YU ISSN 0015—3206
FZKAAA 16 (2) 219 (1984)

LETTER TO THE EDITOR

TRANSVERSE WAVES IN A RELATIVISTIC PLASMA

BHIMSEN K. SHIVAMOGGI

Physical Research Laboratory, Ahmedabad 380 009, *India*

Received 16 May 1983

Revised manuscript received 1 September 1983

UDC 533.95

Original scientific paper

Transverse oscillations in a relativistic plasma are studied. An appropriate evaluation of the dispersion relation for the case of a weakly-relativistic plasma is made for general isotropic distribution functions.

Silin[1], Imre[2] and Buti[3] considered transverse waves in a relativistic plasma. Their treatments are restricted to ultrarelativistic cases and Maxwellian distribution for the plasma particles. Winkles and Eldridge[4], Misra[5] and Mikhailovskii[6] considered the weakly-relativistic cases but restricted the calculation to Maxwellian distribution for the plasma particles. The purpose of this letter is to treat weakly-relativistic cases and consider an isotropic distribution (that includes the Maxwellian distribution as a special case) for the plasma particles.

The motions in a relativistic plasma are governed by (in usual notations) the following equations (Montgomery and Tidman[7] Ch. 10):

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} + e\left(\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}\right) \cdot \frac{\partial f}{\partial \vec{p}} = 0 \tag{1}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{2}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\sum e\, n_0 \int \vec{v} f \,\mathrm{d}\vec{p} \tag{3}$$

where

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}, \quad \vec{v} = \frac{c\vec{p}}{\sqrt{p^2 + m^2c^2}}$$

and the summation sign refers to the ions and electrons.

One has for the perturbations (denoted by a subscript 1) about a field-free equilibrium state (denoted by a subscript $_0$)

$$\frac{\partial f_1}{\partial t} + \vec{v} \cdot \frac{\partial E_1}{\partial \vec{z}} + e\left(\vec{E}_1 + \frac{1}{c}\vec{v} \times \vec{B}_1\right) \cdot \frac{\partial f_0}{\partial \vec{p}} = 0 \tag{4}$$

$$\nabla \times \vec{E}_1 = -\frac{1}{c}\frac{\partial \vec{B}_1}{\partial t} \tag{5}$$

$$\nabla \times \vec{B}_1 = \frac{1}{c}\frac{\partial \vec{E}_1}{\partial t} + 4\pi \sum e n_0 \int \vec{v} f_1 \,\mathrm{d}\vec{p}. \tag{6}$$

Fourier transforming according to

$$\mathscr{V}_1(\vec{k}, \omega) = \int_{-\infty}^{\infty} \mathrm{d}t \int \mathrm{d}\vec{x}\, \mathrm{e}^{-\mathrm{i}(\vec{k}\cdot\vec{x} - \omega t)}\, \mathscr{V}_1(\vec{x}, t) \tag{7}$$

one obtains from Eqs. (4)—(6) (Buti[3]):

$$(k^2 c^2 - \omega^2)\vec{E}_1 - c^2 \vec{k}(\vec{k} \cdot \vec{E}_1) + $$

$$+ \sum \frac{\omega_p^2 \omega m}{n_0} \int \frac{\mathrm{d}\vec{p}\,\vec{p}}{(\vec{k} \cdot \vec{p} - m\omega\sqrt{m^2 c^2 + p^2})} \left[\frac{\partial f_0}{\partial \vec{p}} + \right.$$

$$\left. + \frac{\vec{k} \times \left(\vec{p} \times \dfrac{\partial f_0}{\partial \vec{p}}\right)}{m\omega\sqrt{m^2 c^2 + p^2}}\right] \cdot E_1 = 0. \tag{8}$$

One obtains from (8) for transverse oscillations (Winkles and Eldridge[4]):

$$\omega^2 = k^2 c^2 - \frac{m c \omega_p^2}{n_0} \int_0^{\infty} \mathrm{d}p_\perp \int_{-\infty}^{\infty} \mathrm{d}p_\parallel \cdot 2\pi p_\perp \cdot \frac{1}{2} p_\perp \cdot$$

$$\frac{\left[\dfrac{\partial f_0}{\partial p_\perp} - \dfrac{k c}{\omega\sqrt{m^2 c^2 + p^2}}\left(p_\parallel \dfrac{\partial f_0}{\partial p_\perp} - p_\perp \dfrac{\partial f_0}{\partial p_\parallel}\right)\right]}{\left(\sqrt{m^2 c^2 + p^2} - \dfrac{kc}{\omega} p_\parallel\right)} \tag{9}$$

where we have ignored the ion response and the subscripts $_\perp$ and $_\parallel$ denote components perpendicular to and along $\vec{k}$.

Noting

$$\frac{1}{\frac{kc}{\omega}p_\parallel - \sqrt{m^2c^2+p^2}} = $$

$$= \frac{\frac{\omega}{kcp}\left(\frac{p_\parallel}{p} + \frac{m\omega}{kp}\sqrt{1+\frac{p^2}{m^2c^2}}\right)}{\left(\frac{p_\parallel}{p}\right)^2 - \frac{m^2c^2}{k^2p^2}\left(1+\frac{p^2}{m^2c^2}\right)} =$$

$$= \frac{\frac{\omega}{kcp}\left(\frac{p_\parallel}{p} + \frac{m\omega}{kp}\sqrt{1+\frac{p^2}{m^2c^2}}\right)}{\left[\left(\frac{p_\parallel}{p}\right)^2 - \frac{\omega^2}{k^2c^2}\right] - \frac{m^2\omega^2}{k^2p^2}}$$

the integrand in the second term on the right in (9) will have a singularity if

$$\left(\frac{p_\parallel}{p}\right) > \frac{\omega}{kc}$$

or

$$\omega < kc.$$

Then the integration contour has to be indented around this singularity according to the Landau prescription in the nonrelativistic theory. The consequent damping of the waves is realisable because electrons travel with speeds less than the speed of light and they can resonate only with waves having phase velocities less than the speed of light. In the same vein, waves with phase velocities greater than the speed of light will not undergo any damping.

