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LETTERS TO THE EDITOR

DYSON QUADRUPOLE BOSON REPRESENTATION OF SU (6) HAMILTONIAN

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A new SU (6) Hamiltonian is obtained, to be referred to as Finite Quadrupole Phonon Model (*FQM*), built from the generators of the SU (6) collective algebra in Dyson realization. *FQM* is a particular nonhermitean sixth-order anharmonic quadrupole phonon model. This is the third type of bosonizations characterized by the SU (6) symmetry; the two others are Schwinger realization (*IBM*) and Holstein-Primakoff realization (*TQM*).

In the last decade two new collective models have been introduced, based on SU (6) symmetry: *TQM*[1], a particular quadrupole phonon model, and *IBM*[2], a particular *s*, *d*-boson model. *IBM* and *TQM* correspond to the Schwinger and Holstein-Primakoff realization of the SU (6) quadrupole collective algebra and are equivalent on the levels of matrix elements and operators[2,3-8].

Deductive inference of the SU (6) phenomenological boson models can be based on the SU (6) Quadrupole Collective Algebra (*QCA*), built from the collective quadrupole coordinates, the conjugated momenta and their commutators, derived in Ref. 1. Then the SU (6) collective models can be viewed as resulting from different bosonisations of *QCA*. *IBM* and *TQM* correspond to the Schwinger

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and Holstein-Primakoff realizations of QCA, respectively. In the present paper we consider the third possible boson realization of QCA, namely Dyson realization (DR).

We introduce DR of QCA employing the holomorphically parametrized generalized coherent state [9,10]

$$| C, \vec{j} > = \exp \left(\sum_{\vec{r}(\alpha) \cdot \vec{j} > 0} c_\alpha^* E_{-\alpha} \right) | \vec{j} > . \tag{1}$$

Here $|\vec{j} >$ is the state with highest weight $\vec{j}$; c_α are complex variables labelled by the index $\vec{r}(\alpha)$; $\{H_k (k = 1, ..., 5), E_\alpha (\alpha = \pm 1, ..., \pm 15)\}$ are the generators of the Cartan-Weyl canonical algebra[11] and the root vectors of QCA are used from Ref. 5.

Employing matrix representation of QCA algebra, after straightforward derivation we obtain

$$\vec{j} = \left(\frac{1}{2\sqrt{6}}, \frac{1}{6\sqrt{6}}, \frac{1}{12}, \frac{1}{4\sqrt{15}}, \frac{1}{6\sqrt{20}} \right) \tag{2}$$

$$|\vec{j} > \mathrm{col}\ (1, 0, 0, 0, 0, 0).$$

For convenience we introduce the root lables $\alpha = (\pm \beta, \pm \gamma)$ with $\beta = 1, ..., 5$ and $\gamma = 6. ..., 15$.

The highest weight $\vec{j}$ has divided the roots $\vec{r}(\alpha)$ and analogously the raising (lowering) generators, into 3 groups:

i) Five simple negative roots $\{\vec{r}(-\beta)\}$ for which $\vec{r}(-\beta) \cdot \vec{j} < 0$,

ii) Twenty nonsimple roots $\{\vec{r}(\pm \gamma)\}$ for which $\vec{r}(\pm \gamma) \cdot \vec{j} = 0$,

iii) Five simple positive roots $\{\vec{r}(\beta)\}$ for which $\vec{r}(\beta) \cdot \vec{j} > 0$.

Following Ref. 9, we derive first the differential form (in variables C_β) for Cartan-Weyl canonical generators:

$$H_k | C, \vec{j} > = \mathscr{H}_k^* | C, \vec{j} >, \tag{3}$$

$$E_\alpha | C, \vec{j} > = \mathscr{E}_{-\alpha}^* | C, \vec{j} > . \tag{4}$$

Using explicit expressions for $\{H_k, E_\alpha\}$ and $| C, \vec{j} >$ we obtain

$$\mathscr{H}_k = [12k(k+1)]^{-1/2} \left(N - k C_k \partial_k - \sum_{i=k}^{5} C_i \partial_i \right) \tag{5}$$

$$\mathscr{E}_{\beta} = \partial_{\beta} \tag{6}$$

$$\mathscr{E}_{-\beta} = \frac{C_{\beta}}{12} (N - \sum_{\beta'} C_{\beta'} \partial_{\beta'}) \tag{7}$$

$$\mathscr{E}_{-\gamma} = \frac{1}{\sqrt{12}} C_{\beta_0} \partial_{\beta_0'} \tag{8}$$

$$\mathscr{E}_{\gamma} = \frac{1}{\sqrt{12}} C_{\beta_0'} \partial_{\beta_0}. \tag{9}$$

Here $\partial_k = \frac{\partial}{\partial C_k}$, $\partial_\beta = \frac{\partial}{\partial C_\beta}$ and β_0, β_0' are determined by the unique nonsimple root decomposition in terms of simple positive roots $\vec{r}(\gamma) = \vec{r}(\beta_0) + \vec{r}(\beta_0')$.

