YU ISSN 0015—3206
FZKAAA 15 (4) 419 (1983)

TWISTOR SPACES AS POSSIBLE CARRIERS OF ELECTROWEAK INTERACTIONS

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Received 21 April 1983

UDC 539.12

Original scientific paper

It is suggested that the standard model of electroweak interactions could be recast as a theory on a space of twistors. The following aspects of such a formulation aer discussed in a preliminary way: isospin, hypercharge, and the generation of electromagnetic mass.

1. Introduction

Various modifications of space-time have been exploited in attempts to describe elementary particle interactions. One such approach is based on the space of twistors, and is presented e. g. in Ref. 1. Here we suggest an approach which likewise uses twistors, but which differs in essential ways from that of *loc. cit.* The considerations that follow are fragmentary, but we feel that they are sufficiently suggestive to justify the present note.

These considerations are motivated by the following circumstance. In the standard model ef electroweak interactions[2,3] (cf. also Ref. 4), there is a fundamental difference between negative (left) and positive (right) helicity. In particular, among the leptonic states, only those with negative helicity have nonzero isospin. Now, the twistor framework is one where the negative-helicity states and the positive-helicity states are treated in a different way, once the basic conventions are made. (This framework, moreover, is the only one with this property, as far as we are aware.)

We recall the following facts from twistor analysis[1]. A twistor $Z = (Z^\alpha)$ consists of a pair of spinors (ω, π) on the Minkowski space M_4 or on its complexification M_4^c,

$$Z = (Z^0, \dots, Z^3) = (\omega^0, \omega^1, \pi_{0'}, \pi_{1'}). \tag{1}$$

If we translate by $x \in M_4$ and identify x with a second-rank spinor $x^{AB'}$, then the corresponding transformation of a twistor is

$$Z = (\omega^A, \pi_{B'}) \to (\omega^A + \mathrm{i} x^{AC'} \pi_{C'}, \pi_{B'}). \tag{2}$$

Now, a (holomorphic) solution $\varphi = \varphi(x; A_1', \dots, A_n')$ to the system of wave equations for a free massless particle of helicity $s = \frac{1}{2} n > 0$ is determined by a function f which is holomorphic in a region of the (complex) twistor space T, and which is homogeneous of degree $-n-2$, as follows (we write $\pi_{A'}$ and $\pi(A')$ interchangeably):

$$\varphi = (2\pi \mathrm{i})^{-1} \oint \mathrm{d}(\pi') \, \pi(A_1') \dots \pi(A_n') \, f(\mathrm{i} x^{AC'} \pi_{C'}, \pi_{B'}), \tag{3a}$$

$$\mathrm{d}(\pi') = \pi^{0'} \, \mathrm{d}\pi_{0'} + \pi^{1'} \, \mathrm{d}\pi_{1'}. \tag{3b}$$

The integral is over a suitable closed contour.

For $\varphi(x; A_1, \dots, A_n)$ for helicity $-\frac{1}{2} n < 0$ we have two options. First, we may take (3) with f homogeneous of degree $n-2$, and replace the π by the $\partial/\partial \omega^{A_j}$, $j = 1, \dots, n$. Second, we may utilize the space of dual twistors W, and then there is a direct analogue to (3). Dual twistors are defined by,

$$W = (W_0, \dots, W_3) = (Z^{2*}, Z^{3*}, Z^{0*}, Z^{1*}). \tag{4}$$

Clearly, it is also possible to use dual twistors and appropriate differential operators for positive helicity. We will refer to the different possibilities in the sequel.

It appears that twistors can elucidate the following aspects of electroweak interactions:

(i) *Isospin.* We will discuss this presently.

(ii) *Hypercharge.* The transformation $Z \to e^{i\Theta} Z$ has evident and different effects on functions of different homogeneities, or on functions of Z and of W. Moreover, this transformation leaves the free dynamics invariant. If this transformation leaves also an interaction term invariant, then hypercharge conservation follows.

(iii) *Electromagnetic mass.* In the standard model of electroweak interactions, prior to symmetry breaking, the electromagnetic interaction of the electron field is described by,

$$(\hat{\partial} + e_0 \hat{A}) \psi_L = 0 = (\hat{\partial} + e_0 \hat{A}) \psi_R. \tag{5}$$

As shown in Ref. 4, the field ψ_L and ψ_R do not acquire a mass due to this interaction. In particular, this interaction does not bring about the transitions $\psi_L \leftrightarrow \psi_R$. We discuss below briefly how such transitions may arise if the electromagnetic interactions is expressed in twistorial terms.

(iv) *Integrality of charge.* For completeness we mention that the twistorial formulation of electromagnetic interaction implies that the electric charge must be an integer, in some units[5,6]. However, here the problem of units is not yet understood.

2. *Isospin*

We base our arguments on the following observation. Let us parametrize the isospin phase space by a complex variable ζ, as in geometric quantization (e. g. Ref. 7). Now, if the variables $(W_0, \ldots, W_3, \zeta)$ jointly define a complex structure, then $(Z^0, \ldots, Z^3, \zeta)$ define a complex structure which could be called *unnatural* with respect to the first. On the other hand, $(Z^0, \ldots, Z^3)$, without ζ, define simply the conjugate structure on a subspace. (See e. g. Ref. 8 for complex structures, holomorphic bundles, etc.)

