Zrinka Lovretin Golubić[a]
Denis Dolinar[b]
Davor Zoričić[c]

JEL classification: G11, G12
Preliminary statement
https://doi.org/10.32910/ep.76.1.1

A heuristic approach to the estimation of an efficient benchmark in the Croatian stock market

Abstract

In this paper a heuristic approach, which solely relies on risk parameter estimation, is pursued to estimate the efficient benchmark in the Croatian stock market. Optimisation method focused on risk parity is employed allowing investors to diversify risk by relying on equal risk contribution to achieve optimal portfolio diversification. Six different benchmarks related to risk parity method variations and covariance matrix estimations are examined in order to compare their performance with the capitalization-weighted counterpart. This allows insight regarding the potential sources of differences in their risk-reward characteristics. Results in this study are based on 28 out-of-sample estimations in the period from April 2005 to March 2019. The findings do not show evidence of risk parity method being able to provide exposure to rewarded risk factors in the Croatian stock market. Moreover, regarding the diversification of unrewarded risks even the benchmark portfolio with the lowest reported volatility is more volatile than the CROBEX benchmark. However, if only expansion sub-period is analysed all examined benchmarks outperform the CROBEX benchmark with the factor risk parity portfolio based on two or more components reporting the lowest volatility. Overall the results show that risk parity portfolios do outperform the equally-weighted benchmark and that assuming equal correlations of portfolio constituents or applying statistical shrinkage method for their estimation yields better results than relying on the principal components analysis.

Keywords: efficient benchmark estimation; risk parity; factor risk parity; principal component analysis; statistical shrinkage; asset management

---

[a] Z. Lovretin Golubić, Ph.D., senior teaching and research assistant, University of Zagreb, Faculty of Economics & Business (e-mail: zlovretin@efzg.hr).
[b] D. Dolinar, Ph.D., Associate Professor, University of Zagreb, Faculty of Economics & Business (e.mail: ddolinar@efzg.hr).
[c] D. Zoričić, Ph.D., Associate Professor, University of Zagreb, Faculty of Economics & Business (e-mail: dzoricic@efzg.hr). The paper was received on 03.05.2023. It was accepted for publication on 05.07.2024.

1. INTRODUCTION

Since the optimal market portfolio is unobservable in the real world, it has become standard practice to approximate the market portfolio by stock market indices that are based on market capitalization. Another reason why indices based on market capitalization are used as an approximation in practice is related to the fact that it is relatively easy to trade a portfolio constructed in this way. The research by Haugen & Baker (1991) and Grinold (1992) revealed that stock indices based on market capitalization are not efficient i.e. they do not provide adequate compensation for the systematic risk to the investor. Since research on developed financial markets has shown that indices based on market capitalization are not necessarily efficient, in the last two decades, new approaches have been developed that should offer investors indices that are more efficient. With such approaches, known as the "Smart beta" strategies, it is possible to create a portfolio with better performance than a comparable index based on the market capitalization (cap-weighted benchmark).

However, indices created with "Smart beta" strategies expose investors to new risk factors. The type of systematic risk exposure depends on the chosen construction method. Specific risk exposure depends on the trade-off between parameter estimation risk and optimality risk (Amenc, Goltz & Martellini, 2013, p. 10). Since the estimation of the benchmark with maximum Sharpe ratio (the most desirable portfolio with no optimality risk but highest possible estimation risk) did not yield significant results in the out-of-sample analysis performed in the Croatian stock market (Dolinar, Zoričić & Kožul, 2017), different approaches to estimate the benchmark are taken in this research. The underlying motivation being that better results can be achieved by deviating from optimality conditions which is compensated by lower estimation risk.

In this research risk parity method is used which presents a heuristic (naïve) approach for the construction of the portfolio. Due to limited diversification possibilities with the naïve strategy where each constituent has the same weight in the portfolio (equally-weighted benchmark, EW), Maillard, Roncalli & Teiletche (2010) combined two heuristic approaches (global minimum variance, GMV and EW) into the risk parity strategy. Such a strategy uses risk to determine the weight of each constituent in the portfolio (Maillard et al., 2010, p. 1-2) and mimics the diversification effect of a portfolio with equal weights of the constituents by taking into account individual and joint contributions to portfolio risk. Furthermore, if this diversification approach is pursued by applying the principal component analysis (PCA) to try to capture the unobservable (implicit) underlying risk factors of the portfolio, factor risk parity portfolio can be estimated (Martellini & Milhau, 2018). By constructing such a portfolio equal risk contribution (ERC) is also achieved. Since the risk factors, which should contribute to portfolio risk equally, may or may not be rewarded one can hope that the approach will not only decrease exposure to unrewarded risk factors (diversify away undesired risks) but also increase exposure to rewarded risk factors relative to the cap-weighted benchmark. Also, in this research the performance of the portfolio based on factor risk parity is compared to the risk parity portfolios based on standard covariance historical sample estimators and improved ones based on statistical shrinkage method following the research of Ledoit & Wolf (2004).

The rest of the paper is structured as follows. The second section gives a literature review and the third section describes the methodology and data used in this research. The empirical part is covered in the fourth section with the conclusion given in the final, fifth section.

2. LITERATURE REVIEW

According to the Modern Portfolio Theory (MPT), holding the market portfolio is the optimal investment strategy for an investor. Such a market portfolio represents the best possible choice thus being efficient. According to the most famous asset pricing theory, the Capital Asset Pricing Model (CAPM), all investors desire to hold the same, optimal portfolio, and the weights of investments in such an optimal portfolio are equal to those of the market portfolio

(Amenc, Goltz & Le Sourd, 2006, p. 26). Since a true market portfolio is not observable, it is subject of an approximation in practice. Various proxy portfolios were used in the research, both related to different markets and types of assets. The earliest research used a proxy for the market portfolio indices that included only stocks, and such practice is still most often used today.

