YU ISSN 0015—3206
FZKAAA 16 (3) 309 (1984)

THE ANISOTROPY OF THE SPEED OF LIGHT IN THE FRAMEWORK OF THE SPECIAL THEORY OF RELATIVITY

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Received 29 February 1984

UDC 530.12

Original scientific paper

The reasons why in the framework of the special theory of relativity the speed of light can be by convention assumed to be anisotropic, although the one-way speed of light can not be measured, are discussed in details.

1. Introduction

Relatively recently arose again the discussion about the possibilities to measure the one-way speed of light. Experiments for its measurement are proposed[1] and arguments against the thesis of conventional character of simultaneity have been presented[2]. Although the idea of the conventionality of simultaneity dates back to Poincaré[3], it seems that its consequences are neither widely known nor well understood, and that, for this reason, there is much confusion about this question in the recent literature.

The purpose of this paper is to give the analysis, and interpretation of possible conventions for angular dependence of the speed of light which are compatible with the special theory of relativity and to show distinctly why in its framework it is impossible to single out on the experimental grounds any of them as more adequate.

In the next Section we shall derive, using simple arguments, a general expression for the speed of light as a function of angle between the considered direction and the assumed privileged direction in space, which by convention may be used in the framework of special relativity. Some special cases will also be discussed. Such conventions for the speed of light imply, of course, changes in the description of other physical processes, which for the case of velocities will be discussed in the same Section.

Interpretation of the obtained formulae for the speed of light will be given in Section 3, and with a help of it, physical reasons for which it is not possible to measure the one-way speed of light in the framework of the special theory of relativity, will be underlined.

In the light of the obtained results, we shall discuss in Section 4, the arguments against the hypothesis of conventionality of simultaneity, given in Ref. 2, and show these arguments are wrong. We shall discuss the possibile sense, which the proposed experiments for the measurement of one-way speed of light, may have, from our point of wiev.

We hope that the detailed explanations presented in this paper will contribute to better understanding of the conventional character of simultaneity.

2. *The allowed anisotropy of the speed of light in the framework of the special theory of relativity*

It is well known that according to the special theory of relativity in any inertial frame of reference the standard synchronization of cloks may be introduced, and that in such synhronization the speed of light is constant. We are going to show that besides this synchronization some other, different synchronization, can be introduced which are exactly in the same relation with experiment.

Let us take one inertial frame of reference in which both the standard and nonstandard synchronizations are introduced, so that in this frame we have two systems of coordinates S and $\bar{S}$, respectively. For all the physical quantities in the standard synchronization (in S) we shall use the usual symbols, while in the nonstandard one (in $\bar{S}$) we shall add an overbar to the correspoding symbols. The measurements of all the quantities must be in accord with the synchronization used. We shall see that in the nonstandard synchronization the velocities of all processes, which in the standard synchronization are isotropic, become anisotropic in an universal way.

Since the speed of light is not constant in the nonstandard synchronization it must have a minimum in some direction, which we will call the privileged direction. Let us find the angular dependence $\bar{c}(\bar{\Theta})$, where $\bar{\Theta}$ is the angle between the considered and the privileged direction. Although we are free in the choice of $\bar{c}(\bar{\Theta})$ to a certain degree it must satisfy some general conditions. First of all it must be such that:

a) The time interval required, for light to go around a closed conture is, in any inertial frame, independent of the sense of circulation.

This statement may be considered as an experimental fact.

Because in the new convention we change only the numerical value of the speed of light for different directions, the features of light propagation, which do not depend on this quantity, must remain the same. So, in the new convention the straight lines also remain straight. Especially the parallel straight lines, will remain parallel, so that the Euclidian fifth postulate will remain valid. Due to this fact the one-way speed of light, by which the geometric metric relations (i. e. the measurement of lenghts and angles) are defined, may be chosen in a noncontradictory way only if the sum of angles in a triangle remains π, because this theorem is equivalent to the Euclidian fifth postulate[4]. It may be easily shown that this condition together with the condition a) implies another condition which is more convenient for our purpose, namely that:

b) The right-angled triangle with a cathetus orientated along the privileged direction remains right-angled.

The conditions a) and b) are sufficient for the determination of $\bar{c}(\bar{\Theta})$.

