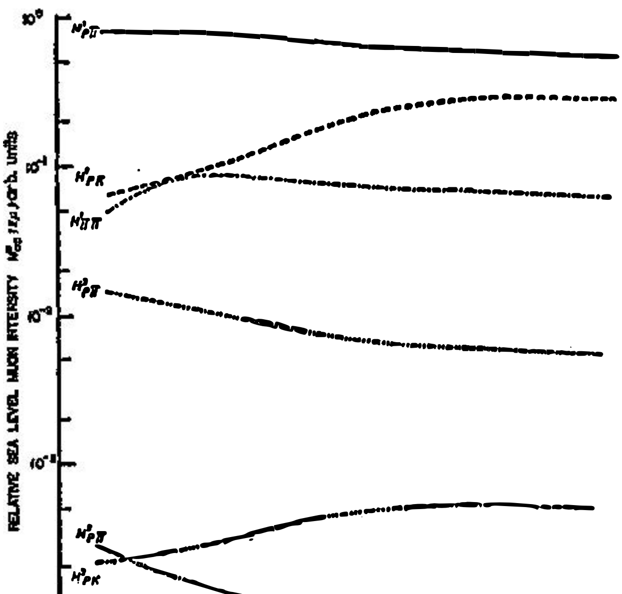

Fig. 2. Derived relative sea level muon intensity $M_{ac}^{N}(E\mu)$ at different generation N of pions: Relative muon intensity $pp \to \pi\pm X$ inclusive reaction $M_{p\pi}^{1}(E\mu)$ represents the muons produced by pions at first generation; $M_{p\pi}^{2}(E\mu)$ represents muons produced by pions at second generation; $M_{p\pi}^{3}(E\mu)$ represents the muons produced by pions at third generation; $M_{pK}^{1}(E\mu)$ and $M_{pK}^{2}(E\mu)$ represent muons produced by kaons at second and third generations, respectively, from $pp \to k\pm X$ inclusive reactions; Muons produced by the pions produced by secondary pions at first generation in the inclusive reaction $\pi p \to \pi\pm X$ represented by the symbol $M_{\pi\pi}^{1}(E\mu)$ in the figure.

We have taken the value of $\lambda_p = 85 \pm 12\ g \cdot cm^{-2}$ from the experiment of Boehmer and Bridge[27] and the value of the $\Lambda_p = 110 \pm 10\ g \cdot cm^{-2}$ from the altitude variation of nuclear active particle data surveyed by Hayakawa[22]. Using the accelerator data Garaffo et al.[27] found the values of the fractional hadronic energy moments Z_{pp} and Z_{pn} which are 0.15 and 0.073, respectively. The kinematical relation for λ_p and Λ_p follows:

$$\lambda_p = \Lambda_p (1 - Z_{pn}) = 0.777\, \Lambda_p \tag{28}$$

and the present chosen the values of λ_p and Λ_p are in accord with the relation (28).