Stambaugh (1982) included both consumer goods and real estate, which resulted in an efficient index. The same could lead to the conclusion that the more different types of assets are included in the index the more efficient such portfolio is and we get closer to the real market portfolio. This is however refuted by the research of Shanken (1985, 1987), Brown & Brown (1987), and Jagannathan & Wang (1996). Even though several types of assets are included in the proxy, the results are either inconclusive or such an index is not efficient. Black, Jensen & Scholes (1972), Fama & MacBeth (1973), Gibbons (1982), Zhou (1981), Gibbons, Ross & Shanken (1989), Harvey & Zhou (1990), Kandel, McCulloch & Stambaugh (1995), and Fama & French (2006) included all stocks listed on the stock exchange in their research. When it comes to the method used to construct the proxy portfolio, research by Haugen & Baker (1991) and Grinold (1992) showed that stock indices based on market capitalization are not efficient. Black et al. (1972), Fama & MacBeth (1973), Gibbons (1982), Shanken (1985, 1987), and Amenc et al. (2006) used the naïve method by assigning equal weights to all stocks in the portfolio which resulted in outperforming the cap-weighted benchmark.

Since customary use of the capitalization-weighted stock market indices as a proxy for the market portfolio yielded significantly suboptimal results, new approaches have been developed. "Smart beta" strategies can be defined in various ways. Malkiel (2014, p. 127) states that "Smart beta" is a technique to enhance returns by assuming additional risk. Although the main aim of the "Smart beta" strategies is to address the basic shortcomings of indices based on market capitalization (exposure to unrewarded risk factors and high concentration in individual stocks), Amenc, Goltz, Lodh & Martellini (2014) highlight that this is incomplete. Alongside efficient diversification of unrewarded factors, the efficient exposure to rewarded risk factors (sometimes referred to as „smart factor investing") needs to be considered in order to address the second shortcoming of the market cap-weighted benchmarks. "Smart beta" benchmark therefore aims to diversify and thus increase the exposure to rewarded risk factors while diversifying away unrewarded risks.

Based on the MPT, the optimal portfolio regarding diversification of unrewarded risk factors for an investor is the Maximum Sharpe Ratio (MSR) portfolio since it maximizes the investor's risk-reward ratio. However, estimating the MSR portfolio is the most challenging since three parameters need to be estimated (returns, volatilities, and correlations between the stock returns). Because of that, other "Smart beta" strategies are focused on the reduction of the estimation risk while accepting certain degree of optimality risk to eliminate the unrewarded risk factors but also to increase the diversification of the rewarded risk factors. Choueifaty & Coignard (2008) created a portfolio that has a maximum diversification ratio (MDR) in order to estimate a maximally diversified portfolio. Estimation of the benchmark with minimum variance, GMV, has a clear built-in bias towards low-volatility stocks which can lead to overexposure to low-volatility industry sectors (Chan, Karceski & Lakonishok, 1999). In order to solve that issue, Christoffersen, Errunza, Jacobs & Langlois (2012) used the assumption that the volatilities between stocks are equal throughout the portfolio and create a maximum decorrelation benchmark (MDC). With the combination of two heuristic approaches (GMV and EW), Maillard et al. (2010) pursued a risk parity strategy that allows investors to target certain levels of risk and share risk throughout the entire investment portfolio as a proxy for optimal portfolio diversification.

As for the analysis performed on the Croatian capital market, most of the research focuses on the efficiency of the CROBEX index, the largest and oldest stock index in the Croatian capital market[1]. Research findings of Zoričić, Dolinar

[1] The CROBEX index is not based solely on market capitalization. The weights of individual components in the index are determined based on the

& Kožul (2014) and Bilić Marjanović, Beljo & Devčić (2017) for the Croatian market suggest that the CROBEX index is not efficient which is in line with the research performed for the developed financial market. Habibović, Zoričić & Lovretin Golubić (2017) tested the efficiency of the CROBEX and the CROBEX10 indices and the research showed that the distance of the CROBEX10 index from its efficient frontier is shorter than in the case of the CROBEX index, which points to its higher efficiency. However, regardless of the higher efficiency, the CROBEX10 index offers fewer opportunities for diversification due to a smaller number of constituents.

Motivated by the fact that the cap-weighted benchmark is not efficient i.e., does not provide adequate compensation for the systematic risk to the investor, some "Smart beta" strategies have been tested on the Croatian stock market. The attempt to estimate the MSR portfolio didn't yield a superior portfolio that could outperform the cap-weighted benchmark on the Croatian stock market (Dolinar et al., 2017). One of the reasons for that could be the estimation of the expected return since such estimations are considered noisier compared to the risk estimates. In addition, it becomes more challenging to estimate the expected return in the undeveloped and less liquid market as Croatian one as shown in the research by Dolinar, Zoričić & Lovretin Golubić (2019). Therefore, the focus of the research has been shifted to the estimation of some other sub-optimal portfolios with less estimation risk. In research by Zoričić, Dolinar & Lovretin Golubić (2018) estimated GMV portfolios outperformed the MSR portfolio but still didn't outperform the cap-weighted benchmark. The same paper confirmed the dominance of the enhanced method for the estimation of covariance matrix, statistical shrinkage, which works even in undeveloped and illiquid settings compared to usage of standard sample covariance. Therefore, by taking the outcomes from previous research performed for the Croatian stock market and with the motivation to estimate the portfolio that could outperform the cap-weighted counterpart, this research is using (factor) risk parity method. Compared to the GMV portfolio, the main task will not be to minimize the portfolio risk but to increase the efficiency of risk exposure on the undeveloped and less liquid Croatian stock market.

free float market capitalization and the weight of individual stock is additionally limited to 10%. The CROBEX index is a price index, which means that dividends are not included in its calculation.

