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THE EFFECT OF COLLISIONS IN THE ENERGY TRANSFER FROM THE ELECTRONS TO THE IONS OF A PLASMA WHEN TWO MONOCHROMATIC ELECTROMAGNETIC WAVES MIX IN IT

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Two monochromatic electromagnetic waves of identical frequencies propagate in a spatially uniform plasma. The two waves enforce the development of an asymmetrical electron distribution function and an increase in the kinetic energy of the electrons. By considering a Fokker-Planck collision term the energy transferred to the ions from the plasma electrons is calculated. As a function of time, the energy expression contains a linear secular term and a sinusoidal term. It is also found that light absorption through collisions is more efficient than expected for a plasma in which only inverse bremsstrahlung takes place.

1. Introduction

In a previous investigation[1] we calculated the power and energy absorbed by a spatially uniform plasma when two monochromatic *em* waves optically mix in it.

We found that the power absorbed by the plasma varies sinusoidally with time.

In that investigation the effect of Coulomb collisions in the process of the energy absorption by the plasma has been neglected by assuming the plasma to be collisionless.

However, collisions are effective over time scales much longer than the electron trapping time τ. The two mixed *em* waves impart energy to the electrons of the plasma enforcing the development of an asymmetrical distribution function. Electron-ion collisions then constitute a dissipation mechanism for the power thus gained by the electrons.

In this paper an attempt is made to estimate the power and energy absorbed by the plasma by including a Fokker-Planck collision term in our previous theory.

2. *Energy absorption by the plasma through Coulomb collisions*

It is assumed that the plasma is initially thermal with a Maxwellian ion distribution function and that a simple Fokker-Planck collision term can adequately describe the situation. Since the electron-electron collisions are not negligible, only a lower limit of the effect can be estimated by the following treatment.

The collision term may be written as follows[2)]

$$\left(\frac{\delta f}{\delta t}\right)_{coll} = -\Gamma \frac{\partial}{\partial \vec{v}} \cdot \left(f \frac{\partial H}{\partial \vec{v}}\right) + \frac{1}{2} \Gamma \frac{\partial^2}{\partial \vec{v} \partial \vec{v}} : \left(f \frac{\partial^2 G}{\partial \vec{v} \partial \vec{v}}\right) \tag{1}$$

where

$$H = n_0 (1 + m/M) \operatorname{erf}(v/a_I)/v, \quad a_I = (2k\, T/M)^{1/2},$$

$$G = \pi^{-1/2} n_0 a_I \exp(-v^2/a_I^2) + \frac{1}{2} n_0 (2v + a_I^2/v) \operatorname{erf}(v/a_I),$$

and

$$\Gamma = 4\pi m^{-2} e^4 \ln \Lambda,$$

Λ being the plasma parameter.

In these expressions n_0 is the plasma number density, k the Boltzmann's constant, T the plasma temperature and m, M the electron and ion mass, respectively. The average power absorbed by the plasma ions over a wavelength $\lambda_0 = 2\pi/k_0$ is then calculated by the relation:

$$P_c(t) = \frac{1}{4} \pi^{-1} n_0 m k_0 \int_{-\pi/k_0}^{\pi/k_0} \mathrm{d}x \int_{-\infty}^{\infty} v^2 \left(\frac{\delta f}{\delta t}\right)_{coll} \mathrm{d}\vec{v} = \pi^{-3/2} n_0^2 m k_0 \Gamma (1 + m/M) \int_{-\pi/k_0}^{\pi/k_0} \mathrm{d}x \left(\left[\int_{-\infty}^{\infty} a_I^{-1} \exp(-v^2/a_I^2) f \mathrm{d}\vec{v} - \frac{1}{2} \pi^{1/2} m (m + M)^{-1} \int_{-\infty}^{\infty} v^{-1} \operatorname{erf}(v/a_I) f \mathrm{d}\vec{v} \right] \right). \tag{2}$$

