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MASS AND PIONIC FORM FACTOR OF THE NUCLEON

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Assuming that particle masses are of dynamical origin, we calculate in a simple model the nucleon mass in terms of its pionic form factor.

1. Introduction

It is an old idea in physics that the masses of fundamental particles arise either entirely or mostly through dynamics. Thus, most of the nucleon mass should be identified with its strong-interaction self-energy. By an additional shift of a few MeV, which is due to the electromagnetic self-energy, the nucleon mass is brought to the observed value.

It is a difficult task to express this idea in the form of a calculation because no reliable theory of strong interactions has been developed so far. For the same reason, the proton-neutron mass difference (believed to be only of electromagnetic origin) could not have been explained in a fully satisfactory way[1].

Here we adopt a semi-field-theoretical approach relying on the analytic *S*-matrix and form factors. Having a direct physical meaning, form factors ensure a natural cutoff which is measurable. Our model is the following.

2. *The model*

Arguing along the line of the time-energy uncertainty principle, it may be assumed that pions represent the dominant component of the nucleon internal structure. This is due to the fact that in a formal expansion of the physical nucleon, states with pions in the cloud have the smallest mass and therefore the longest lifetime. Besides the nucleon there is always at least one pion in the hadronic bag of the physical nucleon. Thus, when a formal expansion of the nucleon propagator is performed over all possible states with required quantum numbers, a single pion may be factorized out of all pions containing states. In analogy with Cottingham's formula, this enables us to write the strong nucleon self-energy in the form (see Fig. 1)

$$\Sigma = \mathrm{i} \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{\delta_{\alpha\beta}}{q^2 - m_\pi^2 + \mathrm{i}\,\varepsilon} T_{\pi N}^{\alpha\beta}(p, q). \tag{1}$$

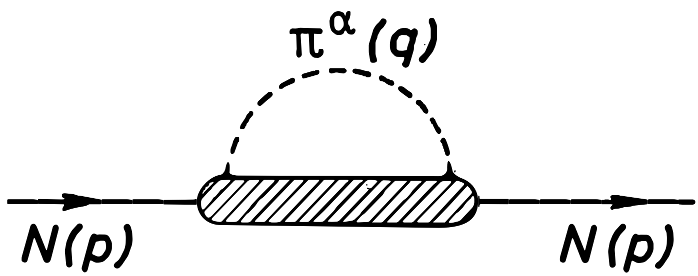

Fig. 1. The virtual pion-nucleon forward scattering amplitude.

In the usual language of *QED*, this is the nucleon quantum field-theoretic self-energy part corresponding to the single-pion radiative corrections. The pion mass in (1) is the one from the free pion propagator and, working with chiral pions, may be put to zero in practical calculations. In (1) $T_{\pi N}^{\alpha\beta}(p, q)$ is the virtual pion-nucleon forward scattering amplitude, conventionally expressed by

$$T_{\pi N}^{\alpha\beta}(p, q) = \mathrm{i} \int \mathrm{d}^4 x \, \mathrm{e}^{\mathrm{i}qx} \, \Theta(-x_0) < p \,|[j_\alpha(x/2), j_\beta(-x/2)]|\, p>. \tag{2}$$

From now on we forget about *QFT* and turn to the framework of the analytic *S*-matrix.

The main obstacle to further procedure is the fact that little is known about the q^2 behaviour of this amplitude (q^2 is the mass of the virtual pion). In the following we argue that in the particular case of interest, the dominant mass contribution comes from the single-nucleon pole in the off-shell πN amplitude. When a complete set of physical states is inserted in (2) and only the single-nucleon contribution is taken into account, the problem is reduced to the evaluation of

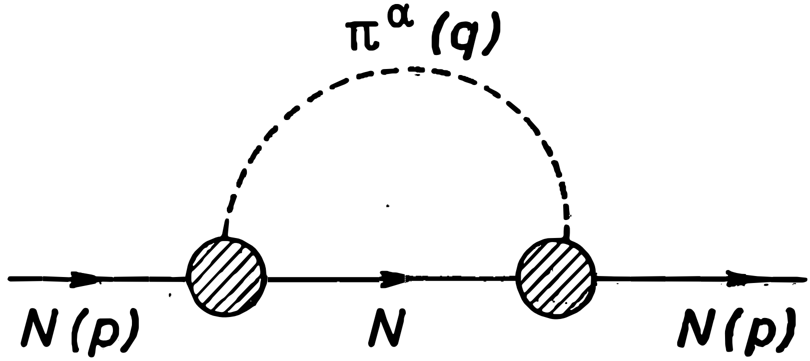

Fig. 2. The generalized Born term in the virtual pion-nucleon forward scattering amplitude. Intermediate nucleon is on its mass shell.

