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SOME CONSIDERATIONS CONCERNING THE FIRST-KIND SUPERCONDUCTORS

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The dependence between interaction constant and the thickness of the superconductive momentum layer was analysed in the paulion as well as in the boson picture. It was shown (in the both approaches) that in better superconductors the same W leads to higher value of q_G. It was also shown that in the boson picture the values of q_G, smaller than those obtained in the paulion picture, lead to the same critical temperature.

1. Introduction

Many papers appeared in last thirty years dealing with superconductivity problem. The *BCS* theory[1] as well as the works of Bogoliubov[2] gave satisfactory explanation of superconductivity phenomena. The question remained: what kinematics obey the operators creating and annihilating the Cooper's pairs. It can be easily shown that the mentioned operators are the Pauli operators in the momentum space. Consequently, they cannot be treated as bosons as it was proposed, for example in Ref. 3. In accordance with Ref. 4, a system of paulions can be treated as boson system but with a series of multiparticle boson interactions.

Our intention is to compare the results obtained in the paulion picture of a first kind superconductor to corresponding ones obtained in the boson picture. Analyses in the boson picture will be given by the use of formulae from Ref. 4. It turns that the both pictures give approximately same results and this will be demonstrated here. The results obtained will be used, together with the experimental data for some characteristic first superconductors, for investigations of mutual relation between the thickness of superconductive layer in momentum space and the constant of electron-phonon coupling.

3. *Paulion and boson picture of the first-kind superconductor*

We start from the *BCS* Hamiltonian, taken in second quantization representation. The form of this Hamiltonian is the following:

$$H = \sum_{\vec{k}} X_{\vec{k}} \left[a^+_{\vec{k}\uparrow} a_{\vec{k}\uparrow} + a^+_{-\vec{k}\downarrow} a_{-\vec{k}\downarrow}\right] - \frac{W}{2N} \sum_{\vec{k}\,\vec{q}} a^+_{\vec{k}\uparrow} a^+_{-\vec{k}\downarrow} a_{-\vec{q}\downarrow} a_{\vec{q}\uparrow}. \tag{2.1}$$

In the formula (2.1) $a^+_{\vec{k}\uparrow}$ and $a_{\vec{k}\downarrow}$ are the creation and annihilation operators of electrons with the wave vector $\vec{k}$ in the given spin state, W is the constant of electron-electron interaction in the momentum layer between $\hbar\,(q_F - q_G)$ and $\hbar\,(q_F + q_G)$, where q_F is boundary momentum of Fermi sphere and q_G defines the thickness of the superconductive momentum layer. The function $X_{\vec{k}}$ is the difference between kinetic energy of electrons and its chemical potential, i. e.

$$X_{\vec{k}} = \frac{\hbar^2 k^2}{2m} - \mu \approx \frac{\hbar^2 k^2}{2m} - \frac{\hbar^2 k_F^2}{2m}$$

where m is the electron mass. The number of atoms in the crystal is denoted by N.

It can be easily shown that the Hamiltonian (2.1) is closed in the fermion subspace

$$h = \{|\, 0_s\, 0_{-s}\rangle,\ |\, 1_s\, 1_{-s}\rangle\},\ s \in (\vec{k}\uparrow, -\vec{k}\downarrow). \tag{2.2}$$

Introducing the operators

$$P_s = a_{-s}\, a_s,\ P_s^+ = a_s^+\, a_{-s} \tag{2.3}$$

one can easily prove that they obey the Pauli-commutation relations in the subspace h. These commutation relations are the following:

$$\begin{gathered}[P_s, P_{s'}^+] = (1 - 2\, P_s^+\, P_s)\, \delta_{s,s'};\ [P_s, P_{s'}] = [P_s^+, P_{s'}^+] = 0 \\ P_s^2 = P_s^{+2} = 0;\ P_s^+\, P_s = a_s^+\, a_s = a_{-s}^+\, a_s.\end{gathered} \tag{2.4}$$

After the substitution of (2.3) into (2.1) we obtain the following form of the Hamiltonian (2.1), now expressed in terms of Pauli-operators P

$$H = \sum_{\vec{k}} 2X_{\vec{k}}\, P^+_{\vec{k}} P^+_{\vec{k}} - \frac{W}{2N} \sum_{\vec{k}\vec{q}} P^+_{\vec{k}} P_{\vec{q}}. \tag{2.5}$$