For electron distribution functions described by

$$f = \pi^{-1} a_e^{-2} \exp(- v_\perp / a_e^2) \hat{f}(x, u, t), \tag{3}$$

where $u_\perp^2 = u_y^2 + u_z^2$ and for $\hat{f}(x, u, t)$ described by

$$\frac{\partial f}{\partial t}(x, u, t) + u \frac{\partial f}{\partial x}(x, u, t) - \frac{e}{m}[E(x, t) + E_0 \sin k_0 x] \frac{\partial f}{\partial u}(x, u, t) = 0 \tag{4}$$

$$\frac{\partial E}{\partial x}(x, t) = 4\pi n_0 e [1 - \int_{-\infty}^{\infty} f(x, u, t)\, \mathrm{d}u], \tag{5}$$

the expression (2) takes the form

$$P_c(t) = 2\pi^{-2} n_0 a_e^{-1} m \Gamma \int_{-\pi/2}^{\pi/2} \Bigg(\int_{TR} \{[a_l a_e^{-1} \exp(- u^2/a_l^2) - \frac{1}{2} \pi m M^{-1} \exp(u^2/a_e^2)$$
$$(1 - \mathrm{erf}(u/a_e))] \exp(-u_0^2/a_e^2)\} \,\mathrm{d}u + \int_{UNTR} \{[a_l a_e^{-1} \exp(-u^2/a_l^2) -$$
$$- \frac{1}{2} \pi m M^{-1} \exp(u^2/a_e^2)(1 - \mathrm{erf}(u/a_e))] \exp(- u_0^2/a_e^2)\} \,\mathrm{d}u] \Bigg) \mathrm{d}\xi. \tag{6}$$

Indices TR and $UNTR$ refer to trapped and untrapped particles, respectively, u is the electron velocity along the x-axis, and $\xi = k_0 x/2$.

The evaluation of Eq. (6) is very difficult as it stands. By considering that $f(x, u, t)$ differs significantly from a Maxwellian distribution function only in the resonant region i. e. $u \simeq 0$, Eq. (4) can however, be approximated by

$$P_c(t) = - 2\pi^{-1} a_e^{-4} n_0^2 m^2 M^{-1} \Gamma \int_{-\pi/2}^{\pi/2} \Bigg(\int_{TR} (u^2 - u_0^2)\, \mathrm{d}u + \int_{UNTR} (u^2 - u_0^2) \mathrm{d}u \Bigg) \mathrm{d}\xi. \tag{7}$$

The simplification arises by assuming that $a_l \ll a$ and that

$$\lim_{a_l \to 0} \frac{1}{2\sqrt{\pi}} a_l^{-1} \exp(- u^2/a_l^2) = \delta(2u). \tag{8}$$

We define here the quantity

$$\varkappa^2 = 2e E_0/(k_0 W + e E_0) \quad \text{and} \quad \varkappa_0^2 = 2e E_0/(k_0 W_0 + e E_0) \tag{9}$$

W being the total energy of a particle, E_0 the amplitude of the excited electrostatic wave, and $W_0 = 1/2\, m a^2$.

For untrapped particles ($\varkappa < 1$) we have,

$$\sin(k_0 x/2) = \sin\xi = \operatorname{sn} F(\xi, \varkappa); \tag{10}$$

$$u = \frac{2}{k_0 \varkappa \tau} \operatorname{dn} F(\xi, \varkappa) \tag{11}$$

$$F(\xi, \varkappa) = \int_0^{\xi} (1 - \varkappa^2 \sin^2 \Theta)^{-1/2} \, d\Theta, \tag{12}$$

whereas for trapped particles ($\varkappa > 1$)

$$\sin\xi = \sin\zeta; \tag{13}$$

$$\mu = \frac{1}{\varkappa} < 1 \tag{14}$$

$$u = \frac{2\mu}{k_0 \tau} \operatorname{cn} F(\zeta, \mu). \tag{15}$$

To perform the integration of Eq. (7), we change variable of integration from ξ, u to μ, F and $\varkappa$, F for trapped and untrapped particles, correspondingly. Considering that only even functions of F contribute to the integrals we obtain:

