YU ISSN 0015—3206
FZKAAA 16 (2) 167 (1984)

NONLINEAR RESONANT THREE-WAVE INTERACTIONS IN A MAGNETISED PLASMA

BHIMSEN K. SHIVAMOGGI

Physical Research Laboratory, Ahmedabad 380 009, *India*

Received 25 November 1982

Revised manuscript received 1 September 1983

Original scientific paper

UDC 533.95

The nonlinear resonant interactions among two circularly-polarised waves and a Langmuir wave in a cold plasma subjected to a magnetic field directed parallel to the direction of propagation of the three waves are considered. The method of multiple time-scales is used. It is found that if two circularly-polarised waves with opposite polarisations and large amplitudes interact, then a Langmuir wave can be excited, and further that it is possible for a large amplitude Langmuir wave to decay into two circularly-polarised waves with the same polarisation. Next, the nonlinear resonant interactions among three extraordinary waves in a warm plasma subjected to a magnetic field directed perpendicular to the direction of propagation of the three waves is considered. The method of coupled-modes is used.

1. Introduction

When two waves with frequencies ω_1, ω_2, and wavevectors $\vec{k}_1$, $\vec{k}_2$, respectively, are present, the nonlinear terms in the plasma equations may contain the product of the wave amplitudes, viz,

$$e^{i(\vec{k}_1 - \vec{k}_2) \cdot \vec{x} - i(\omega_1 - \omega_2) t}$$

which is a beat-frequency wave. If the frequency $\omega_1 - \omega_2 = \omega_3$ and the wave-vector $\vec{k}_1 - \vec{k}_2 = \vec{k}_3$ of the beat happen to be a normal mode of the plasma, i. e.,

$$\varepsilon (\omega_3, \vec{k}_3) = 0$$

$\varepsilon (\omega, \vec{k})$ being the lowest-order dielectric constant of the plasma, then this mode will be generated by the interaction of the first two waves. An exchange of energy and momentum takes place between these three waves and the process is called resonant wave-interaction (Davidson[1] Ch. 6).

The nonlinear interaction between an electron plasma wave and two transverse electromagnetic waves in a cold electron-plasma was studied by Montgomery[2]. This problem is motivated by the experimental possibility of exciting plasma oscillations by properly tuned laser beams (Stern and Tzoar[3]). Sjolund and Stenflo[4] extended the treatment to the case of a warm plasma. Experiments of Stamper et al.[5] showed that intense spontaneously-generated magnetic fields are present in laser-infested plasmas. Sjolund and Stenflo[6] extended the treatment of three-wave interaction to the case of a magnetised plasma with the three waves which are now two circularly-polarised waves and a Langmuir wave propagating parallel to the direction of the applied magnetic field, and used the method of coupled modes. Das[7] considered the three-wave interaction in a magnetised plasma with the magnetic field applied perpendicular to the direction of the three waves (which are now three extraordinary waves) and used the Krylov-Bogoliubov-Mitropolski method. Nonlinear resonant interaction between three ordinary electromagnetic waves propagating perpendicular to the applied magnetic field was treated by Stenflo[8,9], Krishan et al.[10] and Munoz and Degach[11]. Nonlinear resonant interaction between three longitudinal waves in a magnetised plasma was treated by Das and Banerjee[12] and Selim[13].

The present paper develops a method-of-multiple-scale treatment (Nayfeh[14]) to study the nonlinear resonant interaction between two circularly polarised waves and a Langmuir wave all propagating parallel to the direction of the applied magnetic field. This approach reveals several physical aspects of this problem which are completely peculiar to the case of the waves propagating parallel to the direction of the applied magnetic field and are not apparent in the treatment of Sjolund and Stenflo[6]. The present paper next considers the resonant three-wave interaction between extraordinary electromagnetic waves all propagating perpendicular to the direction of the applied magnetic field. A more elegant technique is developed which is a modified version of the method of coupled modes to treat this problem.

2. *Interactions between two circularly-polarised waves and a Langmuir wave in a cold plasma*

Consider a cold electron-fluid plasma subjected to a constant magnetic field $B_0 = B_0 \hat{i}_x$ and embedded in a uniformly smeared-out background of immobile ions. The wave-motion in such a plasma is governed by the following equations:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{v}) = 0 \tag{1}$$

$$\frac{\partial v}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = - \frac{e}{m} \left(\vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right) \tag{2}$$

$$\nabla \times \vec{E} = - \frac{1}{c} \frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\nabla \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} - \frac{4\pi e}{c} n \vec{v} \tag{4}$$

where the usual notations have been used. Let us now consider wave motions in the $\vec{x}$-direction. Eqs. (1)—(4) can be written in a form that exhibts the linearised parts and the nonlinear parts separately

$$\frac{\partial n}{\partial t} + n_0 \frac{\partial v_x}{\partial x} = - \varepsilon \frac{\partial x}{\partial x} (n v_x) \tag{5}$$

$$\frac{\partial v_x}{\partial t} \div \frac{e}{m} E_x = - \varepsilon \left[v_x \frac{\partial v_x}{\partial x} + \frac{e}{mc} (v_y B_z - v_z B_y) \right] \tag{6}$$

$$\frac{\partial v_y}{\partial t} + \frac{e}{m} E_y + \frac{e}{mc} v_z B_0 = - \varepsilon \left[v_x \frac{\partial v_y}{\partial x} + \frac{e}{mc} (v_z B_x - v_x B_z) \right] \tag{7}$$

$$\frac{\partial v_z}{\partial t} + \frac{e}{m} E_z - \frac{e}{mc} v_y B_0 = - \varepsilon \left[v_x \frac{\partial v_z}{\partial x} + \frac{e}{mc} (v_x B_y - v_y B_x) \right] \tag{8}$$

$$\frac{\partial B_x}{\partial t} = 0 \tag{9}$$

$$\frac{\partial B_y}{\partial t} - c \frac{\partial E_z}{\partial x} = 0 \tag{10}$$

$$\frac{\partial B_z}{\partial t} + c \frac{\partial E_y}{\partial x} = 0 \tag{11}$$

$$\frac{\partial E_x}{\partial t} - 4\pi e n_0 v_x = \varepsilon 4\pi e n v_x \tag{12}$$

$$\frac{\partial E_y}{\partial t} + c \frac{\partial B_z}{\partial x} - 4\pi e n_0 v_y = \varepsilon 4\pi e n v_y \tag{13}$$