Using the canonical transformation of operators P to the new Pauli operators Q[5])

$$P_{\vec{k}} = - u_{\vec{k}} v_{\vec{k}} + 2u_{\vec{k}} v_{\vec{k}} Q^+_{\vec{k}} Q_{\vec{k}} + U^2_{\vec{k}} Q_{\vec{k}} - v^2_{\vec{k}} Q^+_{\vec{k}}$$

$$u^*_{\vec{k}} = u_{\vec{k}},\ v^*_{\vec{k}} = v_{\vec{k}},\ u_{-\vec{k}} = u_{\vec{k}},\ v_{-\vec{k}} = v_{\vec{k}},\ u^2_{\vec{k}} + v^2_{\vec{k}} = 1 \tag{2.6}$$

and introducing the Tyablikov's approximation[6]):

$$Q^+ Q Q^+ Q \approx 2 \langle Q^+ Q \rangle Q^+ Q \tag{2.7}$$

we reduce (2.5) to the final, approximate form

$$H_{eff} = \sum_{\vec{k}} \hbar \Omega_{\vec{k}} (\Theta) Q^+_{\vec{k}} Q_{\vec{k}}$$

$$\hbar \Omega_{\vec{k}} (\Theta) = \sqrt{X^2_{\vec{k}} + \Delta^2_{\vec{k}} (\Theta)},\ \Delta_{\vec{k}} (\Theta) = \frac{W}{2N} \sum_{\vec{q}} u_{\vec{q}} v_{\vec{q}} \sigma_{\vec{q}} \tag{2.8}$$

$$\sigma_{\vec{q}} = 1 - 2\overline{N}_{\vec{q}},\ \overline{N}_{\vec{q}} = \langle Q^+_{\vec{q}} Q_{\vec{q}} \rangle.$$

For the Hamiltonian (2.8) the Green's function

$$\Gamma_{\vec{k}} (\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{d}t \ll Q^+_{\vec{k}} (t) \mid Q_{\vec{k}} (0) \gg \mathrm{e}^{i\omega t} \tag{2.9}$$

will be calculated. The result of relatively simple calculations is the following:

$$\Gamma_{\vec{k}} (\omega) = \frac{\mathrm{i}}{2\pi} \frac{\sigma_{\vec{k}}}{\omega - \Omega_{\vec{k}} (\Theta)},\ \overline{N}_{\vec{k}} = \sigma_{\vec{k}} \left(\exp \frac{\hbar \Omega_{\vec{k}} (\Theta)}{\Theta} - 1 \right)^{-1}. \tag{2.10}$$

Using the usual approximations of the superconductivity theory, with the assumption that $\Omega_{\vec{k}} (\Theta)$ approximately does not depend on $\vec{k}$ we obtain the following final result, defining the superconductive critical temperature $\Theta_C = K_B T_C$

$$\Delta (\sigma) = \Delta_0 (\sigma^2 - \sigma_c^2)^{1/2},\ \sigma = \mathrm{th} \frac{\Delta_0 \sigma}{2\Theta}$$

$$\Delta_0 = \frac{M q_F^2 q_G W}{4\pi^2 \varrho},\ \sigma_C = \frac{4\pi^2 \hbar^2 \varrho}{m M q_F W},\ \hbar \Omega (\Theta) = \Delta_0 \sigma. \tag{2.11}$$

In the last formula M is the mass of atom and $\varrho = \frac{MN}{V}$ is the density of the crystal.

The results (2.11) showing that for $\sigma = \sigma_C$ follows $\Theta = \Theta_C$ will be discussed later by the use of well-known experimental data for a sequence of first-kind superconductors. Here we shall proceede our investigations writting the Hamiltonian (2.9) in terms of Bose-operators B and B^+.