$$P_c(t) = -64\pi^{-1} n_0^2 m^2 M^{-1} a_e^{-4} k_0^{-3} \tau^{-3} \Gamma \Big(\int_0^1 \mu^3 \, d\mu \int_0^{K(\mu)} \operatorname{cn}^2 F \, dF +$$

$$+ \int_{\varkappa_0}^1 \varkappa^{-3} \, d\varkappa \int_0^{K(\varkappa)} \operatorname{dn}^2 F \, dF - \int_0^1 \mu^3 \, d\mu \int_0^{K(\mu)} (\operatorname{cn}^2 F \operatorname{cn}^2 z + \operatorname{sn}^2 F \operatorname{dn}^2 F \operatorname{sn}^2 z \operatorname{dn}_2 z)(1 -$$

$$- \mu^2 \operatorname{sn}^2 F \operatorname{sn}^2 z)^{-2} \, dF - \int_{\varkappa_0}^1 \varkappa^{-3} \, d\varkappa \int_0^{K(\varkappa)} (\operatorname{dn}^2 F \operatorname{dn}^2 z +$$

$$+ \varkappa^4 \operatorname{sn}^2 F \operatorname{cn}^2 F \operatorname{sn}^2 z \operatorname{cn}^2 z)(1 - \varkappa^2 \operatorname{sn}^2 F \operatorname{sn}^2 z)^{-2} \, dF \Big). \tag{16}$$

Let C be the contour around the paralellogram whose corners are $-2K$, $2K$, $2\mathrm{i}\,K'$, $2\mathrm{i}\,K' - 2K$ in the F-plane. After expanding the integrand in the last two integrals of (14) in a Fourier series in z the integration can be carried out to give[1)]

$$P_c(t) = -\frac{32 n_0^2 m^2 \Gamma}{\pi a_e^4 k_0^3 \tau^3 M} \Big(\frac{\pi}{4\varkappa_0^2} + \frac{\pi}{8} \ln \varkappa_0 + \frac{8 - \pi}{36} +$$

$$+ 2\pi^2 \sum_{n=1}^{\infty} \Big\{ \int_0^1 \frac{\mu (q^{-2n} + (q^{-n} + q^{-3n}) 3/2)}{K(\mu)(1 - q^{-2n})^2}$$

$$\cos[n \pi t/\tau K(\mu)] \, d\mu + \int_{\varkappa_0}^1 \frac{q^{-2n} + (q^{-n} + q^{-3n}) 3/2}{\varkappa^3 K(\varkappa)(1 - q^{-2n})^2} \cos[n \pi t/\varkappa \tau K(\varkappa)] \, d\varkappa \Big\} \Big). \tag{17}$$

To obtain an expression for the energy transferred to the ions through their collisions with the plasma electrons we integrate Eq. (17) with respect to time and get

$$U_{coll}(t) = -\frac{32 n_0^2 m^2 \Gamma}{\pi a_e^4 k_0^3 \tau^3 M}\Bigg(\left(\frac{\pi}{4\varkappa_0} + \frac{\pi}{8}\ln\varkappa_0 + \frac{8-\pi}{36}\right)t +$$

$$+ \sum_{n=1}^{\infty}\frac{2\pi}{n}\Bigg\{\int_0^1 \frac{\tau\,\mu\,(q^{-2n} + (q^{-n} + q^{-3n})\,3/2)}{(1-q^{-2n})^2}$$

$$\sin\left[n\pi t/\tau\,K(\mu)\right]\mathrm{d}\mu + \int_{\varkappa_0}^1 \frac{\tau\,(q^{-2n} + (q^{-n} + q^{-3n})\,3/2)}{\varkappa^2(1-q^{-2n})^2}\sin\left[n\pi t/\varkappa\tau\,K(\varkappa)\right]\mathrm{d}\varkappa\Bigg\}\Bigg). \qquad (18)$$

