YU ISSN 0015—3206
FZKAAA 15 (4) 425 (1983)

RENORMALIZATION OF THE TWIST-FOUR OPERATOR IN THE PLANAR GAUGE

ANĐELKA ANDRAŠI

Ruđer Bošković Institute, 41000 Zagreb, Croatia, Yugoslavia

and

JOHN C. TAYLOR

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge, CB3 9EW, U. K.

Received 20 June 1983

UDC 539.12

Original scientific paper

We study the renormalization of the twist-four operator and give a complete solution for a single insertion of the operator at an arbitrary momentum. Renormalization constants are evaluated to one-loop level.

1. Introduction

The problem of renormalization of gauge-invariant operators in gauge theories has been studied extensively by a number of authors[1].

The fundamental conjucture states that the most general form of the renormalized action is

$$S^R = S^R_{YM} + \Phi\, \mathcal{O}^R + W^R \Phi\, G^R, \tag{1}$$

where

$$W^R = \frac{\delta S^R_{YM}}{\delta A_\mu}\frac{\delta}{\delta J} + \frac{\delta S^R_{YM}}{\delta J_\mu}\frac{\delta}{\delta A^\mu} + \frac{\delta S^R_{YM}}{\delta \omega}\frac{\delta}{\delta K} + \frac{\delta S^R_{YM}}{\delta K}\frac{\delta}{\delta \omega} \tag{2}$$

and $\Phi\, G$ is the most general polynomial of the right dimensions and ghost number.

We point out that a term $\frac{\delta \Phi \mathcal{O}}{\delta A_\mu} \frac{\delta}{\delta J^\mu}$ has to be included in (2)[2].

2. *Renormalization of the twist-four operator in the planar gauge*

We follow the notation of Ref. 3. The effective Lagrangian including the gauge-fixing term corresponding to the planar gauge is

$$L' = L - (2an^2)^{-1} n_\lambda \underline{A}^\lambda \Box n_\mu \underline{A}^\mu,$$

$$L = -\frac{1}{4} \underline{F}_{\mu\nu} \underline{F}^{\mu\nu} - \frac{1}{4} \Phi \underline{F}_{\mu\nu} \underline{F}^{\mu\nu} \tag{3}$$

$$+ \underline{\eta} (n_\lambda \partial^\lambda \underline{\omega} + g n_\lambda \underline{A}^\lambda \times \underline{\omega})$$

$$+ \underline{J}^\mu (\partial_\mu \underline{\omega} + g \underline{A}_\mu \times \underline{\omega}) - \frac{1}{2} g \underline{K} (\underline{\omega} \times \underline{\omega}), \tag{4}$$

where Φ is the external scalar source.

This Lagrangian satisfies the same BRS identities as the one without the operator:

$$\frac{\delta L'}{\delta A_\mu} \frac{\delta L'}{\delta \underline{J}^\mu} + \frac{\delta L'}{\delta \underline{\omega}} \frac{\delta L'}{\delta \underline{K}} + (a n^2)^{-1} (\Box n_\lambda \underline{A}^\lambda) \frac{\delta L'}{\delta \underline{\eta}} = 0, \tag{5}$$

$$\frac{\delta L'}{\delta \underline{\eta}} = n_\mu \frac{\delta L'}{\delta \underline{J}^\mu}. \tag{6}$$

The divergent parts D to one-loop order obey the BRS identity:

$$WD = \left[\frac{\delta S_{YM}}{\delta \underline{A}_\mu} \frac{\delta}{\delta \underline{J}^\mu} + \frac{\delta S_{YM}}{\delta \underline{J}_\mu} \frac{\delta}{\delta \underline{A}^\mu} + \frac{\delta S_{YM}}{\delta \underline{\omega}} \frac{\delta}{\delta \underline{K}} + \frac{\delta S_{YM}}{\delta \underline{K}} \frac{\delta}{\delta \underline{\omega}} + \frac{\delta \Phi \mathcal{O}}{\delta \underline{A}_\mu} \frac{\delta}{\delta \underline{J}^\mu} \right] D = 0, \tag{7}$$

where

$$W^2 = 0. \tag{8}$$

The general solution of this equation is

$$-\frac{1}{2} a_1 \underline{F}_{\mu\nu} \underline{F}^{\mu\nu} - \frac{1}{2} a_2 n_\mu n_\nu \underline{F}^{\mu\lambda} \underline{F}^\nu_\lambda$$

$$-\frac{1}{2} b_1 \Phi \underline{F}_{\mu\nu} \underline{F}^{\mu\nu} - \frac{1}{2} b_2 \Phi n_\mu n_\nu \underline{F}_{\mu\lambda} \underline{F}^\nu_\lambda + WG, \tag{9}$$

with

$$G = a_3\, \underline{A}_\lambda (\underline{J}^\lambda + n^\lambda\, \underline{\eta}) + a_4\, n^\lambda\, \underline{A}_\lambda (n^\mu\, \underline{J}_\mu + n^2\, \underline{\eta}) + a_5\, \underline{\omega} \cdot \underline{K}$$

$$+ b_3\, \Phi\, \underline{A}_\lambda (\underline{J}^\lambda + n^\lambda\, \underline{\eta}) + b_4\, \Phi\, n^\lambda\, \underline{A}_\lambda (n^\mu\, \underline{J}_\mu + n^2\, \underline{\eta}) + b_5\, \Phi\, \underline{\omega} \cdot \underline{K}. \qquad (10)$$