$$\frac{\partial E_z}{\partial t} - c \frac{\partial B_y}{\partial x} - 4\pi e n_0 v_z = \varepsilon 4\pi e n v_z \tag{14}$$

where the subscript $_0$ denotes the unperturbed values, the unsubscripted quantities refer to the perturbations characterised by the small parameter ε. Note that the left-hand sides in Eqs. (5)—(14) represent the linear problem, and the right-hand sides represent the nonlinear terms. Let us express the quantities $\vec{v}$, $\vec{E}$ and $\vec{B}$ in the following form:

$$\vec{Q} = Q_x \hat{i}_x + \vec{Q}_\perp$$

$$\vec{Q}_\perp = \frac{1}{2}(Q_+ \hat{i}_- + Q_- \hat{i}_+) \tag{15}$$

$$Q_\pm \equiv Q_y \pm i\, Q_z, \quad \hat{i}_\pm \equiv \hat{i}_y \pm \mathrm{i}\, \hat{i}_z.$$

Note that this representation explicitly brings forth the presence of circularly-polarised waves in a magnetised plasma.

Using (15), Eqs. (5)—(14) can be combined to give:

$$\frac{\partial^2 E_x}{\partial t^2} + \omega_p^2 E_x = \varepsilon \cdot 4\pi e \left[\frac{\partial}{\partial t}(n\, v_x) - n_0 \left\{v_x \frac{\partial v_x}{\partial x} + \frac{\mathrm{i}\, e}{2mc}(v_+ B_- - v_- B_+)\right\}\right] \tag{16}$$

$$\frac{\partial^2 E_\pm}{\partial t^2} - c^2 \frac{\partial^2 E_\pm}{\partial x^2} + \omega_p^2 E_\pm \mp 4\pi n_0 e \mathrm{i} \Omega v_\pm = \varepsilon \cdot 4\pi e \left[\frac{\partial}{\partial t}(n\, v_\pm) - \right.$$

$$\left. - n_0 \left(v_x \frac{\partial v_\pm}{\partial x} \pm \frac{\mathrm{i}\, e}{m c} v_x B_\pm\right)\right] \tag{17}$$

where,

$$\omega_p^2 \equiv \frac{4\pi n_0 e^2}{m}, \quad \Omega \equiv \frac{e B_0}{m c}.$$

It is obvious from Eqs. (16) and (17) that three circularly-polarised waves propagating parallel to the applied magnetic field cannot interact. This result was also deduced by Krishan and Fukai[15] who used the quantum-field theory formalism for plasma wave interactions. Therefore, one considers the interaction of two circularly-polarised waves with a Langmuir wave.

Let us consider a right-circularly-polarised electromagnetic wave and a left-circularly-polarised electromagnetic wave with frequencies ω_1, ω_2 and wave numbers k_1, k_2, respectively, propagating in the $\vec{x}$ direction. One has

$$\omega_1^2 - k_1^2 c^2 - \omega_p^2 \frac{\omega_1}{\omega_1 - \Omega} = 0$$

$$\omega_2^2 - k_2^2 c^2 - \omega_p^2 \frac{\omega_2}{\omega_2 + \Omega} = 0. \tag{18}$$

In the following, we exclude the possibility of the cyclotron resonance $\omega_{1,2} = \Omega$. Due to nonlinear interaction between these two waves an electrostatic plasma wave with frequency ω_3 and wave number k_3 propagating in the $\vec{x}$-direction will be excited such that

$$\omega_1 - \omega_2 = \omega_3 = \omega_p, \quad k_1 - k_2 = k_3. \tag{19}$$

In order to treat this three-wave interaction, let us use the method of multiple time-scales (Nayfeh[14]). Thus, seek solutions to Eqs. (16) and (17) of the form

$$Q(x, t; \varepsilon) = Q^{(0)}(x, t, \tilde{t}) + \varepsilon Q^{(1)}(x, t, \tilde{t}) + O(\varepsilon^2) \tag{20}$$

where $\tilde{t} = \varepsilon t$ is the slow-time scale characterising the rate at which energy is exchanged between the three waves.

Using (20), Eqs. (5)—(14) give

$$E^{(0)} = (a_3 e^{i\psi_3} + a_3^* e^{-i\psi_3}) \hat{i}_x +$$

$$+ (a_1 e^{i\psi_1} + a_1^* e^{-i\psi_1}) \hat{i}_+ + (a_2 e^{i\psi_2} + a_2^* e^{-i\psi_2}) \hat{i}_-$$

$$\vec{B}^{(0)} = -\frac{i c k_1}{\omega_1} (a_1 e^{i\psi_1} + a_1^* e^{-i\psi_1}) \hat{i}_+ + \frac{i c k_2}{\omega_2} (a_2 e^{i\psi_2} + a_2^* e^{-i\psi_2}) \hat{i}_-$$

$$\vec{v}^{(0)} = \frac{e}{i m \omega_3} (a_3 e^{i\psi_3} - a_3^* e^{-i\psi_3}) \hat{i}_x +$$

$$+ \left[\frac{e a_1 e^{i\psi_1}}{i m (\omega_1 - \Omega)} - \frac{e a_1^* e^{-i\psi_1}}{i m (\omega_1 - \Omega)} \right] \hat{i}_+ +$$

$$+ \left[\frac{e a_2 e^{i\psi_2}}{i m (\omega_2 + \Omega)} - \frac{e a_2^* e^{-i\psi_2}}{i m (\omega_2 + \Omega)} \right] \hat{i}_-$$

$$n^{(0)} = \frac{e n_0 k_3}{i m \omega_3^2} (a_3 e^{i\psi_3} - a_3^* e^{-i\psi_3}) \tag{21}$$

where,

$$\psi_s = k_s x - \omega_s t, \quad a_s = a_s(\tilde{t}); \quad s = 1, 2, 3$$

and the stars denote the complex conjugates.