The equation (18) states that for t smaller than a few trapping times τ the expression for the energy $U_{coll}(t)$ contains one secular term increasing linearly with time and another two terms varying sinusoidally with time. For $t \ll \tau$ the last two terms phase mix to zero and Eq. (18) takes the form

$$U_{coll}(t) \simeq -\frac{32 n_0^2 m^2 \Gamma}{\pi a_e^4 k_0^3 \tau^3 M}\left(\frac{\pi}{4\varkappa_0^2} + \frac{\pi}{8}\ln\varkappa_0 + \frac{8-\pi}{36}\right)t. \qquad (19)$$

From this expression it seems that $U_{coll}(t)$ increases without limit with time. To find an upper limit for t below which Eq. (19) is valid, we must compare U_{coll} to the energy absorbed by the electrons from the two mixed laser beams[1]. To this end, we form the ratio of the two expressions

$$\frac{|U_{coll}|}{\Delta U(\infty)} = \frac{8 e^2 k_0 \ln\Lambda}{M a_e^2}\cdot(\tau_e/k_0^2\lambda_D^2)\left\{\frac{1}{2\beta} + \frac{1}{4}\ln(2\beta) + \frac{8-\pi}{9\pi}\right\}/\beta F(\beta), \qquad (20)$$

where

$$\tau_e = t/\tau,\ \beta^2 = \frac{e E_0}{k k_0 T},\ F(\beta) = k_0^2\lambda_D^2 E(\infty)/E_0$$

and λ_D is the Debye shielding length.

For $\beta = 0.1$, $F(\beta)$ is approximately 2.3.

Also for a wide range of plasma conditions, $\ln\Lambda$ is of the order of 10. Choosing $k_0^2\lambda_D^2 \simeq 4$ and $k_0 \simeq 10\ \mathrm{cm}^{-1}$ the ratio (20) becomes

$$\frac{U_{coll}}{\Delta U(\infty)} \simeq 10^{-4}\,\tau_e. \qquad (21)$$

It is essential for the validity of the presented investigation, that the Coulomb collisions do not disturb the development of the electron distribution function under the action of the two electromagnetic waves.

A criterion testing whether this is satisfied or not would be that the energy of the electrons given away to the ions of the plasma through collisions should be much smaller than the energy absorbed by the plasma electrons from the two mixing waves i. e.

$$\frac{U_{coll}}{\Delta U(\infty)} \simeq 10^{-4}\, \tau_e \ll 1 \qquad t \ll 10^4\, \tau .$$

Therefore, for time t greater than 3τ and smaller than $10^4\,\tau$, $U_{coll}(t)$ remains much smaller than $\Delta U(\infty)$ and its value is given by Eq. (19), meanwhile for t smaller than 3τ the energy $U_{coll}(t)$ is expressed by the equation (18).

As the plasma electrons transfer part of energy they gain from the mixing light waves to the ions, their distribution function is modified and therefore the temperature is changed. On account of this fact the energy $U_{coll}(t)$ is temperature dependent through a *collisional absorption coefficient* defined by

$$U_{coll} = -A_{coll}\, t. \tag{22}$$

We consider here the case in which a CO_2 laser of power 200 MW is focused down to a diameter of 200 μ in a plasma of number density $n_0 = 10^{14}$ cm^{-3}. For this case we find that the electric field intensity of the laser is $E_{10} = 2.52 \times \times 10^7$ V/cm and that the amplitude of the excited electrostatic wave is $E_0 = = 1.5 \times 10^5$ V/cm. We also have that $k_0 = 1.2 \times 10^4$ cm^{-1}. For these numerical values and for $\varkappa_0^2 = \frac{2e\,E_0}{k\,k_0} \cdot \frac{1}{T}$ the temperature dependence of A_{coll} is described by the relation

$$A_{coll} = 8.8\,(8.85 + 1.5 \ln X)\,(0.677/X - 1.22\, X^{-2} \ln X + 3.25/X)\ \mathrm{J/cm^3\, s}. \tag{23}$$