The counter-terms to the first order in Φ are

$$D = -\frac{1}{2}(a_1 + a_3)\, \underline{F}_{\mu\nu}\, \underline{F}^{\mu\nu} - \frac{1}{2}(a_2 + 2a_4)\, n_\mu\, n_\nu\, \underline{F}^{\mu\lambda}\, \underline{F}^\nu_\lambda$$

$$+ \frac{1}{2} a_3\, g\, \underline{F}^{\mu\nu} (\underline{A}_\nu \times \underline{A}_\mu) + a_4\, n_\mu\, n_\nu\, \underline{F}^{\mu\lambda}\, \partial^\nu\, \underline{A}_\lambda +$$

$$+ \frac{1}{2} g\, a_5\, \underline{K} (\underline{\omega} \times \underline{\omega})$$

$$- a_5\, (\underline{J}^\mu + n^\mu\, \underline{\eta})\, (\partial_\mu\, \underline{\omega} + g\, \underline{A}_\mu \times \underline{\omega})$$

$$- a_3\, (\underline{J}^\mu + n^\mu\, \underline{\eta})\, \partial_\mu\, \underline{\omega}$$

$$- a_4\, (n_\lambda\, \underline{J}^\lambda + n^2\, \underline{\eta})\, n^\mu\, \partial_\mu\, \underline{\omega}$$

$$- \frac{1}{2} b_1\, \Phi\, \underline{F}_{\mu\nu}\, \underline{F}^{\mu\nu} - \frac{1}{2} b_2\, \Phi\, n_\mu\, n_\nu\, \underline{F}^{\mu\lambda}\, \underline{F}^\nu_\lambda$$

$$- \Phi\, [b_3\, \underline{A}^\mu + b_4\, n^\lambda\, \underline{A}_\lambda\, n^\mu]\, [\partial^\nu\, \underline{F}_{\mu\nu} + g\, (\underline{A}^\nu \times \underline{F}_{\mu\nu})]$$

$$- \Phi\, [b_3\, (\underline{J}^\mu + n^\mu\, \underline{\eta}) + b_4\, n^\mu\, (n \cdot \underline{J} + n^2\, \underline{\eta})]\, \partial_\mu\, \underline{\omega}$$

$$+ b_5\, \Phi\, [n_\mu\, \partial^\mu\, \underline{\eta} \cdot \underline{\omega} - g\, \underline{\eta}\, n \quad (\underline{A} \times \underline{\omega})]$$

$$- b_5\, \Phi\, g\, \underline{J}^\mu\, (\underline{A}_\mu \times \underline{\omega})$$

$$+ \frac{1}{2} b_5\, \Phi\, \underline{K}\, (\underline{\omega} \times \underline{\omega})$$

$$- a_3\, \Phi\, \underline{F}^{\mu\nu}\, \partial_\mu\, \underline{A}_\nu - a_3\, g\, \Phi\, \underline{F}_{\mu\nu}\, [\underline{A}_\mu \times \underline{A}_\nu]$$

$$- a_4\, \Phi\, \underline{F}^{\mu\nu}\, n_\nu\, \partial_\mu\, n \cdot \underline{A} - a_4\, g\, \Phi\, \underline{F}^{\mu\nu}\, n_\mu\, [n \cdot \underline{A} \times \underline{A}_\nu]. \qquad (11)$$

To one-loop order we find

$$a_1 = \frac{11}{6}\frac{g^2}{16\pi^2} C_{YM} \ln \lambda^2,$$

$$a_2 = 0,$$

$$a_3 = -\alpha \frac{g^2}{16\pi^2} C_{YM} \ln \lambda^2,$$

$$a_4 = 2\alpha (n^2)^{-1} \frac{g^2}{16\pi^2} C_{YM} \ln \lambda^2,$$

$$a_5 = \alpha \frac{g^2}{16\pi^2} C_{YM} \ln \lambda^2,$$

$$b_1 = b_2 = b_3 = b_4 = b_5 = 0. \tag{12}$$

3. *The twist-four operator in the Lorentz gauges*

In this section we briefly consider the simpler case of the Yang-Mills Lagrangian in the covariant gauges.