3. METHODOLOGY AND DATA

This research covers 64 stocks that were listed on the Zagreb Stock Exchange (ZSE) and included in the CROBEX index at some point in time during the observation period from April 2005 to March 2019[2]. A total of 28 regular revisions of the CROBEX index were considered in which the number of constituents of the CROBEX index varied from 17 to 32 stocks (with an average of 25) in the observed period. The estimation process is based on monthly total excess returns[3] for the last 3 years (36 months) for each stock. The out-of-sample performance for comparison with the performance of the CROBEX index is based on semi-annual returns[4].

The proposed benchmark indices tested in this research are created using several strategies: starting with the most naïve one, the equally-weighted strategy (EW), then diversified risk parity strategy (DRP), and risk parity strategy (RP). The diversified risk parity method uses an inversely proportional approach to risk by giving a smaller weight to components with higher volatility (or risk) and a larger weight to components with lower volatility. In such a portfolio, the risk contribution of each constituent is equal. Still, it is a naïve approach since the correlations of the constituent's returns are not taken into account i.e. they are considered

2 The observation period ends in 2019 so the effect of COVID pandemic is not taken into account.

3 Total return includes both capital gain and dividend yield (where applicable). Excess return is the return above the risk-free rate. As a proxy for the risk-free rate is used the yield on the Croatian three-month treasury bill at the moment of issuing (denominated in local currency).

4 Only capital gain is considered since the CROBEX index is a price index and it doesn't take into account dividend yield. Also, risk free rate is not subtracted in this case.

as being equal. The weight of each constituent in the portfolio is calculated using the following formula:

$$w_i = \frac{\sigma_i^{-1}}{\sum_{i=1}^{N} \sigma_i^{-1}} \tag{1}$$

where σ_i is the standard deviation of the return of each constituent. However, the correlations cannot be ignored, therefore, the strategy which should be referred to as the true risk parity or simply – risk parity takes them into account. In such a portfolio, the risk contribution of each constituent is also equal. If all constituents had correlation coefficients with each other equal to one, the risk parity and the diversified risk parity would yield the same results. However, correlations between the returns are usually not equal to one. If the diversified risk parity method was used, the constituent which has the negative correlation coefficient with the rest of the constituents and high volatility would have a smaller weight in the portfolio and the benefits of diversification present due to the negative correlation coefficient would not be used. In this case, it is more correct to use the risk parity method, given the fact that such a method, after taking into account the volatility of the constituent, assigns a greater weight to components with low correlations. Portfolio volatility $\sigma(w)$ can be defined as a function of the weights vector of the standard deviations and correlations of the constituent's returns (Kind & Poonia, 2015, p. 70-71):

$$\sigma(w) = \sqrt{\boldsymbol{w'\Omega w}} = \\ = \sqrt{\sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \rho_{ij} \sigma_i \sigma_j} \tag{2}$$

where $\mathbf{\Omega}$ is the covariance matrix of the constituent's returns and $\mathbf{w}$ is the weights vector. Since $\sigma(w)$ is a homogeneous function of degree 1 over w, from Euler's theorem for the homogeneous functions it follows:

$$\sigma(w) = \sum_{i=1}^{N} \sigma_i(w) \tag{3}$$

$$\sigma_i(w) = w_i \cdot \partial_{w_i} \sigma(w) = \frac{w_i (\boldsymbol{\Omega w})_i}{\sqrt{\boldsymbol{w'\Omega w}}} \tag{4}$$

where $\sigma_i(w)$ can be interpreted as a contribution of constituent i to the total portfolio risk. Equal risk parity means that $\sigma_i(w) = \sigma_j(w)$ for all i and j or $\sigma_i(w) = \frac{\sigma(w)}{N}$. In order to find the optimal weights for the constituents in the risk parity portfolio, the following non-linear optimization is applied:

$$w_i^* = arg \min_w \sum_{i=1}^{N} \left[w_i - \frac{\sigma(w)^2}{(\boldsymbol{\Omega w})_i N} \right]^2 \tag{5}$$

with the constraint:

$$\sum_{i=1}^{N} w_i^* = 1 \tag{6}$$

where w_i^* is the optimal weight of the constituent in the risk parity portfolio, $\mathbf{\Omega}$ is the estimated covariance matrix of the returns of constituents using historical sample estimators and N is the number of constituents i.e. stock in each composed risk parity portfolio that corresponds to the number of CROBEX index constituents in each revision.

The standard statistical method for the covariance matrix estimation refers to the collection of historical data on stock returns and the calculation of the covariance matrix based on such historical data sample. Such an assessment often involves estimation errors. Estimates within the covariance matrix can be extreme values because they are subject to an extreme amount of estimation errors in case of large portfolios (Ledoit & Wolf, 2004, p. 110). Such a phenomenon is called "error-maximization". For this reason, in order to better estimate the covariance matrix, besides using historical sample estimators, principal component analysis and random matrix theory or statistical shrinkage can be used. In this paper, two methods are used: principal component analysis, resulting in a risk parity portfolio in the factor space, referred to as factor risk parity (FRP portfolio), and statistical shrinkage, resulting in a risk parity portfolio referred to as "shrinked" risk parity portfolio (SRP portfolio).