Using (21), Eqs. (16) and (17) give for the $O(\varepsilon)$ problem:

$$\frac{\partial^2 E_x^{(1)}}{\partial t^2} + \omega_p^2 E_x^{(1)} = \varepsilon\, 2i\, \omega_3 \left(\frac{\partial a_3}{\partial t} e^{i\psi_3} - \frac{\partial a_3^*}{\partial \tilde{t}} e^{-i\psi_3} \right) -$$

$$- \varepsilon \frac{\mathrm{i}\, e\, \omega_p^2}{2m} \left[\left\{ \frac{k_1}{\omega_1 (\omega_2 + \Omega)} - \frac{k_2}{\omega_2 (\omega_1 - \Omega)} \right\} \times \right.$$

$$\left. \times\, a_1\, a_2^*\, \mathrm{e}^{\mathrm{i}(\varphi_1 - \varphi_2)} - \left\{ \frac{k_1}{\omega_1 (\omega_2 + \Omega)} - \frac{k_2}{\omega_2 (\omega_1 - \Omega)} \right\} a_1^*\, a_2\, \mathrm{e}^{-\mathrm{i}(\varphi_1 - \varphi_2)} \right] +$$

$$+ \text{ nonresonant terms} \tag{22}$$

$$\frac{\partial^2 E_\pm^{(1)}}{\partial t^2} - c_2 \frac{\partial^2 E_\pm^{(1)}}{\partial x^2} + \omega_p^2 E_\pm^{(1)} \mp 4\pi n_0\, e\, \mathrm{i}\, \Omega\, v_\pm^{(1)} =$$

$$= \varepsilon\, 2\mathrm{i}\, \omega_{2,1} \left(\frac{\partial a_{2,1}}{\partial \tilde{t}}\, \mathrm{e}^{\mathrm{i}\varphi_{2,1}} - \frac{\partial a_{2,1}^*}{\partial \tilde{t}}\, \mathrm{e}^{-\mathrm{i}\varphi_{2,1}} \right) + \text{ nonresonant terms.} \tag{23}$$

The removal of the secular terms in Eq. (22) and (23) requires

$$\frac{\partial a_{1,2}}{\partial \tilde{t}} \approx 0 \tag{24a}$$

$$\frac{\partial a_3}{\partial \tilde{t}} = \frac{e\, \omega_p^2}{4m\, \omega_3} \left[\frac{k_1}{\omega_1 (\omega_2 + \Omega)} - \frac{k_2}{\omega_2 (\omega_1 - \Omega)} \right] a_1\, a_2^*. \tag{24b}$$

Eqs. (24) may be interpreted in the following manner: If two circularly-polarised waves with opposite polarisations and large amplitudes (so that the latter will change little during the initial stages, Davidson[1] Ch. 6) interact with one another, then Eq. (24c) implies that a Langmuir wave can be excited by this interaction. If the quantity in the rectangular brackets on the right-hand side of Eq. (24c) is made positive by suitably choosing the parameters k_1, k_2 and Ω, then this Langmuir wave has the possibility of growing in time until it saturates due to collisional effects in the plasma.

Let us next consider two right-circularly-polarised electromagnetic waves with frequencies ω_1 and ω_2, and wave-numbers k_1 and k_2, respectively, propagating in the $\vec{x}$-direction. Let these two waves engage in a nonlinear interaction with x Langmuir wave with frequency ω_3 and wave number k_3 and propagating in the $\vec{a}$-direction such that;

$$\omega_1 = \omega_3 - \omega_2, \quad k_1 = k_3 - k_2, \quad \omega_3 = \omega_p. \tag{25}$$

In order to treat this three-wave interaction, let us seek solutions to Eqs. (16) and (17) of the form given in (20).

Using (20), Eqs. (5)—(14) give

$$\vec{E}^{(0)} = (a_3\, \mathrm{e}^{\mathrm{i}\varphi_3} + a_3^*\, \mathrm{e}^{-\mathrm{i}\varphi_3})\, \hat{i}_x + (a_1\, \mathrm{e}^{\mathrm{i}\varphi_1} + a_1^*\, \mathrm{e}^{-\mathrm{i}\varphi_1} + a_2\, \mathrm{e}^{\mathrm{i}\varphi_2} + a_2^*\, \mathrm{e}^{-\mathrm{i}\varphi_2})\, \hat{i}_+$$

$$\vec{B}^{(0)} = -\,\mathrm{i}\left[\frac{c\,k_1}{\omega_1}(a_1\,\mathrm{e}^{\mathrm{i}\psi_1} + a_1^*\,\mathrm{e}^{-\mathrm{i}\psi_1}) + \frac{c\,k_2}{\omega_2}(a_2\,\mathrm{e}^{\mathrm{i}\psi_2} + a_1^*\,\mathrm{e}^{-\mathrm{i}\psi_2})\right]\hat{i}_+$$

$$\vec{v}^{(0)} = \frac{e}{\mathrm{i}\,m\omega_3}(a_3\,\mathrm{e}^{\mathrm{i}\psi_3} - a_3^*\,\mathrm{e}^{-\mathrm{i}\psi_3})\,\hat{i}_x + \left[\frac{e}{\mathrm{i}\,m\,(\omega_1 - \Omega)}(a_1\,\mathrm{e}^{\mathrm{i}\psi_1} - a_2^*\,\mathrm{e}^{-\mathrm{i}\psi_2})\right]\hat{i}_+$$

$$n^{(0)} = \frac{e\,n_0\,k_3}{\mathrm{i}\,m\,\omega_3}(a_3\,\mathrm{e}^{\mathrm{i}\psi_3} - a_3^*\,\mathrm{e}^{-\mathrm{i}\psi_3}) \tag{26}$$

where,

$$\psi_s = k_s\,x - \omega_s\,t, \qquad a_s = a_s(\tilde{t}); \qquad s = 1, 2, 3.$$

Using (26), Eqs. (16) and (17) give for the $O(\varepsilon)$ problem:

$$\frac{\partial^2 E_x^{(1)}}{\partial t^2} + \omega_p^2\,E_x^{(1)} = \varepsilon\,2\mathrm{i}\,\omega_3\left(\frac{\partial a_3}{\partial\tilde{t}}\,\mathrm{e}^{\mathrm{i}\psi_3} - \frac{\partial a_3^*}{\partial\tilde{t}}\,\mathrm{e}^{-\mathrm{i}\psi_3}\right) + \text{nonresonant terms} \tag{27}$$

$$\frac{\partial^2 E_{-1}^{(1)}}{\partial t^2} - c^2\,\frac{\partial^2 E_{-1}^{(1)}}{\partial x^2} + \omega_p^2\,E_{-1}^{(1)} + 4\pi\,n_0\,e\,\mathrm{i}\,\Omega\,v_{-1} =$$