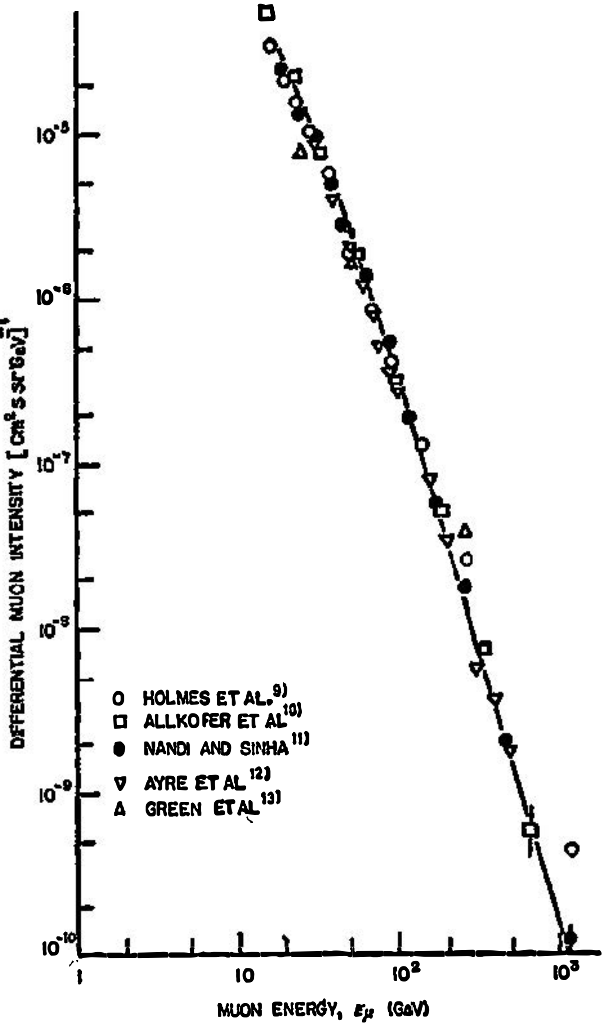

Fig. 3. Differential sea level muon spectrum below 1 TeV energy: Full line is the derived result. Magnetic spectrograph data: 0 — Holmes et al.[9], □ — Allkofer et al.[10], ● — Nandi and Sinha[11], ∇ — Ayre et al.[12], ∇ — Green et al.[13].

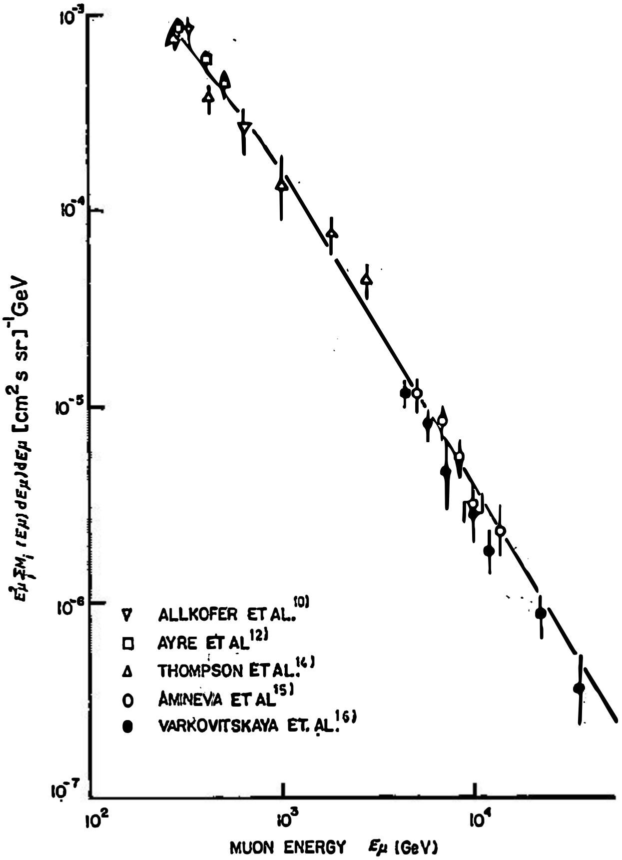

Fig. 4. Differential sea level muon spectrum in the range 0.5—30 TeV: Full line is the derived result. Experimental data: Directly measured from i) Magnetic spectrograph, ∇ — Allkofer et al.[10], □ — Ayre et al.[12], Δ — Thompson et al.[14]; Indirect measurements from the shower observation, o — Amineva et al.[15], ● — Varkovitskaya et al.[16].