The principal component analysis is based on the spectral decomposition of the covariance matrix which aims to explain the covariance structure using only a few linear combinations of original stochastic variables that will form a new set of (unobservable) components (Amenc, Goltz, Martellini & Retkowsky, 2011, p. 56). The result of the conducted principal component analysis is the factor loading matrix **B** used for the estimation of the matrix of returns in the factor space (matrix **F**). After the estimation of the factor returns, the covariance matrix of factor returns $\boldsymbol{\Lambda}$ is calculated. Multiplying factor loadings β_{ij} from the matrix **B** and eigenvalues λ_{ij} from the covariance matrix of factor returns $\boldsymbol{\Lambda}$, covariance matrix of stock returns $\boldsymbol{\Sigma}$ is obtained, for the certain number of components m:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \beta_{11} & \dots & \beta_{1M} \\ \dots & \beta_{22} & \dots \\ \beta_{N1} & \dots & \beta_{NM} \end{bmatrix} \begin{bmatrix} \lambda_{11} & 0 & 0 \\ 0 & \lambda_{22} & 0 \\ 0 & 0 & \lambda_{MM} \end{bmatrix} \begin{bmatrix} \beta_{11} & \dots & \beta_{1M} \\ \dots & \beta_{22} & \dots \\ \beta_{N1} & \dots & \beta_{NM} \end{bmatrix}^T \quad (7)$$

$$\boldsymbol{\Sigma} = \mathbf{B}\boldsymbol{\Lambda}\mathbf{B}^T \quad (8)$$

The statistical shrinkage method reduces the estimation error by pulling the most extreme coefficients towards more central values. The procedure is expressed by the following formula (Dolinar et al., 2017, p. 16; Ledoit & Wolf, 2004, p. 113):

$$\boldsymbol{\Sigma} = \delta \boldsymbol{F} + (1-\delta)\boldsymbol{S} \quad (9)$$

where $\boldsymbol{\Sigma}$ represents the estimation of the true covariance matrix of the expected returns, $\boldsymbol{S}$ represents the sample covariance matrix of returns and $\boldsymbol{F}$ represents the covariance matrix estimator called shrinkage target. To simplify the matrix $\boldsymbol{F}$, work of Ledoit & Wolf (2004) is followed and the constant correlation model[5] is applied. δ represents the shrinkage constant which is used for minimization of the expected value of loss in the process of estimation of the true covariance matrix and is calculated by applying the formulas below (Ledoit & Wolf, 2004, p. 117-118):

$$\hat{\delta} = \max\left\{0, \min\left\{\frac{\hat{\kappa}}{T}, 1\right\}\right\} \quad (10)$$

$$\hat{\kappa} = \frac{\hat{\pi} - \hat{\rho}}{\hat{\gamma}} \quad (11)$$

$$\hat{\pi} = \sum_{i=1}^{N}\sum_{j=1}^{N}\left\{\frac{1}{T}\sum_{t=1}^{T}\left[(r_{i,t}-\bar{r}_i)(r_{j,t}-\bar{r}_j) - s_{ij}\right]^2\right\} \quad (12)$$

$$\hat{\rho} = \sum_{i=1}^{N}\hat{\pi}_{ii} + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\left\{\frac{\bar{u}}{2}\left(\sqrt{\frac{s_{jj}}{s_{ii}}}\hat{\vartheta}_{ii,ij} + \sqrt{\frac{s_{ii}}{s_{jj}}}\hat{\vartheta}_{jj,ij}\right)\right\} \quad (13)$$

$$\hat{\gamma} = \sum_{i=1}^{N}\sum_{j=1}^{N}(f_{ij} - s_{ij})^2 \quad (14)$$

$$\hat{\vartheta}_{ii,ij} = \frac{1}{T}\sum_{t=1}^{T}\left\{\left[(r_{i,t}-\bar{r}_i)^2 - s_{ii}\right]\left[(r_{i,t}-\bar{r}_i)(r_{j,t}-\bar{r}_j) - s_{ij}\right]\right\} \quad (15)$$

where s_{ij} represents values from the sample covariance matrix $\boldsymbol{S}$, f_{ij} represents values from the sample constant-correlation covariance matrix $\boldsymbol{F}$, and $\bar{u}$ is the average correlation coefficient of stock returns in the sample.

After the covariance matrix is estimated using principal component analysis and statistical shrinkage method, the optimization procedure is equivalent to the one described in equation (5), where instead of the covariance matrix of stock returns $\boldsymbol{\Omega}$ new estimated covariance matrix of stock returns $\boldsymbol{\Sigma}$ is used and the benchmarks are denoted as FRP portfolio and SRP portfolio respectively. In addition, the number of components from PCA should be determined and for that purpose, Kaiser's rule is used which retains principal components that have an eigenvalue greater than 1. Based on that, FRP 1 and FRP 2 portfolios are distinguished, meaning that the analysis is carried out based on one component, and more components, respectively.

4. EMPIRICAL RESULTS

Estimation of EW, DRP, RP, FRP 1, FRP 2 and SRP portfolio is always performed for the actual composition of the CROBEX index. For each revision new set of inputs is estimated: estimation of the covariance matrix, marginal and total risk contribution as well as a new out-of-sample output: time-series of returns of the newly estimated benchmark. Estimation of the covariance matrix, marginal and total risk contribution of each stock to the total portfolio risk is calculated for each revision based on the previous three years, i.e. 36 months. Since a total of 14 years are included in the research, rebalancing of the portfolio is performed 28 times for each benchmark. Weights obtained by the optimization process are used for the estimation of monthly returns of EW, DRP, RP, FRP 1, FRP 2, and SRP

5 The average of all the sample correlations is used as the estimator of common constant correlation and the matrix $\boldsymbol{F}$ represents the sample constant-correlation covariance matrix of returns.

Table 1. Performance and robustness of the estimated out-of-sample benchmarks for the entire observation period

	Benchmark	**EW**	**DRP**	**RP**	**FRP 1**	**FRP 2**	**SRP**	**CROBEX**
Entire observation period (March 2005 – March 2019)	Return	-3.29%	-2.06%	-3.12%	-2.89%	-3.37%	-1.96%	-0.35%
	Volatility	24.18%	22.62%	23.49%	23.57%	22.44%	22.60%	21.86%
	Risk-reward ratio	-0.136	-0.091	-0.133	-0.122	-0.150	-0.087	-0.016
	Modified risk-reward ratio[6]	-0.00794	-0.00467	-0.00733	-0.00680	-0.00757	-0.00442	**-0.00077**

Source: Authors' calculation

portfolios for the next 6 months out-of-sample thus estimating the time-series of returns of the of the proposed benchmark indices. Their performance is compared with the performance of the counterpart based on the market capitalization – the CROBEX index. As a key performance measure, the risk-reward ratio is used. Reported portfolio returns refer to the geometric average and were calculated based on semi-annual returns while volatility refers to the standard deviation of the semi-annual returns.