The foregoing observation can be exploited in various ways. For instance: Consider the twistor space T and its dual T^*, with the spin components represented by the $\pi(A'_j)$ and by the $\pi(A_k)$ (rather than by derivatives). Let SU_2 be the isospin group, which defines connections on bundles over T and over T^*. Then, by postulating that a bundle be holomorphic, we limit ourselves either to a bundle over T or to one over T^*. In particular, if leptons with $s = -\frac{1}{2}$ have spin-isospin described by $(W_0, \ldots, W_3, \zeta)$, where ζ is now the bundle variable, then the leptons with $s = \frac{1}{2}$ must be isoscalars. This is the desired conclusion.

This simple scheme calls for some comments. We assumed tacitly that the same complex variable ζ should describe the isospin of both helicities. Moreover, it is clear that for antileptons, complex-conjugate structures should be used. Assumptions such as these are natural-looking, but they may also appear *ad hoc.* However, some such assumptions are needed, e. g. to distinguish from the isospin of gauge fields where the situation is rather different.

3. *Electromagnetic mass*

In describing the interaction with (free) background electromagnetic fields, the difference between those with $s = 1$ and those with $s = -1$ is striking[9-12]. In particular, the presence of fields of both kinds simultaneously presents special complications, and it appears that then the interaction can no longer be described in conformal-covariant terms[11]. One possibility (cf. *loc. cit.*) is to break conformal covariance with the help of the infinity twistor, and to retain Poincaré covariance.

Sush a procedure has not been (apparently) described in print, but the details would not be crucial for the present discussion. It suffices if we assume that the electromagnetic potential has a part Φ which is characterized by the finite Minkowski points, and a part Ω which depends on the points at infinity. Then the equation for ψ_L in (5) reduces to

$$(\mathrm{i}^{-1} \nabla^{A}_{B'} + e_0 \Phi^{A}_{B'}) \pi_A = - e_0 (\Omega^{A}_{B'}, \pi_A). \tag{6}$$

(For such forms of equations cf. Ref. 10 Sec. 9.) The r. h. s. is a spinor with index B', and it is natural to identify it with a component of ψ_R. Then we have a coupling $\psi_L \leftrightarrow \psi_R$.

We do not suggest here any criteria for determining the two parts Φ and Ω Note, however, that by taking f in (3a) independent of $x^{AC'}$ (i. e. having singu. larities at infinity) we obtain a constant. Let us classify it as $\Omega^{A}_{B'}$, and the r. h. s then looks like a mass term. The generation of the contribution $m\overline{\psi}\psi$ to the Lagrangian is now a reasonable hypothesis.

Acknowledgment

The author is very grateful to Professors P. Budinich and L. Fonda for their hospitality at SISSA.

References

1) L. P. Hughston, *Twistors and Particles* (Springer-Verlag, Berlin etc., 1979; Lecture Notes in Physics **97**);

2) S. Weinberg, Phys. Rev. Letters **19** (1967) 1264;

3) Abdus Salam, in *Elementary Particle Theory* (Nobel symposium 8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367;

4) E. S. Abers and B. W. Lee, Phys. Reports **9** C (1973) 1, especially Secs. 4, 7;

5) R. Penrose and G. A. J. Sparling, in *Advances in Twistor Theory*, edited by L. P. Hughston and R. S. Ward (Pitman Publ. Ltd, London, 1979), p. 136; R. Penrose, in *Complex Manifold Techniques in Mathematical Physics*, edited by D. E. Lerner and P. D. Sommers (Pitman Publ. Ltd, London, 1979), p. 66;

6) P. Green, in *Complex* ... (Ref. 5) p. 231;

7) J. Sniatycki, *Geometric Quantization and Quantum Mechanics* (Springer-Verlag, New York—Heidelberg—Berlin, 1980; Applied Math. Sciences v. **30**), Chap. 11;

8) R. O. Wells, Jr. *Differential Analysis on Complex Manifolds* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1973);

9) R. S. Ward, in *Advances* ... (Ref. 5) p. 132;

10) M. G. Eastwood, R. Penrose and R. O. Wells Jr., Commun. Math. Phys. **78** (1981) 305;

11) R. Penrose, in *Advances* ... (Ref. 5), p. 152, and in *Complex* ... (Ref. 5) p. 69, p. 78;

12) L. P. Hughston, in *Advances* ... (Ref. 5) p. 162.

PROSTORI TVISTORA KAO MOGUĆI NOSIOCI ELEKTROSLABIH MEĐUDJELOVANJA

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Originalni znanstveni rad

Predlaže se da se standardni model elektroslabih međudjelovanja dade predstaviti kao teorija u prostoru tvistora. Slijedeći aspekt takve formulacije se razmatraju na preliminaran način: izospin, hipernaboj i generiranje elektromagnetske mase.