$$= \varepsilon\,2\mathrm{i}\,\omega_1\left(\frac{\partial a_1}{\partial\tilde{t}}\,\mathrm{e}^{\mathrm{i}\psi_1} - \frac{\partial a_1^*}{\partial\tilde{t}}\,\mathrm{e}^{-\mathrm{i}\psi_1}\right) + \varepsilon\,\frac{\mathrm{i}\,e\,\omega_p^2}{m\,\omega_3}\left[\left\{-\frac{k_3\,(\omega_3 - \omega_2)}{\omega_3\,(\omega_2 - \Omega)} + \frac{k_2}{\omega_2 - \Omega} - \right.\right.$$

$$\left.\left. - \frac{k_2}{\omega_2}\right\} \times \{a_3\,a_2^*\,\mathrm{e}^{\mathrm{i}(\psi_3 - \psi_2)} - a_3^*\,a_2\,\mathrm{e}^{-\mathrm{i}(\psi_3 - \psi_2)}\}\right] + \text{nonresonant terms} \tag{28}$$

$$\frac{\partial^2 E_{-2}^{(1)}}{\partial t^2} - c^2\,\frac{\partial^2 E_{-2}^{(1)}}{\partial x^2} + \omega_p^2\,E_{-2}^{(1)} + 4\pi\,n_0\,e\,\mathrm{i}\,\Omega\,v_{-2}^{(1)} =$$

$$= \varepsilon\,2\mathrm{i}\,\omega_2\left(\frac{\partial a_2}{\partial\tilde{t}}\,\mathrm{e}^{\mathrm{i}\psi_2} - \frac{\partial a_2^*}{\partial\tilde{t}}\,\mathrm{e}^{-\mathrm{i}\psi_2}\right) +$$

$$+ \varepsilon\,\frac{\mathrm{i}\,e\,\omega_p^2}{m\,\omega_3}\left\{\left[-\frac{k_3\,(\omega_3 - \omega_1)}{\omega_3\,(\omega_1 - \Omega)} + \frac{k_1}{\omega_1 - \Omega} - \frac{k_1}{\omega_1}\right] \times\right.$$

$$\left. \times \left[a_3\,a_1^*\,\mathrm{e}^{\mathrm{i}(\psi_3 - \psi_1)} - a_3^*\,a_1\,\mathrm{e}^{-\mathrm{i}(\psi_3 - \psi_1)}\right]\right\} + \text{nonresonant terms.} \tag{29}$$

The removal of the secular terms in Eqs. (27)—(30) then requires

$$\frac{\partial a_1}{\partial\tilde{t}} = \frac{e\,\omega_p^2}{2m\,\omega_1\,\omega_3^2}\left[\frac{k_3\,\omega_1\,\omega_2 - k_2\,\omega_3\,\Omega}{\omega_2\,(\omega_2 - \Omega)}\right] a_3\,a_2^* \tag{30a}$$

$$\frac{\partial a_2}{\partial \tilde{t}} = \frac{e\,\omega_p^2}{2m\,\omega_2\,\omega_3^2}\left[\frac{k_3\,\omega_1\,\omega_2 - k_1\,\omega_3\,\Omega}{\omega_1\,(\omega_1 - \Omega)}\right] a_3 a_1^* \tag{30b}$$

$$\frac{\partial a_3}{\partial \tilde{t}} \approx 0. \tag{30c}$$

Eq. (30) may be interpreted as follows: If a Langmuir wave with a large amplitude interacts with two circularly-polarised waves with the same polarisation and small amplitudes, then the amplitude of the Langmuir wave will change only negligibly with time (which is what Eq. (30c) implies). If the quantities in the rectangular brackets on the right hand sides of Eqs. (30a) and (30b) are made positive by suitably choosing the parameters k_1, k_2 and Ω, then these equations indicate the possibility of these circularly-polarised waves growing in time (this would be like the decay of a large-amplitude Langmuir wave into two circularly-polarised waves with the same polarisation).

These results have been missed by Sjolund and Stenflo[6] in their treatment using a different method.

3. Interactions among three extraordinary waves in a warm plasma

Consider a warm electron-plasma subjected to a constant magnetic field $\vec{B}_0 = B_0\hat{i}_z$ and embedded in a uniformly smeared-out background of ions. The wave motion in such a plasma is governed by the following equations

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}) = 0 \tag{31}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\,\nabla)\,\vec{v} = -\frac{e}{m}\left(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right) - \frac{a^2}{n}\nabla n \tag{32}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{33}$$

$$\nabla \times \vec{B} = -\frac{1}{c}\frac{\partial \vec{E}}{\partial t} - \frac{e}{c}\,n\,\vec{v} \tag{34}$$

where a is the thermal velocity.

Let us now consider wave motions in the $\vec{x}$-direction. Eqs. (31)—(34) can be written in a form that exhibits the linearised parts and the nonlinear parts separately

$$\frac{\partial n}{\partial t} + n_0\frac{\partial v_x}{\partial x} = -\,\varepsilon\,\frac{\partial}{\partial x}(n\,v_x) \tag{35}$$

$$\frac{\partial v_x}{\partial t} + \frac{e E_x}{m} + \Omega v_y + \frac{a^2}{n_0}\frac{\partial n}{\partial x} = -\varepsilon\left(v_x \frac{\partial v_x}{\partial x} + \frac{e}{mc} v_y B_z - \frac{a^2}{n_0} n \frac{\partial n}{\partial x}\right) \quad (36)$$

$$\frac{\partial v_y}{\partial t} + \frac{e E_y}{m} - \Omega v_x = -\varepsilon\left(v_x \frac{\partial v_y}{\partial x} - \frac{e}{mc} v_x B_z\right) \quad (37)$$

$$\frac{\partial E_x}{\partial x} - e n_0 v_x = \varepsilon (e n v_x) \quad (38)$$

$$\frac{\partial E_y}{\partial t} + c \frac{\partial B_z}{\partial x} - e n_0 v_y = \varepsilon (e n v_y) \quad (39)$$

$$\frac{1}{c}\frac{\partial B_z}{\partial t} + \frac{\partial E_y}{\partial x} = 0 \quad (40)$$

where the subscript $_0$ denotes the unperturbed values, and the unsubscripted quantities refer to the perturbations characterised by the small parameter ε, and $\Omega = \frac{e B_0}{m c}$. We have assumed the polarisation of the extraordinary modes to be such that $E_z = B_y = v_z = 0$. Note that the left-hand sides in Eqs. (35)—(40) represent the linear problem and the right-hand side represent the nonlinear terms.