If the entire observation period is considered, the Table 1 results expose the failure of all of the benchmarks' out-of-sample estimation for the analysed data. This is in line with previous research performed out-of-sample on the Croatian stock market for MSR benchmark estimation, for a similar observation period (Dolinar et al., 2017). Generally, out-of-sample estimation is very sensitive to parameters used as inputs and may involve large estimation errors. Even though applied strategies have considerably less parameter estimation risk as opposed to optimality risk, they didn't succeed to produce benchmarks with better performance compared to the CROBEX index. A significant drop in benchmarks' return and increase of volatility in the case of all except DRP and SRP benchmarks, dismisses any possibility of outperforming the CROBEX index.

In addition to presenting the results for the entire period for which the analysis is carried out, to test the robustness of the results, the entire observation period is divided into several sub-periods[7]. Results are presented in the Table 2.

Due to significant changes and fluctuations in the financial market, robustness analysis re-

[6] Due to the appearance of the negative values of portfolio returns in the observed period, the ratio is modified as proposed by Israelsen (2005). When negative returns are present, the risk-reward ratio should be modified in order to accurately measure the performance of a portfolio. If two portfolios exhibit the same negative return, a portfolio with larger volatility could be favoured, since its risk-reward ratio will be higher, i.e. less negative. Therefore, the modified risk-reward ratio is used by adding an exponent to its denominator as follows: $\frac{R_p}{\sigma_p^{\left(\frac{R_p}{absolute(R_p)}\right)}}$, where σ_p is the standard deviation of portfolio return (volatility measure) and R_p is portfolio return. When portfolio return is positive there is no difference between the standard and modified risk-reward ratio.

[7] The expansion period is considered to last until the end of the first revision in 2007 (September 2007) since if the following CROBEX revision is included CROBEX return falls and volatility increases. The rest of the observation period is considered the post-expansion period. The post-expansion period is further divided into two sub-periods. The crisis period which begins with the second revision in 2007 (September 2007) and lasts to the end of the second revision in 2015 (March 2016) is characterised by CROBEX revision periods leading to the lowest CROBEX return accompanied by the highest volatility. The post-crisis period lasts from the first revision in 2016 (March 2016) until the end of the observation period (until the end of the second revision in 2018, i.e. until March 2019).

Table 2. Robustness analysis of the estimated out-of-sample benchmark indices

	Benchmark	EW	DRP	RP	FRP 1	FRP 2	SRP	CROBEX
Expansion (March 2005 – September 2007)	Return	21.28%	21.51%	20.91%	22.03%	18.06%	21.69%	19.89%
	Volatility	12.94%	12.43%	13.54%	14.35%	12.07%	13.12%	15.92%
	Risk-reward ratio	1.645	1.731	1.544	1.535	1.497	1.654	1.249
	Modified risk-reward ratio	**1.64464**	**<u>1.73106</u>**	**1.54411**	**1.53517**	**1.49674**	**1.65366**	1.24942
Crisis (September 2007 – March 2016)	Return	-10.07%	-8.67%	-9.63%	-9.64%	-9.65%	-8.46%	-6.20%
	Volatility	25.50%	23.93%	24.85%	24.23%	23.78%	23.72%	23.60%
	Risk-reward ratio	-0.395	-0.362	-0.387	-0.398	-0.406	-0.357	-0.263
	Modified risk-reward ratio	-0.02567	-0.02075	-0.02392	-0.02335	-0.02296	-0.02007	**-0.01463**
Post-crisis (March 2016 – March 2019)	Return	-1.59%	-0.26%	-1.92%	-1.53%	-1.08%	-0.53%	1.37%
	Volatility	17.74%	14.51%	15,96%	17.22%	17.42%	14.79%	12.15%
	Risk-reward ratio	-0.090	-0.018	-0.120	-0.089	-0.062	-0.036	0.113
	Modified risk-reward ratio	-0.00282	-0.00037	-0.00306	-0.00264	-0.00188	-0.00078	**0.11317**

Source: Authors' calculation

sults allow additional insight. In the expansion period, all estimated benchmarks succeeded to produce a portfolio with better performance compared to the cap-weighted benchmark. In all cases, the average return is higher and volatility is decreased relative to CROBEX with the notable exception of the FRP 2 benchmark for which the return is lower. The main aim of the risk parity method is not to minimize the portfolio risk yet to ensure risk control (i.e. increase efficiency of risk exposure) by aiming to achieve an equal contribution of risk to the portfolio risk. Although, the FRP 2 benchmark stands out regarding the portfolio volatility reduction it is outperformed by all other benchmarks in terms of overall performance measured by the risk-reward ratio, with DRP and SRP benchmark portfolios leading the way. While it seems that FRP 2, as the factor risk parity benchmark based on two or more components, is able to capture and diversify the unrewarded risk factors it doesn't seem to offer exposure to rewarded risk factors. Therefore, the biggest reduction in volatility is accompanied by the substantial reduction in return. Even though risk reduction in the case of other benchmarks is not as pronounced, they seem to offer exposure to additional sources of risk premium outperforming the CROBEX even in respect to the return achieved in the expansion period.