Using the formulae from Ref. 4

$$Q_{\vec{k}} = \left[\sum_{\nu=0}^{\infty} \frac{(-2)^\nu}{(1+\nu)!} B_{\vec{k}}^{+\nu} B_{\vec{k}}^{\nu}\right]^{1/2} B_{\vec{k}} \equiv Q_{\vec{k}}^{(B)}$$
$$Q_{\vec{k}}^{+} Q_{\vec{k}} = \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{(1+\nu)!} B_{\vec{k}}^{+\nu+1} B_{\vec{k}}^{\nu+1} \tag{2.12}$$

we obtain the Hamiltonian (2.8) in Bose-representation

$$H_{eff} = \sum_{\vec{k}} \hbar \Omega_{\vec{k}}(\Theta) \sum_{\nu=0}^{\infty} \frac{(-1)^\nu}{(1+\nu)!} B_{\vec{k}}^{+\nu+1} B_{\vec{k}}^{\nu+1}. \tag{2.13}$$

In further we follow the approach used in Ref. 7. Instead of the boson equivalent of Γ, i. e.:

$$\Lambda(\vec{k}, t) = \langle\langle [\sum_{\nu=0}^{\infty} B_{\vec{k}}^{+\nu}(t) B_{\vec{k}}^{\nu}(t)]^{1/2} B_{\vec{k}}(t) \mid B_{\vec{k}}^{+}(0) [\sum_{\nu=0}^{\infty} B_{\vec{k}}^{+\nu}(0) B_{\vec{k}}^{\nu}(0)] \rangle\rangle \tag{2.14}$$

the artificial Green's function

$$\Psi(\vec{k}, t) = \langle\langle \sqrt{1 + \hat{n}_{\vec{k}}}\, Q_{\vec{k}}^{(B)}(t) \mid Q_{\vec{k}}^{+(B)}(0) \sqrt{1 + \hat{n}_{\vec{k}}} \rangle\rangle, \quad \hat{n}_{\vec{k}} = B_{\vec{k}}^{+} B_{\vec{k}} \tag{2.15}$$

will be calculated. During the calculations of Ψ only the terms proportional to $G_{\vec{k}}(t) D_{\vec{k}}(t) G_{\vec{k}}(t)$, where

$$G_{\vec{k}}(t) = \langle\langle B_{\vec{k}}(t) \mid B_{\vec{k}}^{+}(0) \rangle\rangle, \quad D_{\vec{k}}(t) = \langle\langle B_{\vec{k}}^{+}(t) \mid B_{\vec{k}}(0) \rangle\rangle \tag{2.16}$$

were taken account. The higher order terms in G and D are neglected.

The approximative connection betwen Fourier components of $\Psi(\vec{k}, t)$ and $\Lambda(\vec{k}, t)$ (according to Ref. 7) is the following

$$\Lambda(\vec{k}, \omega) = \sigma_B(\vec{k})\, \Psi(\vec{k}, \omega). \tag{2.17}$$

Using (2.17) and the results from Ref. 7 for Ψ we finally find the following expression for boson Green's function of the svystem

$$G(\vec{k}, \omega) = \frac{i}{2\pi} \frac{A_{\vec{k}} - \frac{1}{2} A^4_{\vec{k}} \sigma_B^2(\vec{k})}{\omega - \Omega_{\vec{k}} + i\delta}, \quad A_k = \frac{1}{\sigma_B(\vec{k}) \left[1 + \left(\frac{1 - \sigma_B^2(\vec{k})}{2}\right)^2\right]}. \tag{2.18}$$

With help of (2.18), after neglecting of space dispersion, the folowing final result for ordering parameter is obtained

$$\frac{1}{\sigma_B} = 1 + \left\{ \frac{2}{\sigma_B \left[1 + \left(\frac{1 - \sigma_B^2}{2} \right)^2 \right]} - \frac{\sigma_B^3}{\left[1 + \left(\frac{1 - \sigma_B^2}{2} \right)^2 \right]^4} \right\} \frac{1}{\exp \frac{\hbar \Omega (\Theta)}{\Theta} - 1} \cdot \tag{2.19}$$

The formula (2.19) is the boson-equivalent of the formula (2.11) and these results will be compared to in the following Section.

3. *Discussion of results and conclusion*

The results (2.11) and (2.19) will be used for finding of the dependence between constant of electron-phonon interaction W and the value q_G, defining the thickness of the superconductive layer in the momentum space.

The experimental data for first-kind superconductors Pb, In, Sn, Tl, Zn, Al, Ga and Cd are taken from Ref. 8 and they are exposed in Table 1.