Without the loss of generality we can use a triangle ABC as a closed conture. For the difference of times $\tau^{\pm}$, which the light needs to go around the triangle in two opposite directions, we have

$$\tau^{+} - \tau^{-} = \overline{AB}\left[\frac{1}{\bar{c}_{AB}} - \frac{1}{\bar{c}_{BA}}\right] + \overline{BC}\left[\frac{1}{\bar{c}_{BC}} - \frac{1}{\bar{c}_{CB}}\right] + \overline{AC}\left[\frac{1}{\bar{c}_{AC}} - \frac{1}{\bar{c}_{CA}}\right].$$

This difference is zero, i. e. the condition a) is fulfilled, if the speed of light satisfies the relation

$$\frac{1}{\bar{c}(\bar{\Theta})} = \frac{1}{c}\,[f(\bar{\Theta}) + cR\cos\bar{\Theta}], \quad \text{with } f(\bar{\Theta} + \pi) = f(\bar{\Theta}) \text{ and } R = const. \tag{1}$$

If changing the sychronization we do not change the units of time, we can easily obtain the relation between lenghts in S and $\bar{S}$, because the time which the light needs to cover a certain distance in one direction and back, is the same in both systems, and equal to

$$\frac{2l}{c} = \frac{\bar{l}}{\bar{c}(\bar{\Theta})} + \frac{\bar{l}}{\bar{c}(\bar{\Theta} + \pi)} = \frac{2\bar{l}}{c} f(\bar{\Theta}) \tag{2}$$

so that

$$l = \bar{l} f(\bar{\Theta}). \tag{3}$$

[The last equality in (2) is obtained using (1)].

We shall determine $f(\bar{\Theta})$ with the help of the condition b).

Using (3) and expressing the catheti of the right-angled triangle in $\bar{S}$ through the hypotenuse and the angle $\bar{\Theta}$, we get from the Pithagorian theorem

$$f^2(\bar{\Theta}) = f^2(0)\cos^2\bar{\Theta} + f^2\left(\frac{\pi}{2}\right)\sin^2\bar{\Theta}$$

which may be written as $f(\bar{\Theta}) = P\sqrt{1 - Q\sin^2\bar{\Theta}}$, where P and Q are constants. So the general relation for the anisotropic speed of light becomes

$$\frac{1}{\bar{c}(\bar{\Theta})} = \frac{1}{c}\left[P\sqrt{1 - Q\sin^2\bar{\Theta}} + cR\cos\bar{\Theta}\right]. \tag{4}$$

The relations between lenghts and times in systems S and $\bar{S}$ are defined through the constants P, Q and R. As we shall see the most interesting case for interpretation is that when the constants P, Q and R are chosen so that $P^2 Q = c^2 R^2$ and $P/(P^2 - c^2 R^2) = 1$. Then after introduction of a new constant v, through $Q = v^2/c^2$, the expression (4) becomes:

$$\bar{c}(\bar{\Theta}) = c\left[\left(1 - \frac{v^2}{c^2}\sin^2\bar{\Theta}\right)^{1/2} - \frac{v}{c}\cos\bar{\Theta}\right]. \tag{5}$$

With such a choice for $\bar{c}(\bar{\Theta})$, the formula (3) becomes

$$l = \bar{l}\,\frac{\left(1 - \frac{v^2}{c^2}\sin^2\bar{\Theta}\right)^{1/2}}{\left(1 - \frac{v^2}{c^2}\right)}. \tag{6}$$

From (6) we may directly obtain the formula for $\cos\Theta$ which we will need latter (let us recall that Θ and $\bar{\Theta}$ are angles measured relative to the privileged direction in S and $\bar{S}$, respectively)

$$\cos\Theta = \frac{\cos\bar{\Theta}}{\left(1 - \frac{v^2}{c^2}\sin^2\bar{\Theta}\right)^{1/2}}. \tag{7}$$

Orienting the coordinates x and $\bar{x}$ along the privileged direction we obtain

$$x = \frac{\bar{x}}{\left(1 - \frac{v^2}{c^2}\right)}. \tag{8}$$

The relation between $\bar{t}$ and t may be obtained as follows

$$\bar{t} = \frac{\bar{l}}{\bar{c}} = \frac{\bar{l}}{c\left[\left(1 - \frac{v^2}{c^2}\sin^2\bar{\Theta}\right)^{1/2} - \frac{v}{c}\cos\bar{\Theta}\right]} =$$

$$-\frac{\bar{l}\left(1-\frac{v^2}{c^2}\sin^2\bar{\Theta}\right)^{1/2}}{c\left(1-\frac{v^2}{c^2}\right)}+\frac{v}{c^2}\frac{\bar{l}\cos\bar{\Theta}}{\left(1-\frac{v^2}{c^2}\right)}$$

which, with the help of (6) and (8), becomes

$$\bar{t}=t+\frac{v}{c^2}x. \tag{9}$$

If in S we consider any physical process which is described isotropically i. e. has a constant velocity u, the corresponding velocity in $\bar{S}$ becomes

$$\bar{u}(\bar{\Theta})=\frac{\Delta\bar{l}}{\Delta\bar{t}}=\frac{\Delta l\left(1-\frac{v^2}{c^2}\right)^{1/2}\Big/\left(1-\frac{v^2}{c^2}\sin^2\bar{\Theta}\right)^{1/2}}{\Delta t+\frac{v}{c^2}\Delta x}.$$