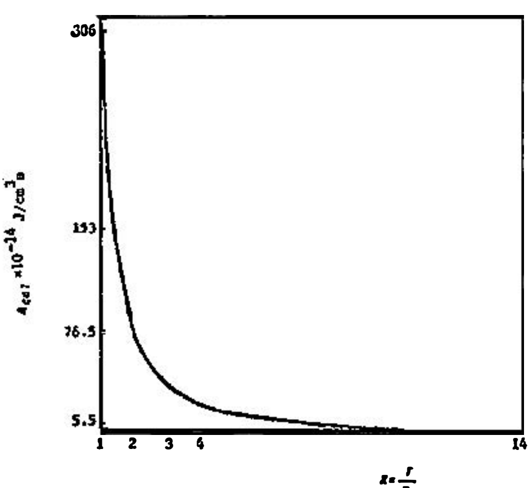

Fig. 1. The collisional absorption coefficient A_{coll} is plotted versus $X = T/T_0$.

Here $X = T/T_0$ and T_0 is the plasma temperature at time $t = 0$. Next we plot the collisional absorption coefficient A_{coll} as a function of X and the result is shown in Fig. 1. It is seen that A_{coll} is decreasing with temperature. This is expected because for small T more electrons get trapped and gain energy from the wave field. In this way they become faster and therefore more effective in their collisions with the ions in transferring part of their energy to them.

In addition to the described absorption process the inverse bremsstrahlung will contribute to the light energy absorption by the plasma.

The absorption coefficient for this mechanism can be written as[3]

$$a = 0.7\, n_0^2 g\,(T, \omega)/\omega^2\, T^{3/2}\ \mathrm{cm}^{-1}$$

where $g\,(\tau, \omega)$ is the Gaunt factor.

Assuming that both mixing waves are of the same intensity and polarized in the same direction, the rate of energy increase due to inverse bremsstrahlung is given by the realtion

$$A_{ib} = 1.75\ 10^{-30}\, n_0\, E_0\, e^2\, g\,(T, \omega)/k_0\,(k\,T)^{3/2}\ \mathrm{J/cm^3\ s}. \qquad (24)$$

For the same situation as in the case of collisional absorption, Eq. (24) takes the form:

$$A_{ib} = 10^{-1}\,(20 - 1.3 \ln X)/X^{3/2}\ \mathrm{J/cm^3\ s}. \qquad (25)$$

For comparison purposes only A_{ib} has been numerically computed for various values of X and the results are plotted and shown in Fig. 2. By comparing the two figures 1 and 2, one finds that beam mixing greatly dominates the inverse bremsstrahlung.

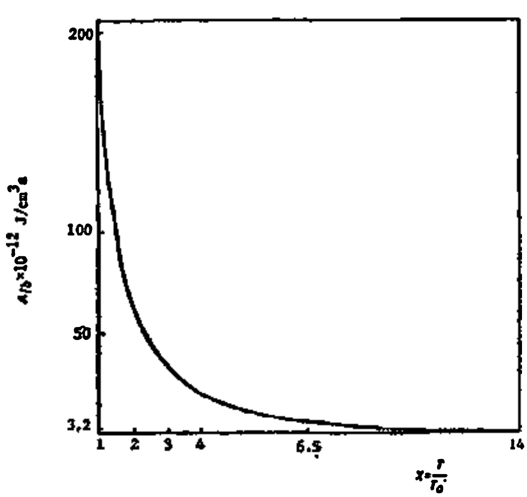

Fig. 2. The inverse bremsstrahlung absorption A_{ib} is plotted versus $X = T/T_0$.

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UTJECAJ SUDARA NA PRIJENOS ENERGIJE ELEKTRON-ION UZROKOVAN MIJEŠANJEM DVAJU MONOKROMATSKIH ELEKTROMAGNETSKIH VALOVA U PLAZMI

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Razmatran je proces prijenosa energije ionima preko kolizija s elektronima koji su rezonantno ubrzani istovremenim djelovanjem dvaju elektromagnetskih valova u homogenoj plazmi. Koristeći Fokker-Planckov kolizioni član izračunata je efikasnost ovakvog prijenosa energije kao i koeficijent apsorpcije zračenja.