Let us consider linearised extraordinary modes of the form $e^{i k_T x}$ and introduce

$$a_T = v_y + \frac{i \omega_T}{e n_0}\left(\frac{\Omega^2}{D_T} + 1\right) E_y + \frac{i k_T c}{e n_0}\left(\frac{\Omega^2}{D_T} + 1\right) B_z + + \frac{a^2}{n_0}\frac{i k_T \Omega}{D_T} n + \frac{i \omega_T \Omega}{D_T} v_x - \frac{e}{m}\frac{\Omega}{D_T} E_x \quad (41)$$

where ω_T and k_T satisfy the linear dispersion relation,

$$(\omega^2 + k^2 c^2 - \omega^2)(\omega^2 + k^2 a^2 - \omega^2) + \Omega^2 (k^2 c^2 - \omega^2) = 0 \quad (42)$$

or

$$\omega_T^2 = \frac{1}{2}[k_T^2 (a^2 + c^2) + 2\omega_p^2 + \Omega^2] + \frac{1}{2}[\{k_T^2 (a^2 + c^2) + 2\omega_p^2 + \Omega^2\}^2 - - 4(\omega_p^2 + k_T^2 a^2)(\omega_p^2 + k_T^2 c^2) - 4 k_T^2 c^2 \Omega^2]^{1/2} \quad (43)$$

and

$$D_T \equiv \omega_p^2 + k_T^2 a^2 - \omega_T^2, \quad \omega_p^2 = \frac{4\pi e^2 n_0}{m}.$$

Using Eqs. (35)—(42), one obtains

$$\frac{\partial a_T}{\partial t} = \mathrm{i}\, \omega_T\, a_T \tag{44}$$

so that a_T is a normal mode of the linearised problem associated with Eqs. (35)—(40). Note that in the absence of the applied magnetic field (i. e., $\Omega \Rightarrow 0$), (41) represents transverse electromagnetic waves (hence the justification for the use of a subscript T in (41)).

Similarly, let us consider another class of linearised extraordinary modes of the form $e^{-ik_L x}$ and introduce

$$a_L = n + \frac{\omega_L n_0}{k_L a^2} v_x + \frac{\mathrm{i}\, e\, n_0}{m\, k_L a^2} E_x - \frac{\mathrm{i}\, n_0}{k_L a^2} \frac{D_L}{\Omega} v_y +$$

$$+ \frac{\omega_L}{k_L a^2 \Omega} (\Omega^2 + D_L) E_y + \frac{c}{e\, a^2 \Omega} (\Omega^2 + D_L) B_z \tag{45}$$

where ω_L and k_L satisfy the linear dispersion relation (42),

$$\omega_L^2 = \frac{1}{2} [k_L^2 (a^2 + c^2) + 2\omega_p^2 + \Omega^2] - \frac{1}{2} [\{k_L^2 (a^2 + c^2) + 2\omega_p^2 + \Omega^2\}^2 -$$

$$- 4 (\omega_p^2 + k_L^2 a^2)(\omega_p^2 + k_L^2 c^2) - 4 k_L^2 c^2 \Omega^2]^{1/2} \tag{46}$$

and

$$D_L \equiv \omega_p^2 + k_L^2 a^2 - \omega_L^2 .$$

Using Eqs. (35)—(40), (42) and (45), one obtains

$$\frac{\partial a_L}{\partial t} = \mathrm{i}\, \omega_L\, a_L \tag{47}$$

so that a_L is another normal mode of the linearised problem associated with Eqs. (35)—(40). Note that in the absence of the applied magnetic field (i.e. , Ω, $D_L \Rightarrow 0$), (45) represents Langmuir waves (hence the justification for the use of a subscript L in (45)).

Note that in the original method applied for the case without the applied magnetic field (Sjolund and Stenflo[6)]) a_T is a linear combination of v_y, E_y and B_z, and a_L is a linear combination of n, v_x and E_x. In contrast, for the present case with the applied magnetic field, this method has to be modified in that a_L and a_T are both linear combination of all the variables in question — v_y, E_y, B_z, n, v_x and E_x.

The two sets of extraordinary modes given by (41) and (45) are uncoupled in the linearised problem. The coupling between the two sets of extraordinary modes given by (41) and (45) becomes effective in the nonlinear problem. When the nonlinear terms are included, one obtains, from Eqs. (35)—(40)

$$\frac{\partial a_T}{\partial t} - \mathrm{i}\,\omega_T\, a_T = \left(-v_x \frac{\partial v_y}{\partial x}\right) + \frac{e}{mc}\, v_x\, B_z + \frac{\mathrm{i}\,\omega_T}{n_0}\left(\frac{\Omega^2}{D_T} + 1\right) n\, v_y -$$

$$-\frac{a^2}{n_0}\frac{\mathrm{i}\,k_T\,\Omega}{D_T}\frac{\partial}{\partial x}(n\,v_x) - \frac{\mathrm{i}\,\omega_T \Omega}{D_T}\left(v_x \frac{\partial v_x}{\partial x} + \frac{e}{mc}\, v_y\, B_z - \frac{a^2}{n_0} n \frac{\partial n}{\partial x}\right) -$$

$$-\frac{e^2}{m}\frac{\Omega}{D_T}(n\,v_x) \tag{48}$$

$$\frac{\partial a_L}{\partial t} - \mathrm{i}\,\omega_L\, a_L = -\frac{\partial}{\partial x}(n\,v_x) - \frac{\omega_L\, n_0}{k_L\, a^2}\left(v_x \frac{\partial v_x}{\partial x} + \frac{e}{mc}\, v_y\, B_z - \frac{a^2}{n_0} n \frac{\partial n}{\partial x}\right) +$$

$$+\frac{\mathrm{i}\,\omega_p^2}{k_L\, a^2}(n v_x) - \frac{\mathrm{i}\, n_0}{k_L\, a^2}\frac{D_L}{\Omega}\left(-v_x \frac{\partial v_y}{\partial x} + \frac{e}{mc} v_x\, B_z\right) + \frac{\omega_L}{k_L \Omega\, a^2}(\Omega^2 + D_L)\, n\, v_y. \tag{49}$$

Let us consider the extraordinary waves of the form $e^{-\mathrm{i}(k_{T_0} x - \omega_{T_0})}$ and $e^{-(k_{T_1} x - \omega_{T_1} t)}$ propagating in the $\vec{x}$-direction, with (ω_{T_0}, k_{T_0}) and (ω_{T_1}, k_{T_1}) related to each other according to (43). Due to nonlinear resonant interaction between these two extraordinary waves, let a third extraordinary wave of the form $e^{-\mathrm{i}(k_L x - \omega_L t)}$ propagating in the $\vec{x}$-direction, be excited such that

$$\omega_{T_1} - \omega_{T_1} = \omega_L, \; k_{T_0} - k_{T_1} = k_L \tag{50}$$

and (ω_L, k_L) are related to each other according to (16).