TABLE 1.

element	$M \cdot 10^{-27}$ kg	$\varrho \cdot 10^3$ kg/m^3	$q_F \cdot 10^{10}$ m^{-1}	T_C/K
Pb	136.12	11.34	0.835	7.22
In	81.34	7.29	0,797	3.37
Sn	83.00	5.76	0.867	3.74
Tl	134.46	11.87	0.774	2.39
Zn	49.80	7.13	0.834	0.93
Al	21.58	2.70	0.928	1.20
Ga	51.46	5.91	0.878	1.10
Cd	79.68	8.65	0.742	0.56

The numerical calculations of

$$q_G = f(W) \tag{3.1}$$

are graphically expressed in Fig. 1.

In Fig. 1, the results obtained by the use of formulae (2.11) (the paulion picture) are denoted by full lines. The corresponding results obtained in the boson picture (from the formulae (2.19)) are expressed by broken lines.

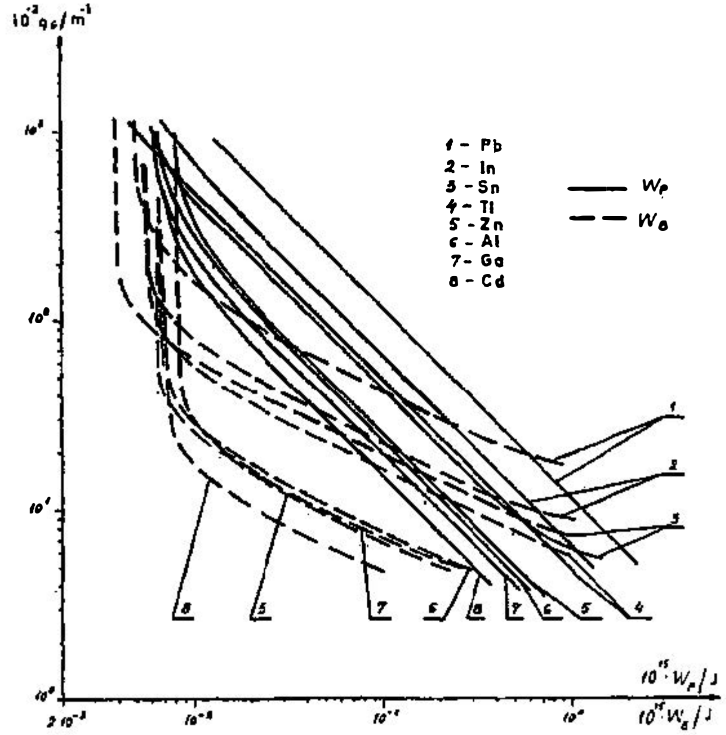

Fig. 1.

Analysing the results obtained one can conclude the following:

a) For the same value of interaction constant W the magnitude of q_G is higher for superconductors having the higher critical temperature. This is valid for the both formulae ((2.11) and (2.19)) with some rare exclusions. The general conclusion follows: in defining of magnitude of q_G, and consequently of Θ_C, the interaction constant W plays dominant role.

b) In boson picture the magnitude of q_G is generally smaller then in the paulion one, for the same W. Since Θ_C is proportional to q_G one can conclude that the approach leads to more optimistic conclusion than the paulion one: the same Θ_C can be reached with lower q_G than predicted by the paulion approach. Taking into account that in calculations with bosons the more approximations were used than in the paulion approach, we are of opinion that the results obtained according to (2.11) earn more confidence than those from the formula (2.19).

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NEKA RAZMATRANJA O SUPERPROVODNIKU PRVE VRSTE

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U radu je tražena zavisnost između konstante efektivne elektron-elektron interakcije i širine superkonduktivnog sloja Fermi sfere, i to kako u paulinskoj tako i u bozonskoj slici. Pokazano je i u jednoj i u drugoj slici da se za iste vrednosti W dobivaju veće vrednosti za q_G za supraprovodnike sa većom kritičnom temperaturom (osim manjih izuzetaka). Dalje je pokazano da u bozonskoj slici vrednosti q_G koje su manje od onih dobijenih u paulinskoj slici dovode do iste kritične temperature.