The relative muon flux at different parents generation N produced via different channels viz., $pp \rightarrow \pi^\pm X$ inclusive reaction yields the muon flux at generation N viz. by $M_{p\pi}^N (E_\mu, Y_0)$, the reaction $pp \rightarrow K^\pm X$ contributes the muon flux $M_{pK}^N (E_\mu, Y_0)$ and the reaction $\pi p \rightarrow \pi^\pm X$ channel at first generation produces $M_{\pi\pi}^1 (E_\mu, Y_0)$ flux, and the results have been displayed in Fig. 2. It is evident from Fig. 2 that muon flux at sea level increases with energy for all cases at first generation viz. for channels $pp \rightarrow \pi^\pm X$, $K^\pm X$ and $\pi p \rightarrow \pi^\pm X$ channels viz. $M_{p\pi}^1 (E_\mu, Y_0)$ $M_{pK}^1 (E_\mu, Y_0)$ and $M_{\pi\pi}^1 (E_\mu, Y_0)$ along with the $M_{pK}^2 (E_\mu, Y_0)$ for $pp \rightarrow K^\pm X$ channel. On the other hand muon flux created at second and third generation via $pp \rightarrow \pi^\pm X$ channel viz. for $M_{p\pi}^2 (E_\mu, Y_0)$ and $M_{p\pi}^3 (E_\mu, y_0)$ decreases with energy. The sea level muon spectrum estimated from the different source functions (for pion and kaons intensity) below 1 TeV energy has been plotted in Fig. 3 along with the magnetic spectrograph data of different authors[9-13]. The calculated spectrum is in good agreement with the experimental data. Above 1 TeV muon energy the derived differential sea level muon spectrum has been presented in Fig. 4 along with the measured data of Thompson et al.[14], Amineva et al.[15] and Varkovitskaya et al.[16]. It is evident from the figure that the agreement of the experimental data with the calculated results in the spectral range 1 — 30 TeV is satisfactory.

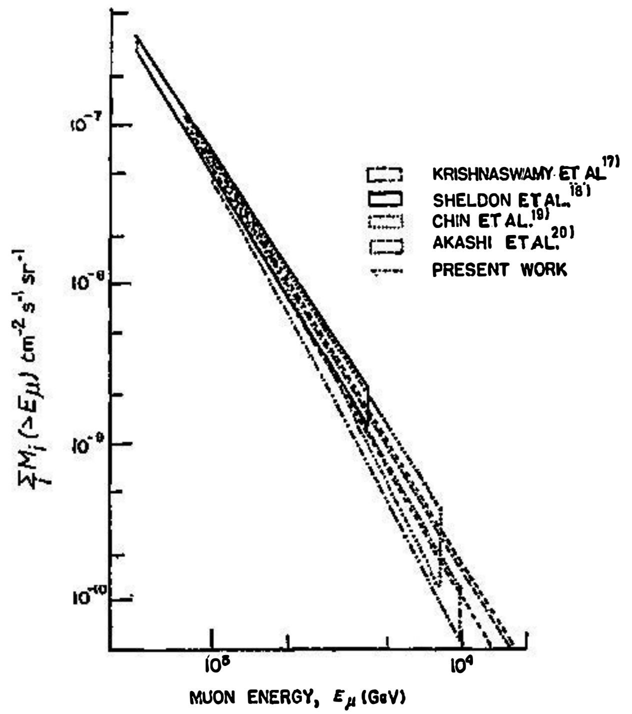

Fig. 5. Integral sea level muon spectrum up to 30 TeV energy: Chain curve —·—·—·—· is the derived present result. Experimental data: the shaded area — — — Krishnaswamy et al.[17], ———— Sheldon et al[18] from deep underground measurements; ··········· Chin et al.[19], —··—··— Akashi et al.[20] from the shower observations.

The calculated differential spectrum in Fig. 4 follows the form:

$$\sum_i M_i (E_\mu)\, dE_\mu = 9.5\, E_\mu^{-3.6}\, dE_\mu\, (\text{cm}^2\ \text{s sr GeV})^{-1}. \qquad (29)$$

The estimated integral sea level muon spectrum follows:

$$\sum_i M_i (> E_\mu) = 3.65\, E_\mu^{-2.65}\, (\text{cm}^2\ \text{s sr})^{-2} \qquad (30)$$

and this integral muon spectrum has been plotted in Fig. 5 along with the results from direct deep underground measurements of Krishnaswamy et al.[17] and Sheldon et al.[18] and indirect muon intensity measurements from shower obser-

TABLE 4.