However, in the following sub-periods all the benchmarks prove to be unable to capture either rewarded or unrewarded risk factors failing to outperform the cap-weighted benchmark both regarding risk and return. The surprising lack of increase in efficiency related to the efficient diversification of unrewarded risk factors even in the case of FRP 2 can be explained by the fact that principal factors are often hard to interpret and are unstable across different periods (Carli, Deguest & Martellini, 2014). The reduction in return of the analysed portfolios on the other hand seems much more challenging since, unlike in the case of volatility, none of the benchmarks come even close to the CROBEX in the crisis period. Overall, the analysis of sub-periods confirms the dominance of the DRP and SRP benchmarks already noted in the results presented in the Table 1.

The results of this research, therefore, do not provide evidence of more efficient diversification relative to the cap-weighted benchmark based on the ERC approach regardless of the covariance estimation method for the tested risk parity portfolios. There doesn't seem to be any evidence of exposure to rewarded risk factors in particular, while the capture of unrewarded risk factors doesn't seem to be robust enough to be reflected in the benchmark portfolios' volatility in the ERC approach. The results indicate that the estimation of covariances is poor enough that they don't even have to be estimated. Namely, the DRP benchmark which doesn't require the estimation of covariances and relies on historical sample estimators for variances matches closely the performance of the SRP portfolio which relies on their estimation by applying the statistical shrinkage method. When considering these results one should also take into account that a paper by Zoričić, Dolinar & Lovretin Golubić (2018) showed that GMV portfolios based on statistical shrinkage estimation of covariances yield superior results to the ones based on historical sample estimators. Therefore, the results of this paper suggest that in order to rely on the risk parity portfolios and ERC approach in the illiquid and undeveloped market, enhancement of the PCA method should be pursued. For instance, the minimum linear torsion (MLT) method proposed by Meucci, Santangelo & Deguest (2015) and Martellini & Milhau (2018) could be further examined in such environment due to its advantages regarding the ability to capture and interpret the underlying (rewarded and unrewarded) risk factors.

In order to complete this analysis, a comment should be added in respect of the (de)concentration[8] of estimated benchmarks. Research has shown that the cap-weighted benchmarks in the developed markets are limiting exposure to long-term rewarded risk factors and are highly concentrated implying that the exposure to unrewarded risk factors is also inefficient (Amenc et al., 2006). When it comes to the cap-weighted index on the Croatian stock market, however, the deconcentration is relatively high. Therefore, the naïve strategy maximizing deconcentration, i.e. EW benchmark, does not outperform the cap-weighted benchmark. However, it is interesting to note that all other benchmarks which focus on ERC (and not EW) also result in higher deconcentration (in the range of 78.85% to 87.12%) relative to CROBEX (52.8%). The stated levels correspond to effective number of stocks in the range of 19.32 – 21.25 on average as opposed to 12.81 related to CROBEX index. Previous papers such as Zoričić et al. (2018) and Zoričić, Dolinar & Lovretin Golubić (2020) have shown that the analysed benchmark portfolios were unable to beat the CROBEX index when the optimisation approach lead to higher concentrations of the proposed benchmarks. In this paper the optimisation approach lead to multiple benchmarks, with deconcentrations falling in the range between the CROBEX and the EW portfolio, however they are still unable to outperform the CROBEX index. This provokes the question whether the concentration of the estimated benchmark should be abandoned as an argument for the (un)success of such benchmark to outperform the cap-weighted benchmark on the Croatian stock market or even more enhanced estimation methods should be tested taking into account the specifics of the Croatian stock market.

5. CONCLUSION

This research deals with the estimation of an efficient benchmark in the Croatian stock market by pursuing the heuristic approach where the weights of constituents are determined based on individual and joint contributions to portfolio risk. By constructing an RP portfolio, the intention is to construct a portfolio based on ERC principle in the hope of diversifying away unrewarded risks while also providing exposure to rewarded risks. Since underlying risk exposures are unobservable in nature, in order to improve the ability of capturing them factor risk parity portfolios are used by employing the PCA method. In this re-

[8] The concentration level of the portfolio is calculated by using the following equation:
Concentration = $(N \cdot \sum_{i=1}^{N} w_i^2)^{-1}$.

search the performance of FRP portfolios and RP portfolios based on historical sample estimators and improved covariance estimators relying on statistical shrinkage method is compared.

The results have shown that all estimated benchmarks succeeded to produce a portfolio with better performance compared to the cap-weighted benchmark only during the expansion period. The DRP portfolio which doesn't take into account correlations of constituents' returns and is based on historical sample estimators dominates other benchmarks. It is closely followed by the RP benchmark relying on improved covariance matrix estimation, the statistical shrinkage (the SRP benchmark). Thus, results for the expansion period provide for the investors an insight into the possibility of estimation of benchmarks with better performance compared to the cap-weighted counterpart on the Croatian stock market. This leads them to their utmost goal and that is a higher risk-reward ratio with efficient diversification of unrewarded risk factors. However, investors should be aware that the conditions on the Croatian stock market are far from those present at the time of expansion so their strategy should be adaptable to market conditions.

Results reveal that the PCA method does not successfully capture underlying risk factors in the illiquid and undeveloped financial market corroborating the findings of Zoričić et al. (2020). Since according to Carli et al. (2014) the principal factors are hard to interpret and are often unstable across different periods further research should be focused on enhanced methods like the MLT proposed by Meucci et al. (2015) and Martellini & Milhau (2018). Overall, the findings of this research do not provide evidence of more efficient diversification relative to the cap-weighted benchmark based on the ERC approach regardless of the covariance estimation method. The exposure to additional rewarded risk factors seems to be lacking while diversifying away the unrewarded risk factors doesn't seem to be robust enough. Unlike in other similar research papers, such as Zoričić et al. (2018) and Zoričić et al. (2020), due to the ERC approach pursued in this paper the proposed benchmark portfolios result in substantially lower portfolio concentrations relative to the CROBEX benchmark. However, the tested portfolios were unsuccessful in outperforming the cap-weighted benchmark in all instances, therefore supporting the view that better parameter estimation techniques should be in focus of future research rather than portfolio concentration issues.