$$\begin{aligned} L = &-\frac{1}{4} \underline{F}_{\mu\nu} \underline{F}^{\mu\nu} - \frac{1}{2\alpha} (\partial_\mu \underline{A}_\mu)^2 - \frac{1}{4} \Phi \underline{F}_{\mu\nu} \underline{F}_{\mu\nu} \\ &- \underline{\eta}\, \partial_\mu (\partial^\mu \underline{\omega} + g\, \underline{A}^\mu \times \underline{\omega}) \\ &+ \underline{J}^\mu (\partial_\mu \underline{\omega} + g\, \underline{A}_\mu \times \underline{\omega}) - \frac{1}{2} g\, \underline{K} \underline{(\omega} \times \underline{\omega}). \end{aligned} \tag{13}$$

The solution of the BRS identities is now

$$D = -a_1 \underline{F}_{\mu\nu} \underline{F}^{\mu\nu} - b_1 \Phi \underline{F}_{\mu\nu} \underline{F}^{\mu\nu} + WG, \tag{14}$$

where

$$\begin{aligned} G = &\, a_2 \underline{A}_\lambda (\underline{J}^\lambda + \partial^\lambda \underline{\eta}) + a_3\, \underline{\omega} \cdot \underline{K} \\ &+ b_2 \Phi \underline{A}_\lambda (\underline{J}^\lambda + \partial^\lambda \underline{\eta}) + b_3 \Phi\, \underline{\omega} \cdot \underline{K}. \end{aligned} \tag{15}$$

The divergent counter-terms displayed explicitly are

$$D = -a_1 \underline{F}_{\mu\nu} \underline{F}^{\mu\nu}$$

$$
\begin{aligned}
&- a_2 \underline{A}^\mu \{\partial^\nu \underline{F}_{\mu\nu} + g(\underline{A}^\nu \times \underline{F}_{\mu\nu})\} \\
&- a_2 (\underline{J}_\mu + \partial_\mu \underline{\eta}) \partial^\mu \underline{\omega} \\
&+ a_3 (\Box \underline{\eta} + \partial_\mu \underline{J}^\mu) \underline{\omega} \\
&- a_3 (J^\mu + \partial^\mu \underline{\eta}) g (\underline{A}_\mu \times \underline{\omega}) \\
&- \frac{1}{2} g a_3 \underline{K} (\underline{\omega} \times \underline{\omega}) \\
&- b_1 \Phi \underline{F}_{\mu\nu} \underline{F}^{\mu\nu} \\
&- b_2 \Phi \underline{A}^\mu \{\partial_\nu \underline{F}^\nu_\mu + g(\underline{A}_\nu \times \underline{F}^\nu_\mu)\} \\
&- b_2 \Phi (\underline{J}_\mu + \partial_\mu \underline{\eta}) \partial^\mu \underline{\omega} \\
&+ b_3 \Phi (\Box \underline{\eta} + \partial^\mu \underline{J}_\mu) \cdot \underline{\omega} \\
&- b_3 \Phi (\underline{J}_\mu + \partial_\mu \underline{\eta}) g (\underline{A}^\mu \times \underline{\omega}) \\
&- \frac{1}{2} g b_3 \Phi \underline{K} (\underline{\omega} \times \underline{\omega}) \\
&- a_2 \Phi \underline{F}_{\mu\nu} \partial_\mu \underline{A}_\nu - a_2 \Phi g \underline{F}^{\mu\nu} (\underline{A}_\mu \times \underline{A}_\nu).
\end{aligned} \tag{16}
$$

The constants to one-loop order are

$$
\begin{aligned}
a_1 &= \frac{11}{12} \frac{g^2}{16\pi^2} C_Y \ln \lambda^2, \\
a_2 &= \left(- \frac{3}{4} - \frac{1}{4}\alpha\right) \frac{g^2}{16\pi^2} C_{YM} \ln \lambda^2, \\
a_3 &= \frac{1}{2} \alpha \frac{g^2}{16\pi^2} C_{YM} \ln \lambda^2, \\
b_1 &= 0, \\
b_2 &= - \frac{3}{8} \frac{g^2}{16\pi^2} C_{YM} \ln \lambda^2, \\
b_3 &= 0.
\end{aligned} \tag{17}
$$

4. Conclusion

We have given a systematic approach to the renormalization of the twist-four operator. Contrary to the previous authors[4,5], we have demonstrated that the counter-terms can be found as the solution of the *BRS* identities.

We have displayed the divergent parts with constants calculated to one--loop order.

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ANĐELKA ANDRAŠI

Institut »Ruđer Bošković«, 41000 Zagreb

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JOHN C. TAYLOR

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street, Cambridge, CB3 9EW, U. K.

UDK 539.12

Originalni znanstveni rad

Proučavana je renormalizacija tvist-4 operatora i dano je potpuno rješenje za jedno uključenje operatora pri proizvoljnom transferu impulsa. Konstante renormalizacije izračunate su do reda jedne petlje.