Now $\Delta x/\Delta t=\Delta l/\Delta t\cos\Theta=v\cos\Theta$, and using (7), we get

$$\bar{u}(\bar{\Theta})=\frac{u\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{v^2}{c^2}\sin^2\bar{\Theta}\right)^{1/2}+\frac{uv}{c^2}\cos\bar{\Theta}}. \tag{10}$$

This is one of the consequences of the nonstandard convention for the one way velocity of light. In the next section, using the obtained results we shall show that, inside one frame of reference, it is not possible to decide experimentally whether the usual convention $c=\text{const}$, or any of the general conventions (4), is more adequate.

3. *The interpretation of the obtained results*

Let us introduce together with the system of coordinates $\bar{S}$ another system of coordinates $\bar{S}_1$ which differs from S only by units of lenght and time. These new units are multiplied by the same factor so that we have

$$\bar{t}_1=\frac{\bar{t}}{\sqrt{1-\frac{v^2}{c^2}}},\quad \bar{l}_1=\frac{\bar{l}}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{11}$$

With this change of units, all the expressions for velocities will remain unchanged.

Let us consider first the case when the speed of light is given with (5). From (11), (9) and (8) we have

$$\bar{t}_1 = \frac{t + \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \bar{x}_1 + v\,\bar{t}_1 = \frac{x + vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \bar{y}_1 = y, \; \bar{z}_1 = z.$$

Now, introducing $x' = \bar{x}_1 + v\,\bar{t}_1$, $y' = \bar{y}_1$, $t' = \bar{t}_1$ we immediately see that (x', t') and (x, t) are connected through Lorentz transformations while (x', t') and $(\bar{x}_1, \bar{t}_1)$ are connected through Galilean transformations. In the system of coordinates $(\bar{x}_1, \bar{t}_1)$ the speed of light is described anisotropically by (5), and we see that this anisotropy may be interpreted as a consequence of Galilean transfromations. In other words, the anisotropy of the speed of light in one system, may always be interpreted as a consequence of the use of the same synchronization as in another system which moves with the constant velocity v relatively to the first one, and in which the propagation of light is described isotropically. [It may easily be seen that also the most general case (4) may be reduced to the case (5) by introduction of the appropriate change of the local time (in the form (9)), and by changing the unit of lenght. Due to this, apart from the above mentioned trivial transformations, this case may be interpreted in the same way.]

Contrary to the statements of many textbooks that Galilean transformation contradict the special theory of relativity, all the effects of special relativity may be exactly reproduced by these transformations[5]. In this way the interpretation of the obtained formulae for the speed of light shows that the new convention, as far as experiment is concerned, is equivalent to the standard one, i. e. it is compatible with the special relativity. The standard convention is of course, simpler.

Now we are in the position to clearly see the logical status of the one way speed of light in the framework of the special relativity. First of all this speed has conventional character. In order to be noncontradictory, this convention for the speed of light must satisfy the conditions a) and b), or some of their equivalents. We have shown that these conditions are necessary. The interpretation shows that at the same time, they are sufficient.

It is now easy to understand why in the framework of the special theory of relativity it is not possible to measure the one-way speed of light. Indeed, if this were possible, it would also mean that it is possible to measure the velocity of any frame of reference relatively to the privileged one, which in that case would be the unique frame in which the light propagates isotropically. We can also give more concrete reasons for the above statement. We might, for example, expect that it would be possible to measure the one way speed of light by direct comparison of the velocities of different physical processes in opposite directions. But, by using formula (10), we obtain for the difference of time intervals needed for some process to propagate over the same distance in two opposite directions:

$$\frac{\bar{l}}{\bar{u}(\bar{\Theta})} - \frac{\bar{l}}{\bar{u}(\bar{\Theta} + \pi)} = \frac{v/c^2}{\left(1 - \frac{v^2}{c^2}\right)} \bar{l} \cos \bar{\Theta}.$$

This difference is the same for all physicall processes as it does not depend of the velocity of the process $\bar{u}(\bar{\Theta})$. For the sum of the same time intervals we obtain

$$\frac{\bar{l}}{\bar{u}(\bar{\Theta})} + \frac{\bar{l}}{\bar{u}(\bar{\Theta} + \pi)} = \frac{2\bar{l}\left(1 - \frac{v^2}{c^2}\sin^2\bar{\Theta}\right)^{1/2}}{u\left(1 - \frac{v^2}{c^2}\right)} = \frac{2l}{u}.$$

This sum does not depend on any of the anisotropy parameters. So we see that by comparison of the velocities of different processes we can not obtain any quantity which would allow us to measure any parameter of anisotropic propagation whatsoever.