Now, one obtains for the linearised problem associated with Eqs. (35)—(40),

$$v_{yT} = a\, a_T$$

$$E_{yT} = \left(\frac{\mathrm{i}\,\omega_T\, e\, n_0}{k_T^2\, c^2 - \omega_T^2}\right) a\, a_T$$

$$B_{zT} = \left(\frac{\mathrm{i}\, e\, n_0\, k_T\, c}{k_T^2\, c^2 - \omega_T^2}\right) a\, a_T$$

$$n_T = \left(\frac{k_T\, n_0 \Omega}{\mathrm{i}\, D_T}\right) a a_T$$

$$v_{xT} = \left(\frac{\omega_T \Omega}{\mathrm{i}\, D_T}\right) \alpha\, a_T$$

$$E_{xT} = \left(-\frac{e\, n_0\, \Omega}{D_T}\right) \alpha\, a_T$$

$$a_T = a_T(t)\, \mathrm{e}^{\mathrm{i}(\omega_T t - k_T x)} + a_T^*(t)\, \mathrm{e}^{-\mathrm{i}(\omega_T t - k_T x)} \tag{51}$$

and

$$v_{xL} = \beta\, a_L$$

$$n_L = \left(\frac{k_L\, n_0}{\omega_L}\right) \beta\, a_L$$

$$E_{xL} = \left(-\frac{\mathrm{i}\, e\, n_0}{\omega_L}\right) \beta\, a_L$$

$$v_{yL} = \left(\frac{\mathrm{i}\, D_L}{\omega_L\, \Omega}\right) \beta\, a_L$$

$$E_{yL} = \left(\frac{D_L}{\Omega} \frac{e\, n_0}{\omega_L^2 - k_L^2 c^2}\right) \beta\, a_L$$

$$B_{zL} = \left(\frac{D_L}{\omega_L\, \Omega} \frac{e\, n_0\, k_L\, c}{\omega_L^2 - k_L^2 c^2}\right) \beta\, a_L$$

$$a_L = a_L(t)\, \mathrm{e}^{\mathrm{i}(\omega_L t - k_L x)} + a_L^*(t)\, \mathrm{e}^{-\mathrm{i}(\omega_L t - k_L x)} \tag{52}$$

where,

$$\alpha \equiv \left[1 + \frac{(\omega_T^2 + k_T^2 c^2)\left(\frac{\Omega^2}{D_T} + 1\right)}{\omega_T^2 - k_T^2 c^2} + \frac{\Omega^2}{D_T^2}(D_T + 2\omega_T^2)\right]^{-1}$$

$$\beta \equiv \left[\frac{(\omega_p^2 + \omega_L^2 + k_L^2 a^2)\, n_0}{k_L\, \omega_L\, a^2} + \frac{n_0\, D_L}{k_L\, \omega_L\, a^2\, \Omega}\left\{D_L + \frac{(\Omega^2 + D_L)(\omega_L^2 + k_L^2 c^2)}{\omega_L^2 - k_L^2 c^2}\right\}\right]^{-1}$$

and $a_T(t)$ and $a_L(t)$ are slowly-varying functions of time.

Using (51) and (52), Eqs. (48) and (49) give, upon keeping only the resonant terms on their right-hand sides

$$\frac{\partial a_{T_0}}{\partial t} - \mathrm{i}\, \omega_{T_0}\, a_{T_0} = \alpha_1\, \beta \left[\mathrm{i}\, k_{T_1} + \mathrm{i}\, k_L \frac{\omega_{T_1}}{\omega_L} \frac{D_L}{D_{T_1}} + \mathrm{i}\, k_{T_1} \frac{\omega_p^2}{k_{T_1}^2 c^2 - \omega_{T_1}^2} + \right.$$

$$+ \mathrm{i}\, k_L \frac{\omega_{T_1}}{\omega_L} \frac{D_L}{D_{T_1}} \frac{\omega_p^2}{k_L^2 c^2 - \omega_L^2} + \mathrm{i}\, \omega_{T_0} \left(\frac{\Omega^2}{D_{T_0}} + 1 \right) \left(\frac{k_L}{\omega_L} + \frac{k_{T_1}}{\omega_L} \frac{D_L}{D_{T_1}} \right) +$$

$$+ \frac{\mathrm{i}\, a^2 k_{T_0}^2 \Omega^2}{D_{T_0} D_{T_1}} \left(k_{T_1} + \frac{\omega_{T_1}}{\omega_L} k_L \right) + \frac{\mathrm{i}\, \omega_{T_0} \Omega^2}{D_{T_0}} \left\{ k_{T_0} \frac{\omega_{T_1}}{D_{T_1}} + k_L \frac{\omega_p^2}{\Omega^2} \frac{D_L}{\omega_L (k_L^2 c^2 - \omega_L^2)} + \right.$$

$$\left. + k_{T_1} \frac{\omega_p^2}{\Omega^2} \frac{D_L}{\omega_L (k_{T_1}^2 c^2 - \omega_{T_1}^2)} - k_{T_0} k_{T_1} k_L \frac{a^2 n_0}{\omega_L D_{T_1}} \right\} +$$

$$+ \mathrm{i}\, \omega_p^2 \frac{\Omega^2}{D_{T_0}} \left(\frac{k_{T_1}}{D_{T_1}} + k_L \frac{\omega_{T_1}}{\omega_L D_{T_1}} \right) \Bigg] a_{T_1} a_L^* \tag{53a}$$

$$\frac{\partial a_{T_1}}{\partial t} - \mathrm{i}\, \omega_{T_1} a_{T_1} = a_0 \beta \Bigg[\mathrm{i}\, k_{T_0} + \mathrm{i}\, k_L \frac{\omega_{T_0}}{\omega_L} \frac{D_L}{D_{T_0}} + \mathrm{i}\, k_{T_0} \frac{\omega_p^2}{k_{T_0}^2 c^2 - \omega_{T_0}^2} +$$