For isotropic distributions, the second term on the right in (9) becomes

$$2\pi\frac{mc\omega_p^2}{n_0}\int\limits_0^\infty \mathrm{d}p_\perp \int\limits_{-\infty}^\infty \mathrm{d}p_\parallel \frac{\left(\frac{p_\perp^2}{2}/\sqrt{m^2c^2+p^2}\right)\frac{\partial f_0}{\partial p_\perp}}{1-\frac{kc}{\omega}p_\parallel/\sqrt{m^2c^2+p^2}}.$$

Let us consider waves with $\omega > kc$ so that ω is real; this then becomes

$$2\pi\frac{\omega_p^2}{n_0}\int\limits_0^\infty \mathrm{d}p_\perp \int\limits_{-\infty}^\infty \mathrm{d}p_\parallel \frac{\partial f_0}{\partial p_\perp}\frac{p_\perp^2/2}{\sqrt{1+p^2/m^2c^2}}\left[1+\frac{\left(\frac{p_\parallel}{m}\right)/w}{\sqrt{1+p^2/m^2c^2}}+\right.$$

$$+ \frac{\left(\frac{p_{\parallel}}{m}\right)^2 / w^2}{(1 + p^2/m^2 c^2)} + \frac{\left(\frac{p_{\parallel}}{m}\right)^3 / w^3}{(1 + p^2/m^2 c^2)^{3/2}} + \dots \Bigg] =$$

$$= 2\pi \frac{\omega_p^2}{n_0} \int\limits_0^{\infty} p_{\perp} \, \mathrm{d}p_{\perp} \int\limits_{-\infty}^{\infty} \mathrm{d}p_{\parallel} \frac{f_0}{2} \left[\frac{2\left(1 + \frac{p_{\perp}^2 + p_{\parallel}^2}{m^2 c^2}\right) - \frac{p_{\perp}^2}{m^2 c^2}}{(1 + p^2/m^2 c^2)^{3/2}} + \right.$$

$$\left. + \frac{(p_{\parallel}/m)^2}{w^2} \frac{2\left(1 + \frac{p_{\perp}^2 + p_{\parallel}}{m^2 c^2}\right) - \frac{3p_{\perp}^2}{m^2 c^2}}{(1 + p^2/m^2 c^2)^{5/2}} + \dots \right] = \omega_p^2 \left(1 - \frac{5}{2} \frac{v_T^2}{c^2}\right)$$

$$K T = 2\pi \int\limits_0^{\infty} p_{\perp} \, \mathrm{d}p_{\perp} \int\limits_{-\infty}^{\infty} \mathrm{d}p_{\parallel} \frac{p_{\perp}^2}{2} f_0$$

$$\frac{1}{2} K T = 2\pi \int\limits_0^{\infty} p_{\perp} \, \mathrm{d}p_{\perp} \int\limits_{-\infty}^{\infty} \mathrm{d}p_{\parallel} \frac{p_{\parallel}^2}{2} f_0$$

$$v_T^2 = \frac{K T}{m}, \quad w \equiv \frac{\omega}{k}.$$

Thus (9) becomes

$$\omega^2 = k^2 c^2 + \omega_p^2 \left(1 - \frac{5}{2} \frac{v_T^2}{c^2}\right) \tag{10}$$

This agrees with the one given by Winkles and Eldridge[4], Misra[5] and Mikhailovskii[6] who used the Maxwellian distribution for the particles.

Now for longitudinal oscillations in a weakly-relativistic plasma one has (Misra[5], Mikhailovskii[6] and Shivamoggi[8])

$$\omega^2 = \omega_p^2 \left(1 + \frac{3k^2 v_T^2}{\omega_p^2} - \frac{5}{2} \frac{v_T^2}{c^2}\right). \tag{11}$$

Observe that the relativistic contributions in (10) and (11) are identical. This is to be expected since for small wave numbers the distinction between transverse and longitudinal waves should disappear.

References

1) V. P. Silin, Soviet Phys. JETP **11** (1960) 1136;
2) K. Imre, Phys. Fluids **5** (1962) 1;
3) B. Buti, Phys. Fluids **6** (1963) 89;

4) B. B. Winkles and O. Eldridge, Phys. Fluids **15** (1972) 1790;
5) P. Misra, J. Plasma Phys. **14** (1975) 529;
6) A. B. Mikhailovskii, Plasma Phys. **22** (1980) 133;
7) D. Montgomery and D. A. Tidman, *Plasma Kinetic Theory*, McGraw Hill, New York, 1964;
8) B. K. Shivamoggi, Fizika **15** (1983) 83.

TRANSVERZALNI VALOVI U RELATIVISTIČKOJ PLAZMI

BHIMSEN K. SHIVAMOGGI

Physical Research Laboratory, Ahmedabad 380 009, *India*

UDK 533.95

Originalni znanstveni rad

Razmatrano je širenje transverzalnih elektromagnetskih valova kroz slabo relativističku plazmu. Pokazano je da se poznati izraz za odgovarajuću disperzionu relaciju za Maxwellovu plazmu dobiva i u općenitijem slučaju kada je energetska raspodjela čestica plazme određena proizvoljnom izotropnom funkcijom.