4. *The analysis of some of the proposed experiments to measure the one-way speed of light*

In this Section we shall discuss the papers of Nisim—Sabat[1] and Fung and Hsieh[2].

For our analysis the most interesting conclusion of Ref. 1 are that one can readily measure the speed of light in one direction »if it is different from $c/(1 - \alpha\cos\Theta)$« and that »a wide variety of experiments fail to yield a value for α«.

To see the meaning of these conclusions from our point of view, let us introduce the hybrid system of coordinates S_H, in which $t_H = \bar{t}$ and $l_H = l$. With such assumptions, lenghts in S_H may be measured with »standard meter«. Keeping in mind the relation between S_H, S and $\bar{S}$ and using formula (9), we obtain for the speed of light in S_H

$$c_H(\Theta) = \frac{\Delta l}{\Delta \bar{t}} = \frac{\Delta l}{\Delta t + \frac{v}{c^2}\Delta x} = \frac{c}{1 + \frac{v}{c}\cos\Theta}. \qquad (12)$$

[This formula may also be obtained directly from (4) taking $Q = 0$ and choosing P and R properly]. Now we see that the formula (12) differs from the above mentioned formula from Ref. 1 only by the sense of orientation of the privileged direction and also by the explicite interpretation for α given in (12). In the adopted way of measuring lenghts, (12) as a special case of (4), is the unique form of anisotropy of the speed of light which is compatible with the special relativity. So, if it would be possible to describe the anisotropy of the speed of light in the form different from (12) (keeping the same units of lenght) it would mean that special relativity is not valid in this case, and that one can measure the velocity of any inertial frame relatively to the privileged one and so obtain α (or vice versa). If, on the other hand, special relativity is valid and the system S_H is used, the velocity of light must necessarily be given by (12) and then it is, as we have seen, in principle, impossible to determine α experimentaly.

The main conclusion of Ref. 2, namely that the Einstein's postulate on the constancy of the speed of light can in principle be tested experimentally, is from our point of view completely wrong. Let us look at their arguments more carefully.

Treating the reflection by the plane mirror Fung and Hsieh use the principle of least time for the description of the ray propagation in an inappropriate way. The formula which they use is valid in the case of two media in which the speeds of light differ, but are constant inside each medium. In the case of the anisotropic description of light propagation we do not have the analogous situation. When one uses the principle of least time correctly, taking for the speed of light for example (4), one easily sees that all the angular dependences combine in such a way which gives an expression independent of any anisotropy parameter, contrary to the expression which Fung and Hsieh use in order to prove that the speed of light is isotropic.

Their conclusion from the first part of the same Section is also incorrect. Namely, although the reversibility of the ray part allows one to use their Fig. 2 in both cases, the interpretation must be different from the one they supply. First of all, contrary to the assumption of Fung and Hsieh, the orthogonality of two straight lines can not be defined independently from the convention of simultaneity. Indeed, the angles between some direction and the privileged one are given by formula (7). With the help of the same formula the angles between any two directions may be obtained, and in a general case the orthogonal lines do not remain orthogonal, when one changes the convention for simultaneity. Due to this, although the straight lines given in their Fig. 2 may represent the directions of the propagation of the wave fronts when the standard synchronization is used, their meaning changes when the synchronization is changed. In the last case, these same lines may represent the limits of the part of the light ray which is considered. With such interpretation using our results, straightforward calculation shows that the quantities a and b from their Fig. 2 are different for oposite directions of light propagation. So, the argument of Fung and Hsieh fails.

The situation is similar with their proposed experiment to measure the anisotropy coefficient ε from »ε—Lorentz« transformations. In order to discuss their conclusions we shall first derive the general expression for ε.