$$+ \mathrm{i}\, k_L \frac{\omega_{T_0}}{\omega_L} \frac{D_L}{D_{T_0}} \frac{\omega_p^2}{k_L^2 c^2 - \omega_L^2} + \mathrm{i}\, \omega_{T_1} \left(\frac{\Omega^2}{D_{T_1}} + 1 \right) \left(\frac{k_L}{\omega_L} - \frac{k_{T_0}}{\omega_L} \frac{D_L}{D_{T_0}} \right) +$$

$$+ \frac{\mathrm{i}\, a^2 k_{T_1}^2 \Omega^2}{D_{T_0} D_{T_1}} \left(k_{T_0} + \frac{\omega_{T_0}}{\omega_L} k_L \right) + \frac{\mathrm{i}\, \omega_{T_1} \Omega^2}{D_{T_1}} \left\{ k_{T_1} \frac{\omega_{T_0}}{D_{T_0}} + k_L \frac{\omega_p^2}{\Omega^2} \frac{D_L}{\omega_L (k_L^2 c^2 - \omega_L^2)} - \right.$$

$$\left. - k_{T_0} \frac{\omega_p^2}{\Omega^2} \frac{D_L}{\omega_L (k_{T_0}^2 c^2 - \omega_{T_0}^2)} + k_{T_0} k_{T_1} k_L \frac{a^2 n_0}{\omega_L D_{T_0}} \right\} +$$

$$+ \mathrm{i}\, \omega_p^2 \frac{\Omega^2}{D_{T_1}} \left(\frac{k_{T_0}}{D_{T_0}} + k_L \frac{\omega_{T_0}}{\omega_L D_{T_0}} \right) \Bigg] a_{T_1} a_L^* \tag{53b}$$

$$\frac{\partial a_L}{\partial t} - \mathrm{i}\, \omega_L a_L = a_0 a_1 \Bigg[\frac{\mathrm{i}\, n_0 \Omega^2}{k_L a^2 D_{T_0} D_{T_1}} (\omega_p^2 + k_L^2 a^2)(k_{T_0} \omega_{T_1} + k_{T_1} \omega_{T_0}) -$$

$$- \frac{\mathrm{i}\, \omega_L n_0}{k_L a^2} \left\{ k_{T_0} \frac{\omega_p^2}{k_{T_0}^2 c^2 - \omega_{T_0}^2} - k_{T_1} \frac{\omega_p^2}{k_{T_1}^2 c^2 - \omega_{T_1}^2} + \right.$$

$$\left. + \frac{k_L \Omega^2}{D_{T_0} D_{T_1}} (- \omega_{T_0} \omega_{T_1} + k_{T_0} k_{T_1} n_0 a^2) \right\} -$$

$$- \frac{\mathrm{i}\, n_0 D_L}{k_L a^2 D_{T_0} D_{T_1}} \left(- k_{T_1} \omega_{T_0} + k_{T_0} \omega_{T_1} - k_{T_1} \omega_{T_0} \frac{D_{T_1}}{D_{T_0}} \frac{\omega_p^2}{k_{T_1}^2 c^2 - \omega_{T_1}^2} - \right.$$

$$\left. - k_{T_0} \omega_{T_1} \frac{D_{T_0}}{D_{T_1}} \frac{\omega_p^2}{k_{T_0}^2 c^2 - \omega_{T_0}^2} \right) +$$

$$+ \frac{\mathrm{i}\, \omega_L n_0}{k_L a^2} (\Omega^2 + D_L) \left(\frac{k_{T_1}}{D_{T_1}} - \frac{k_{T_0}}{D_{T_0}} \right) \Bigg] a_{T_0} a_{T_1}^* \tag{53c}$$

where,

$$D_{T_0} \equiv D(\omega_{T_0}, k_{T_0}), \quad D_{T_1} \equiv D(\omega_{T_1}, k_{T_1})$$

$$\alpha_0 \equiv \alpha(\omega_{T_0}, k_{T_0}), \quad \alpha_1 \equiv \alpha(\omega_{T_1}, k_{T_1}).$$

In terms of a physical quantity like n_T and n_L (as given in (51) and (52)), Eqs. (53) can be written in the following form

$$\frac{\partial n_{T_0}}{\partial \tilde{t}} \equiv \frac{\partial n_{T_0}}{\partial t} - \mathrm{i}\,\omega_{T_0} n_{T_0} = -\,p\, n_{T_1} n_L \tag{54a}$$

$$\frac{\partial n_{T_1}}{\partial \tilde{t}} \equiv \frac{\partial n_{T_1}}{\partial t} - \mathrm{i}\,\omega_{T_1} n_{T_1} = q\, n_{T_0} n_L^* \tag{54b}$$

$$\frac{\partial n_L}{\partial \tilde{t}} \equiv \frac{\partial n_L}{\partial t} - \mathrm{i}\,\omega_L n_L = r\, n_{T_0} n_{T_1}^* \tag{54c}$$

where p, q and r are constants which are determined from the right-hand side in Eqs. (53).

One then obtains for the slow variations in time

$$\frac{\partial^2 n_{T_0}}{\partial \tilde{t}^2} = -\,p\,q\, n_{T_0} |n_L|^2 - p\,r\, n_{T_0} |n_{T_1}|^2 \tag{55a}$$

$$\frac{\partial^2 n_{T_1}}{\partial \tilde{t}^2} = -\,p\,q\, n_{T_1} |n_L|^2 + q\,r\, n_{T_1} |n_{T_0}|^2 \tag{55b}$$

$$\frac{\partial^2 n_L}{\partial \tilde{t}^2} = -\,p\,r\, n_L |n_{T_1}|^2 + q\,r\, n_L |n_{T_0}|^2. \tag{55c}$$