the generalized Born term. In Fig. 2 we show this term with the intermediate nucleon on its mass shell. We obtain

$$\Sigma = -3\mathrm{i}\frac{g_{\pi N}^2}{(2\pi)^4}\int \mathrm{d}^4 q \frac{\overline{u}(p)\,\gamma_\mu q_\mu u(p)\, G_{\pi N}^2(q^2)}{(q^2 - m_\pi^2)\,[(p-q)^2 - m^2]}, \tag{3}$$

where the nucleon pionic form factor is defined by

$$\langle N(p+q)\,|\,j_\alpha(0)\,|\,N(p)\rangle = \mathrm{i}g_{\pi N}\, G_{\pi N}(q^2)\,\overline{u}(p+q)\,\gamma_5\,\tau_a\, u(p). \tag{4}$$

The approximation adopted here can be made plausible only along the line of the time-energy uncertainty principle, $\Delta E \cdot \Delta T = \hbar$. The physical nucleon spends most of its time (ΔT) in the state of smallest energy variation (ΔE), the πN state. With the belief that this approximation holds, we now have to face the central unknown quantity, namely, the pionic form factor of the nucleon, $G_{\pi N}(q^2)$.

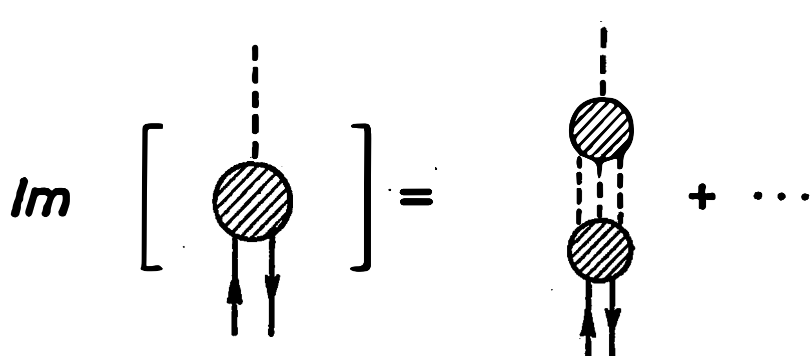

Fig. 3. The imaginary part of the πNN vertex shown in a symbolic way.

Very little is known about this form factor. Already the leading low-energy contribution to its imaginary part involves inelastic amplitudes (Fig. 3). Some recent calculations[2,3] have been performed only in the vicinity of the soft-pion limit, whereas the integration in (3) runs over the whole q space. Some previous parametrizations[4] included higher values of $-q^2$; however, they contained a hard-core term, and can hardly be considered presently as realistic. Being constant at infinity, such a form factor does not ensure the cutoff of the mass integral.

Within a class of confining potentials, recent nonrelativistic *QCD* calculations imply a $G_{\pi N}(q^2)$ which falls off (exponentially) extremely rapidly[5]. Having in mind the known analyticity properties, it is difficult to adopt such a type of behaviour, when one has to extend the integration over the whole q space.

A further piece of information about $G_{\pi N}(q^2)$ is available in the soft-pion region where the *PCAC* hypothesis correlates the pionic form factor of the nucleon to its axial-vector form factors[6]. In no case, however, can this soft-pion information be extrapolated into the whole q space.

Having in mind that we insist on the verification of the idea of dynamical mass rather than on the numerical accuracy, we adopt the following simple model. When we evaluate the mass integral (3) by using Feynman's standard method of symmetric integration, the virtual pion mass q^2 always stays in the region of pion-nucleon scattering owing to the $q_0 \to i\, q_0$ rotation. In this region, $G_{\pi N}(q^2)$ is real and, being far from the cut which starts at $q^2 = 9\, m^2$, may be parametrized by the closest simple pole. There is only one pion-like particle found below 2 GeV, namely, π (1300)[7]. Denoting its mass by λ, we write

$$G_{\pi N}(q^2) = \frac{\lambda^2 - m_\pi^2}{\lambda^2 - q^2}. \tag{5}$$

By evaluating the integral (3) and putting $\sum = m$, we obtain the sum rule

$$1 = \frac{3}{8\pi}\frac{g_{\pi N}^2}{4\pi}\left[\frac{\lambda^2 - m_{\pi N}^2}{m^2}\left(1 - \frac{\lambda^2}{2m^2}\ln\frac{\lambda^2}{m^2} - \frac{\lambda^2}{m^2}\frac{2m^2-\lambda^2}{4m^2-\lambda^2}\, Y_\lambda \operatorname{arctg} Y_\lambda\right) - \right.$$
$$\left. - \frac{m_\pi^2}{m^2}(I_\lambda - I_m)\right], \tag{6}$$

with the abbreviated notation

$$Y_\lambda = \left(\frac{4m^2}{\lambda^2} - 1\right)^{1/2}, \quad I_\lambda = \left(1 - \frac{\lambda^2}{2m^2}\right)\ln\frac{\lambda^2}{m^2} - \frac{\lambda^2}{m^2}\, Y_\lambda \operatorname{arctg} Y_\lambda.$$