Let the light signal be emited form the origin of the chosen system of coordinates S, in the moment $t_1(0)$ and let it reach the point $(\bar{l}, \bar{\Theta})$ in the moment $t_2(\bar{l}, \bar{\Theta})$. In the same moment another light signal is emited from this pointh which reaches the origine in the moment $t_3(0)$. The times $\bar{t}_1(0)$ and $\bar{t}_3(0)$ are measured directly with the same clock, which for the determination of the time $\bar{t}_2(\bar{l}, \bar{\Theta})$ some convention about the speed of light in opposite directions must be used. Between those times there is a relation

$$\bar{t}_2(\bar{l}, \bar{\Theta}) = \bar{t}_1(0) + \varepsilon(\bar{\Theta})\,[\bar{t}_3(0) - \bar{t}_1(0)] \tag{13}$$

where

$$0 < \varepsilon(\bar{\Theta}) < 1.$$

The value $\varepsilon = 1/2$ corresponds to the assumption that the speed of light is the same in all directions. In general case we get from (13), using for the speed of light formula (5),

$$\varepsilon(\overline{\Theta}) = \frac{1}{2} + \frac{1}{2}\frac{v}{c}\cos\overline{\Theta}\left[1 - \frac{v^2}{c^2}\sin^2\overline{\Theta}\right]^{1/2}. \qquad (14)$$

Fung and Hsieh use the already defined hybrid systems S_H in which the speed of light is given by (12), so in this case ε becomes

$$\varepsilon_H(\Theta) = \frac{1}{2} + \frac{1}{2}\frac{v}{c}\cos\Theta.$$

From the last formula we see that this is exactly the case for which Fung and Hsieh claim that it can not be measured using their scheme. [It is also impossible to measure ε in the more general case (14). The lenght in this case must of course be calculated using (6). Apart from other reasons, this impossibility is also garanteed by the condition a)].

So we see that the anisotropy which can be experimentaly tested by the Fung and Hsieh scheme is incompatible with the special relativity, while the anisotropy which is allowed in the special relativity can not be measured in their scheme. Due to this, their conclusion that any anysotropy of the speed of light can be measured is erroneous.

Fung and Hsieh give another example which in their opinion shows that the thesis of conventional character of simultaneity is false. They claim that if this thesis were correct then the particles of identical masses and indentical average or round-trip velocites would have different one-way velocities in different directions, what would cause different rises of temperature after their stopping in identical calorimeters. In our opinion this conculusion is wrong. In fact, as we have seen earlier, in one frame of reference which we can think of as the physical laboratory we can introduce two systems of coordinates; one, in which the propagation of light is described isotropically, and the other in which this is not the case. We have also seen that in the latter system we have some freedom of choice of the speed of light (formula (4)). However, once we have made this choice, we are obliged to transform consistently to a new system the velocities of all physical processes. The same is true, of course, for all other physical quantities. We have also seen that both descriptions reproduce exactly the same physical effects.

If the particles from the example of Fung and Hsieh, in the system in which the isotropic description of the light propagation is adopted, produce the same temperature rise in corresponding calorimeters then it will not be possible to realize the energy transfer between these calorimeters. Since the absence of energy transfer is an absolute physical fact, in a sense that is it not dependent on the way of description, the same conclusion must remain valid also in the system with the anisotropic description. For this reason, if we in such a system keep the same temperature scale, we only have to transform the specific heat consistently in an analogous way as we transformed velocities. So we see that this example of Fung and Hsieh can not be used as an argument against the thesis of conventional character of simultaneity.

5. *Conclusion*

We have shown that in the framework of the special theory of relativity one can by convention assume that the speed of light is dependent on the direction in space. We have found the general expression for this anisotropic speed. Also, we have shown that, in the framework of special relativity, it is in principle impossible to measure one-way speed of light. Due to this fact, the assumptions of all the experiments which are being proposed with a goal to measure the one-way speed of light remaining in the framework of the special relativity, necessarily contain principal errors. The analysis of such experiments may be of interest only in order to find concrete points where the corresponding arguments fail, just as we have done in the case treated in previous Section.

It should be understood that a potential positive result of experiments for the measurement of one-way speed of light, would mean the breaking of the laws of both the classical mechanics and the special relativity. Therefore, in such a case, it would be of course necessary to show that such a result does not contradict to the numerous experimental cofirmations of the special theory of relativity given so far.

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NEIZOTROPIJA BRZINE SVETLOSTI U OKVIRIMA SPECIJALNE TEORIJE RELATIVNOSTI

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Originalan naučni rad

Detaljno su diskutovani razlozi zbog kojih se u okvirima specijalne teorije relativnosti ne može meriti brzina svetlosti u jednom smeru, mada se po dogovoru može usvojiti da je brzina svetlosti neizotropna.