If one assumes that

$$|n_{T_0}| \gg |n_{T_1}|,\ |n_L|$$

then n_{T_0} can be taken to be constant. If further

$$n_{T_1}, n_L \sim \mathrm{e}^{\sigma t} \tag{56}$$

then Eqs. (55) give for the growth rate

$$\sigma = \sqrt{q\,r}\,|n_{T_0}|.$$

Note that in the absence of the magnetic field (i. e., Ω, $D_L \Rightarrow 0$), Eqs. (53) reduce to

$$\frac{\partial a_{T_0}}{\partial t} - \mathrm{i}\,\omega_{T_0} a_{T_0} = \frac{\mathrm{i}\,k_L^2 a^2 \omega_p^2 \omega_{T_0}}{4\omega_L^2 \omega_{T_1}^2 n_0} a_{T_1} a_L$$

$$\frac{\partial a_{T_1}}{\partial t} - \mathrm{i}\,\omega_{T_1} a_{T_1} = \frac{\mathrm{i}\,k_L^2 a^2 \omega_p^2 \omega_{T_1}}{4\omega_L^2 \omega_{T_0}^2 n_0} a_{T_0} a_L^*$$

$$\frac{\partial a_L}{\partial t} - \mathrm{i}\,\omega_L a_L = \frac{\mathrm{i}\,n_0 \omega_L \omega_p^4}{4\omega_{T_0}^2 \omega_{T_1}^2} a_{T_0} a_{T_1}^* \tag{57}$$

where,

$$\omega_T^2 = \omega_p^2 + k_T^2 c^2, \quad \omega_L^2 = \omega_p^2 + k_L^2 a^2.$$

Eqs. (57) are the same as those deduced by Sjolund and Stenflo[4].

Now, in order to compare the present results with those deduced by Das[8] using the Krylov-Bogoliubov-Mitropolski method, first note that the latter's amplitude variables a_1, a_2, a_3 are related to the present amplitude variables a_{T_0}, a_L, a_{T_1} changed to a_{T_1}, a_L, a_{T_3} as follows:

$$a_{T_{1,3}} = \mathrm{i}\,\frac{\omega_{T_{1,3}}^2 - k_{T_{1,3}}^2 c^2}{e\, n_0\, \omega_{T_{1,3}}} \left[1 + \frac{\Omega^2}{D_{T_{1,3}}^2}(k_{T_{1,3}}^2 a^2 + \omega_{T_{1,3}}^2 + \omega_p^2) + \right.$$

$$\left. + \left(1 + \frac{\Omega^2}{D_{T_{1,3}}}\right)\left(\frac{\omega_{T_{1,3}}^2 + k_{T_{1,3}}^2 c^2}{\omega_{T_{1,3}}^2 - k_{T_{1,3}}^2 c^2}\right)\right] a_{1,3} \equiv \gamma_{1,3}\, a_{1,3}$$

$$a_L = \mathrm{i}\,\frac{k_L}{e}\left[1 + \frac{1}{k_L^2 a^2}(\omega_L^2 + \omega_p^2 + D_L^2/\Omega^2) + \right.$$

$$\left. + \frac{D_L/\Omega^2}{\omega_L^2 - k_L^2 c^2}(\Omega^2 + D_L)\left[\frac{\omega_L^2}{k_L^2 a^2} + \frac{c^2}{a^2}\right)\right] \equiv \gamma_2\, a_2. \tag{58}$$

If one writes Eq. (53c) as

$$\frac{\partial a_L}{\partial t} - \mathrm{i}\,\omega_L a_L = \mu\, a_{T_1} a_{T_3}^* \tag{59}$$

then, using (54), this becomes

$$\frac{\partial a_2}{\partial t} - \mathrm{i}\,\omega_L a_2 = \frac{\gamma_1 \gamma_3^*}{\gamma_2} \mu\, a_1 a_3^*. \tag{60}$$

This does not seem to agree with the one given by Das[8].

4. Discussion

In this paper, first we considered the nonlinear resonant interactions between two circularly-polarised waves and a Langmuir wave in a cold plasma subjected to a magnetic field. The method of multiple scales was used. We found that if two circularly-polarised waves with opposite polarisations and large amplitudes interact with one another, then a Langmuir wave can be excited. We also showed that it is possible for a large-amplitude Langmuir wave to decay into two circularly-polarised waves with the same polarisation in a magnetised plasma. Next, we treated the nonlinear resonant interactions between three extraordinary waves in a warm plasma subjected to a magnetic field. The method of coupled modes was modified suitably to be able to treat this problem. Using these results, one may study different wave-coupling mechanisms. For instance, when one of the extraordinary waves (the pump wave) has such a large amplitude as not to be affected by the presence of the other waves in the plasma, one can excite a signal extraordinary wave and an idler extraordinary wave.

Acknowledgments

The author is thankful to the referee for his constructive criticism.

References

1) R. C. Davidson, *Methods in Nonlinear Plasma Theory*, Academic Press, New York, 1972;

2) D. Montgomery, Physica **31** (1965) 693;

3) R. A. Stern and N. Tzoar, Phys. Rev. Lett. **16** (1966) 785;

4) A. Sjolund and L. Stenflo, Physics **35** (1967) 499;

5) J. A. Stamper, K. Papadopoulos, R. N. Sudan, S. O. Dean, E. A. McLean and J. M. Lawson, Phys. Rev. Lett. **26** (1971) 1012;

6) A. Sjolund and L. Stenflo, Z. Physik **204** (1967) 211;

7) K. P. Das, Phys. Fluids **14** (1971) 124;

8) L. Stenflo, J. Plasma Phys. **5** (1971) 413;

9) L. Stenflo, J. Plasma Phys. **7** (1972) 107;

10) S. Krishan, D. Ramkin and A. A. Selim, J. Plasma Phys. **9** (1973) 23;

11) P. Munoz and S. Dagach, J. Plasma Phys. **17** (1977) 51;

12) K. P. Das and D. Banerjee, J. Plasma Phys. **16** (1976) 95;

13) A. A. Selim, J. Plasma Phys. **19** (1978) 405;

14) A. H. Nayfer, *Perturbation Methods*, Wiley-Interscience, New York, 1973.

15) S. Krishan and J. Fukai, Phys. Fluids **14** (1971) 1158.

NELINEARNA INTERAKCIJA TRIJU VALOVA U MAGNETIZIRANOJ PLAZMI

BHIMSEN K. SHIVAMOGGI

Physical Research Laboratory, Ahmedabad 380 009, *India*

UDK 533.95

Originalni znanstveni rad

Razmatrana je nelinearna interakcija triju valova u magnetiziranoj plazmi. Metodom višestrukih vremenskih skala ispitana je nelinearna interakcija Langmuirovog, longitudinalnog vala s dva paralelna, suprotno polarizirana elektromagnetska vala koji se prostiru uzduž vanjskog magnetskog polja. Razmatrana je također nelinearna interakcija triju neredovna vala u toploj plazmi koji se prostiru normalno na smjer magnetskog polja.