Authors	Energy range (TeV)	Procedure	
Bhattacharyya and Pal[1]	1 – 10	Theoretical	2.70
Krishnaswamy et al.[17]	0.2 – 40	Deep underground measurements	2.60 ± 0.05
Sheldon et al.[18]	< 1 > 1	Deep underground measurements	2.40 2.65
Chin et al.[19]	0.5 – 8	Shower observations	2.50 ± 0.20
Mitzutani et al.[20]	1 – 10 5 – 10	Shower observations	2.70 ± 0.10 2.80 ± 0.25
Amineva et al.[30]	4 – 7 7 – 30	Shower observation	2.65 ± 0.10 2.85 ± 0.20
Baschiera et al.[31]	0.5 – 5	Deep underground measurements	2.67 ± 0.05
Kiraly and Wolfendale[32]	1 – 8	Deep underground measurements	2.60
Ng and Wolfendale[33]	1 – 10	World survey	2.67
Fomin et al.[34]	1 – 20	Shower observations	2.75
Present work	0.6 – 30	Theoretical	2.60

Integral muon spectral indices found by different authors.

vations after Chin et al.[19] and Akashi et al.[20]. It is evident from the figure that our derived result (single chain curve in Fig. 5) is well in agreement with the depth intensity measurements of Krishnaswamy et al.[17] and Sheldon et al.[18] and shower measurements of Chin et al.[19]. Table 4 shows the integral muon spectral index found by different authors at super high energies (above 1 TeV) along with the present result.

Our derived integral spectral index is well in accord with the findings of Krishnaswamy et al.[17] and Kiraly and Wolfendale[31]. It may be noticed that when the muon intensity at different generations have been considered the calculated sea level muon spectrum is flattened to some extent viz. the spectral index $\gamma = 2.7$ (in Ref. 1) is diminished to $\gamma = 2.6$. However the agreement of the calculated muon spectra with the measured data indicates that the effect of intranuclear cascading on the spectrum of energetic secondaries from nucleon-air nucleus interactions is small. Hence it confirms the application of $p - p$ collision accelerator data in cosmic ray propagation studies is reasonable. The present investigation indicates that even up to 30 TeV muon energy the intranuclear cascading may be negligible and Feynman scaling application in this analysis is justified.

We have used the primary cosmic ray spectrum[6] derived from the direct mass composition measurements of proton and other nuclei intensity data[7,8] as the source of hadrons for the analysis of the present muon propagation in earth's atmosphere. The only assumption has been made that a heavy nucleus of mass A behaves as A independent nucleons in its interactions in the atmosphere. The spectral shape of our derived primary spectrum is also valid up to 300 TeV energy (without any break or change in spectral index). It is shown that the primary nucleon of TeV energies can explain muon spectrum up to 30 TeV energy adequately and this fact supports the hypothesis of limiting fragmentation.

4. *Conclusion*

The recently determined primary spectrum[6] from the direct measured proton and nuclei intensity data[7,8] can be used in the framework of Feynman scaling with accelerator data explain the sea level muon spectrum adequately up to 30 TeV energy. The fair agreement of the calculated muon spectrum with magnetic spectrograph data, deep underground muon telescope and emulsion chamber data confirm the validity of the primary spectrum and Feynman scaling hypothesis up to 300 TeV median primary energy reasonably.

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MUONSKI SPEKTAR OKOMIT NA MORSKU RAZINU PRI VRLO VISOKIM ENERGIJAMA IZRAČUNAT IZ NEDAVNOG PRIMARNOG SPEKTRA KOZMIČKIH ZRAKA

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Izračunat je muonski spektar okomit na morsku razinu iz primarnog spektra kozmičkih zraka. Pretpostavivši valjanost akceleratorskih podataka u fragmentacijskom području za $x > 0{,}1$ u okviru Feynmanova scalinga i rabeći jednadžbu atmosferske difuzije ocijenjeno je da muonski tijek za prvu generaciju roditelja iz izvora inkluzivnih reakcija $pp \to \pi^{\pm} X$, $pp \to K^{\pm} X$, $\pi p \to \pi^{\pm} X$ i drugu generaciju iz inkluzivne reakcije $pp \to K^{\pm} X$ raste s energijom muona.