Now we employ the fact that multiplications by C_β and differentiations ∂_β are Bargmann space representatives of b_β^+ and b_β, respectively[9] and utilize

$$c_\beta \to b_\beta^+, \ \partial_\beta \to b_\beta. \tag{10}$$

Inserting (10) into (5) — (9) we obtain Dyson realization of QCA generators in the canonical form:

$$\mathscr{H}_k^{DR} = [12k(k+1)]^{-1/2} (N - k\, b_\mu^+ b_\mu - \hat{N}) \tag{11}$$

$$\mathscr{E}_\beta^{DR} = \frac{1}{\sqrt{12}} b_\mu \tag{12}$$

$$\mathscr{E}_{-\beta}^{DR} = \frac{1}{\sqrt{12}} b_\mu^+ (N - \hat{N}) \tag{13}$$

$$\mathscr{E}_{-\gamma}^{DR} = \frac{1}{\sqrt{12}} b_\nu^+ b_\mu \tag{14}$$

$$\mathscr{E}_\gamma^{DR} = \frac{1}{\sqrt{12}} b_\mu^+ b_\nu \tag{15}$$

with

$$\hat{N} = \sum_\nu b_\nu^+ b_\nu.$$

Here we have employed a one to one correspondence[12] between the root labels and the L-projection of the $\varkappa = 2$ angular momentum labeling quadrupole boson (phonon) operators introduced in DR.

By substituting DR for $\{H_k, E_\alpha\}$ in Eqs. (29) of Ref. 5, which express QCA generators in terms of canonic forms, we obtain DR of physical QCA generators:

$$\hat{q}_\mu^{DR} = b_\mu^+ (N - \hat{N}) b_{\tilde{\mu}}^- \tag{16}$$

$$\hat{p}_\mu^{DR} = i\,[b_{\tilde{\mu}}^{+} - (N - \hat{N})\, b_\mu] \tag{17}$$

$$i\,[\hat{q}_\mu, \hat{p}_{\mu'}]^{DR} = b_\mu^+ b_{\mu'} + b_{\tilde{\mu}'}^{+} b_{\tilde{\mu}}^- - 2\delta_{\mu\mu'} (\hat{N} - N) \tag{18}$$

$$[\hat{q}_{\mu'}, \hat{q}_{\mu'}]^{DR} = b_\mu^+ b_{\tilde{\mu}'}^- - b_{\mu'}^+ b_{\tilde{\mu}}^-. \tag{19}$$

Substituting (16) — (19) into H_{QCA}[1], we obtain Dyson realization of QCA, to be referred to as H_{FQM} (Finite Quadrupole Phonon Model — FQM):

$$\begin{aligned} H_{FQM} = H_{QCA}^{DR} &= ie \sum_\mu [\hat{q}_{\mu'}, \hat{p}_\mu]^{DR} + s\,\hat{q}^{DR} \cdot \hat{q}^{DR} + v\,\hat{p}^{DR} \cdot \hat{p}^{DR} + \frac{1}{2}\,i\,w\,\hat{q}^{DR} \cdot \\ &\cdot ([\hat{q}, \hat{p}]^{DR})_2 - \frac{1}{4} \sum_L \frac{1}{2} [1 + (-)^L]\, d_L \{[\hat{q}, \hat{p}]^{DR}\}_L \cdot \{[\hat{q}, \hat{p}]^{DR}\}_L - \\ &- \frac{1}{4} \sum_L \frac{1}{2} [1 - (-)^L]\, d_L \{[\hat{q}^{DR}, \hat{q}^{DR}]\}_L \cdot \{[\hat{q}^{DR}, \hat{q}^{DR}]\}_L = \\ &= 2e\hat{N} - 10e (N - \hat{N}) + d_0 (5 - 2\hat{N})(N - \hat{N}) + \\ &+ (s + v)\,[(5 + 2\hat{N})(N - \hat{N}) + \hat{N}] + \\ &+ (s - v)\,[b^+ \cdot b^+ (N - \hat{N})(N - \hat{N} - 1) + \tilde{b} \cdot \tilde{b}] + \\ &+ w\,[(b^+ b^+)_2 \cdot \tilde{b}\,(N - \hat{N}) + b^+ \cdot (\tilde{b}\tilde{b})_2] + \\ &+ \sum_L (-)^L d_L (b^+ \tilde{b})_L \cdot (b^+ \tilde{b})_L. \end{aligned} \tag{20}$$

Here $[e, s, v, w, d_L]$ are parameters in the Hamiltonian.

H_{FQM} is a nonhermitean sixth-order quadrupole phonon Hamiltonian. For $s = v$, $w = 0$ the nonhermitean terms disappear, leaving an SU (5) type Hamiltonian.

We point out that H_{FQM}, besides being interesting in itself, plays an important role as an integral part of the deductive inference of the full scope phenomenological boson models based on SU (6) symmetry: Orthogonalizing Dyson realization (= FQM) one can derive Holstein-Primakoff realization (= TQM). Acting with the appropriate bijective dicontinuous intertwinning operator on Holstein-Primakoff realization one obtains Schwinger realization (= IBM); on the other hand, the Schwinger realization of QCA can be directly constructed. This complete SU (6) collective scope as well as its extensions to boson fermion system[7] will be fully laborated in a forthcoming publications[12].

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DYSONOVA KVADRUPOLNO BOZONSKA REPREZENTACIJA ZA SU (6) HAMILTONIJAN

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Dobiven je novi SU (6) Hamiltonijan, konačni kvadrupolno-fononski model (*FQM*), građen iz generatora SU (6) kolektivne algebre u Dysonovoj realizaciji. *FQM* je specifični nehermitski anharmonijski kvadrupolno-fononski model šestog reda. To je treći tip bozonizacije koji je karakteriziran sa SU (6) simetrijom; druga dva tipa su Schwingerova realizacija (*IBM*) i Holstein-Primakoffova realizacija (*TQM*).