Finally, this research has its limitations. One of the limitations is the negligence of transaction costs that come when rebalancing portfolios. The goal of this paper was never to deploy an efficient marketable benchmark in real life. The goal was to confirm that cap-weighted indices are generally inefficient, as shown in studies for the developed markets, and that risk parity schemes might overperform (at least in perfect conditions). Contrary to expectation, our research showed that even if there were no frictions on the market, the risk parity portfolio would underperform, i.e. the cap-weighted index shows superior performance for the whole observation period.

References

Amenc, N., Goltz, F., & Le Sourd, V. (2006). *Assessing the Quality of Stock Market Indices: Requirements for Asset Allocation and Performance Measurement*, An EDHEC Risk & Asset Management Research Centre Publication, Retrieved from: https://risk.edhec.edu/assessing-quality-stock-market-indices

Amenc, N., Goltz, F., Martellini, L., & Retkowsky, P. (2011). Efficient Indexation: An Alternative to Cap-Weighted Indices. *The Journal of Investment Management,* 9(4), 1-23.

Amenc, N., Goltz, F., & Martellini, L. (2013). *Smart Beta 2.0.*, EDHEC-Risk Institute, Retrieved from: https://risk.edhec.edu/publications/smart-beta-2-0

Amenc, N., Goltz, F., Lodh, A., & Martellini, L. (2014). Towards smart equity factor indices: Harvesting risk premia without taking unrewarded risks. *The Journal of Portfolio Management,* 40(4), 106-122. https://doi.org/10.3905/jpm.2014.40.4.106

Bilić Marjanović, M., Beljo, I., & Devčić, K. (2017). Analiza primjenjivosti CAPM-a na tržištu kapitala Republike Hrvatske. *Oeconomica Jadertina,* 7(2), 5-16.

Black, F., Jensen, M., & Scholes, M. (1972). The capital asset pricing model: some empirical tests. In M.C. Jensen (Ed.), *Studies in the The-*

ory of Capital Markets (p. 79-124.). New York, NY: Praeger.

Brown, K.C., & Brown, G.D. (1987). Does the composition of the market portfolio really matter. *Journal of Portfolio Management,* 13(2), 26-32.

Carli, T., Deguest, R., & Martellini, L. (2014). *Improved Risk Reporting with Factor-Based Diversification Measures*, EDHEC-Risk Institute Publication, Retrieved from: https://risk.edhec.edu/sites/risk/files/edhec_publication_improved_risk_reporting_f.pdf

Chan, L., Karceski J., & Lakonishok, J. (1999). On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model. *Review of Financial Studies,* 12(5), 937-974.

Choueifaty, Y., & Coignard, Y. (2008). Toward Maximum Diversification. *Journal of Portfolio Management,* 35(1), 40-51. https://doi.org/10.3905/JPM.2008.35.1.40

Christoffersen, P., Errunza, V., Jacobs, K., & Langlois, H. (2012). Is the Potential for International Diversification Disappearing? A Dynamic Copula Approach. *The Review of Financial studies,* 25(12), 3711-3751. https://doi.org/10.1093/rfs/hhs104

Dolinar, D., Zoričić, D., & Kožul, A. (2017). Towards the Estimation of an Efficient Benchmark Portfolio: The Case of Croatian Emerging Market. *Zagreb international review of economics & business,* 20(SCI), 13-23. https://doi.org/10.1515/zireb-2017-0002

Dolinar, D., Zoričić, D., & Lovretin Golubić, Z (2019). Application of semi-deviation as a proxy for the expected return estimation in the Croatian equity market. *Croatian Review of Economic, Business and Social Statistics,* 5(1), 9-20. https://doi.org/10.2478/crebss-2019-0002

Fama, E. F., & French, K. R. (2006). The value premium and the CAPM. *The Journal of Finance,* 61(5), 2163-2185. https://doi.org/10.1111/j.1540-6261.2006.01054.x

Fama, E. F., & MacBeth, J. D. (1973). Risk, Return, and Equilibrium: Empirical Tests. *Journal of Political Economy,* 81(3), 607-636. https://doi.org/10.1086/260061

Gibbons, M. R. (1982). Multivariate tests of financial models: A new approach. *Journal of Financial Economics,* 10(1), 3-27. https://doi.org/10.1016/0304-405X(82)90028-9

Gibbons M., Ross, S. A., & Shanken, J. (1989). A test for the efficiency of a given portfolio. *Econometrica,* 57(5), 1121-1152. https://doi.org/10.2307/1913625

Grinold, R. C. (1992). Are Benchmark Portfolios Efficient?. *The Journal of Portfolio Management,* 19(1), 34-40.

Habibović, A., Zoričić, D., & Lovretin Golubić, Z. (2017). Efficiency of CROBEX and CROBEX10 stock market indices. *UTMS Journal of Economics,* 8(3), 271-280.

Harvey, C. R., & Zhou, G. (1990). Bayesian inference in asset pricing tests. *Journal of Financial Economics,* 26(2), 221-254. https://doi.org/10.1016/0304-405X(90)90004-J

Haugen, R. A., & Baker, N. L. (1991). The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios. *The Journal of Portfolio Management,* 17(3), 35-40. https://doi.org/10.3905/jpm.1991.409335

Israelsen, C. L. (2005). A refinement to the Sharpe ratio and information ratio. *Journal of Asset Management,* 5(6), 423-427. https://doi.org/10.1057/palgrave.jam.2240158

Jagannathan, R, & Wang, Z. (1996). The conditional CAPM and the cross-section of expected returns. *The Journal of finance,* 51(1), 3-53. https://doi.org/10.1111/j.1540-6261.1996.tb05201.x

Kandel, S., McCulloch, R., & Stambaugh, R. F. (1995). Bayesian Inference and Portfolio Efficiency. *Review of Financial Studies,* 8(1), 1-53.