Given the pion-nucleon coupling constant and the pionic form factor (here parametrized by λ), the sum rule (6) determines the nucleon hadronic mass. When the values $\lambda^2 = 1.69\,\mathrm{GeV}^2$ [7] and $(g_{\pi N}^2/4\pi) = 14.3$ [8] are used, the solution of the above transcendental equation for the nucleon mass is $m = 867$ MeV. Having in mind the crudeness of our approximation, this result is satisfactory. Furthermore, it is expected that inclusion of heavier nucleon isobars, in particular the Ropper resonance, will raise this value, but not too much because of their larger masses and more rapidly decreasing form factors at large q^2.

It is worth stressing that the result of the present calculation depends only slightly on the particular choice of the form factor provided that it has the correct behavior at low q^2 (the normalization and the slope at $q^2 = m_\pi^2$) and that it is sufficiently strongly bounded at high q^2, so that the convergence of the mass integral is ensured.

Although it is difficult to understand the ansatz in Ref. 2 in view of the axiomatic analyticity properties of $G_{\pi N}(q^2)$, for the sake of comparison, we have evaluated the mass integral (3) by using the quoted[2] best value of the free parameter found within the region $0 \geqslant q^2 > -0.1$ GeV^2. It leads to $m = 526$ MeV, a considerably lower value for the nucleon mass. This value is still not too low if we have in mind the possibility of the extrapolation of this model outside of the narrow strip considered above.

We make the following comments. If our model evaluation holds with a sufficient degree of confidence, as we hope it does, the result obtained supports the idea of dynamical mass generation. The essential assumption made here about quantum-mechanical fluctuations of the physical nucleon is that the πN state dominates in the expansion of the propagator — the nucleon spends most of its lifetime in a πN state. In other words, the πN state is the dominant component of the nucleon structure. This assumption can presently be supported in no other way but by the $\Delta E \cdot \Delta T = \hbar$ principle.

In a similar sense we may interpret and trust the parametrization (5) of the nucleon pionic form factor, which is very poorly known. A more realistic model would be to consider the 3π intermediate state in Fig. 3 not as a bound state π (1300), which we apply here, but as a general 3π state. This leads to the product of two $\pi\pi \to \pi\pi$ amplitudes; in one of them, one pion is identified with the nucleon-antinucleon pair in the s state. Of course, this pion is highly off its mass shell. The necessary analytic continuation may then be easily performed by using the well-known Veneziano ansatz for the $\pi\pi \to \pi\pi$ amplitude[9]. As it has already been shown[10], this ansatz works very well just for pionic systems. However, this model of the pionic form factor is solvable only in a numerical way and is, therefore, outside the scope of this paper.

To conclude, this simple model renormalizes the nucleon field-theoretical self-energy in a semi-phenomenological way through introduction of the pionic form factor. Here it plays the role of a measurable cutoff. The result of the calculation supports the possibility of ascribing almost all the mass of a nucleon to its strong self-energy. It is, however, not clear to what extent the time-energy uncertainty principle is sufficient to argue away the contributions of higher components of the nucleon structure.

Finally, since we have worked within the S-matrix framework, reference to the quark structure of the nucleon is neither possible nor necessary.

References

1) See, for example, A. Ze, Physics Reports **3C** (1972) 129;

2) K. Bongardt, H. Pilkuhn, and H. G. Schlaile, Phys. Lett. **52B** (1974) 271;

3) J. Korner, private communications;

4) E. Ferrari and F. Selleri, Nuovo Cimento **27** (1963) 1450;
5) R. Tegan, R. Brockmann, and W. Weise, Z. Physik **A307** (1982) 339; E. Werner, *Proceedings of the Schladming Winter School* 1982, to be published;
6) See, for example, S. L. Adler and R. F. Dashen, in *Current Algebra and Applications to Particle Physics* (Benjamin Inc., New York, 1968);
7) Particle Data Group, Review of Particle Properties, Phys. Lett. **111B** (April 1982);
8) M. M. Nagels, Th. A. Rijken, J. J. De Swart, G. C. Oades, J. L. Petersen, A. C. Irving, C. Jarlskog, W. Pfeil, H. Pilkuhn and H. P. Jakov, Nucl. Phys. **147B** (1979) 189;
9) G. Veneziano, Nuovo Cimento **57A** (1968) 190;
10) C. Lovelace, Phys. Lett. **28B** (1968) 264.

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Pretpostavivši da su mase fundamentalnih čestica dinamičkog porijekla, računali smo u jednostavnom modelu masu nukleona u zavisnosti o njegovom pioničkom form-faktoru.