Kind, C., & Poonia, M. (2015). Comparing diversification management strategies. *Global Economy and Finance Journal,* 8(1), 67-81. https://doi.org/10.21102/GEFJ.2015.03.81.05

Ledoit, O., & Wolf, M. (2004). Honey, I shrunk the sample covariance matrix. *Journal of Portfolio Management,* 30(4), 110-119. https://doi.org/10.2139/ssrn.433840

Maillard, S., Roncalli, T., & Teiletche, J. (2010). The Properties of Equally Weighted Risk Contribution Portfolios. *Journal of Portfolio Management,* 36(4), 60-70. https://doi.org/10.2139/SSRN.1271972

Malkiel, B. G. (2014). Is Smart Beta Really Smart?. *The Journal of Portfolio Management,* 40(5), 127-134. https://doi.org/10.3905/jpm.2014.40.5.127

Martellini, L., & Milhau, V. (2018). Proverbial Baskets Are Uncorrelated Risk Factors! A Factor-Based Framework for Measuring and Managing Diversification in Multi-Asset Investment Solutions. *The Journal of Portfolio Management,* 44(2), 8-22.

Meucci, A., Santangelo, A., & Deguest, R. (2015). Measuring Portfolio Diversification Based on Optimized Uncorrelated Factors. *Risk,* 11(29), 70-75.

Shanken, J. (1985). Multivariate tests of the zero-beta CAPM. *Journal of financial economics,* 14(3), 327-348. https://doi.org/10.1016/0304-405X(85)90002-9

Shanken, J. (1987). Multivariate proxies and asset pricing relations: Living with the Roll critique. *Journal of Financial Economics,* 18(1), 91-110. https://doi.org/10.1016/0304-405X(87)90062-6

Stambaugh, R. F. (1982). On the exclusion of assets from tests of the two-parameter model: A sensitivity analysis. *Journal of financial economics,* 10(3), 237-268. https://doi.org/10.1016/0304-405X(82)90002-2

Zhou, G. (1981). Small sample tests of portfolio efficiency. *Journal of Financial Economics,* 30(1), 165-191.

Zoričić, D., Dolinar, D., & Kožul, A. (2014). The Market Index Benchmark and Adequate Compensation for Systematic Risk in an Illiquid and Undeveloped Financial Market. In D. Miloš Sprčić (Ed.), *Risk Management: Strategies for Economic Development and Challenges in the Financial System* (p. 257-277). New York, NY: Nova Science Publishers.

Zoričić, D., Dolinar, D., & Lovretin Golubić, Z. (2018). A test of global minimum variance portfolio in the Croatian capital market. In A. Mašek Tonković, & B. Crnković (Eds.), *Book of proceedings of 7th International Scientific Symposium ECONOMY OF EASTERN CROATIA - VISION AND GROWTH* (p. 1165-1173). Osijek: University of Josip Juraj Strossmayer in Osijek, Faculty of Economics in Osijek.

Zoričić, D., Dolinar, D., & Lovretin Golubić, Z. (2020). Factor-based optimization of a fundamentally-weighted portfolio in the illiquid and undeveloped stock market. *Journal of Risk and Financial Management,* 13(12), 302. https://doi.org/10.3390/jrfm13120302

Heuristički pristup procjeni efikasnog *benchmark* indeksa na hrvatskom dioničkom tržištu

Sažetak

U radu se primjenjuje heuristički princip koji se isključivo temelji na procjeni parametara rizika kako bi se procijenio efikasan benchmark indeks na hrvatskom dioničkom tržištu. Metoda optimizacije koja se primjenjuje usmjerena je na paritet rizika. Kako bi se postigla optimalna diverzifikacija portfolija paritet rizika omogućuje investitorima diverzifikaciju rizika kroz jednak doprinos promatranih dionica u riziku ukupnog portfoliija. U radu se ispituje šest različitih indeksa povezanih s varijacijama metode pariteta rizika i procjenama matrice kovarijanci kako bi se njihove performanse usporedile s indeksom koji se temelji na tržišnoj kapitalizaciji. Na taj način omogućen je uvid u potencijalne izvore razlika u njihovim karakteristikama mjereno odnosom rizika i nagrade. Istraživanje se temelji na 28 procjena izvan uzorka za razdoblje od travnja 2005. godine do ožujka 2019. godine. Rezultati ne pokazuju da bi metoda pariteta rizika mogla omogućiti izloženost nagrađenim faktorima rizika na hrvatskom tržištu dionica. Štoviše, vezano uz diverzifikaciju nenagrađenih faktora rizika, čak je i benchmark indeks s najnižom volatilnošću volatilniji u odnosu na indeks CROBEX. Međutim, ako se analizira samo razdoblje ekspanzije unutar ukupnog promatranog razdoblja, svi procijenjeni benchmark indeksi ostvaruju bolje performanse u odnosu na indeks CROBEX pri čemu portfolio procijenjen metodom faktorskog pariteta rizika koji se temelji na dvije ili više komponenti pokazuje najnižu volatilnost. Općenito, rezultati istraživanja pokazuju kako indeksi temeljeni na paritetu rizika nadmašuju performanse indeksa s jednakim udjelima sastavnica te da pretpostavka o jednakim korelacijama sastavnica indeksa ili primjena metode statističkog sažimanja za njihovu procjenu daje bolje rezultate od korištenja metode glavnih komponenti.

Ključne riječi: procjena efikasnog *benchmark* indeksa; paritet rizika; faktorski paritet rizika; metoda glavnih komponenti; statističko sažimanje; upravljanje imovinom