YU ISSN 0015—3206
FZKAAA 16 (4) 371 (1984)

WARD IDENTITIES IN THE LIGHT-CONE GAUGE

ANĐELKA ANDRAŠI

Ruđer Bošković Institute, 41000 Zagreb, Croatia, Yugoslavia

Received 23 March 1984

Revised manuscript received 9 July 1984

UDC 393.12

Original scientific paper

We check Ward identities in a special noncovariant gauge, defined by $n^\mu A_\mu = 0$, where n is a fixed vector subject to the condition $n^\mu n_\mu = 0$. Although this gauge would appear to be a special case of the planar gauge, one cannot pass continuously in the limit $n^2 \to 0$, from the latter to the light-cone gauge. The renormalization constants differ from the planar gauge; however, they give the same ratios, i. e. Ward identities identical to the ones in the axial gauge. The β-function is gauge invariant.

1. Introduction

Recently much attention has been paid to a special non-covariant gauge, the light-cone gauge, defined by

$$n^\mu A_\mu = 0, \tag{1.1}$$

where n is a fixed vector subject to the condition

$$n^\mu n_\mu = 0. \tag{1.2}$$

Mandelstam[1] has demonstrated the ultraviolet finiteness of the $N = 4$ supersymmetric Yang-Mills theory using the light-cone gauge.

A major difficulty in both the light-cone and axial gauges is the existence of $(k \cdot n)^{-1}$ terms in the A propagator. These terms give rise to ill-defined Feynman integrals. We shall use the *principal value prescription*[2] to define the integrals. Once such a consistent prescription is established, we can exploit Lorentz invariance for the actual evaluation of the integrals. All the integrals manifestly proportional to n^2 are set to zero.

2. *Renormalization constants for the gluon field*

The Lagrangian we consider consists of a set of Hermitian gauge fields $A^a_\mu(x)$ and a multiplet of spin one-half massless fields $\psi^j(x)$, and possesses local gauge invariance with respect to a compact semisimple Lie group G of dimension r. The bare Lagrangian is therefore

$$\mathscr{L}_B = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \overline{\psi} \gamma_\mu D_\mu \psi, \tag{2.1}$$

where

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{2.2}$$

and

$$(D_\mu \psi)^i = \partial_\mu \psi^i - \mathrm{i} g (R^a)^i_j \psi^j A^a_\mu. \tag{2.3}$$

Here f^{abc} (a: 1, 2, ..., r) are the real, totally antisymmetric structure constants of G; $(R^a)^i_j$ is the matrix representation of the a^{th} generator of G on the fermion multiplet (i: 1, 2, ..., $d(R)$, where $d(R)$ is the dimension of the representation).

In practice, we can use the SU (2) group.

The propagator for the gluon in the light-cone gauge is

$$G^{ab}_{\mu\nu} = (k^2 + \mathrm{i}\varepsilon)^{-1} [-g_{\mu\nu} + (n \cdot k)^{-1} (n_\mu k_\nu + n_\nu k_\mu)]. \tag{2.4}$$

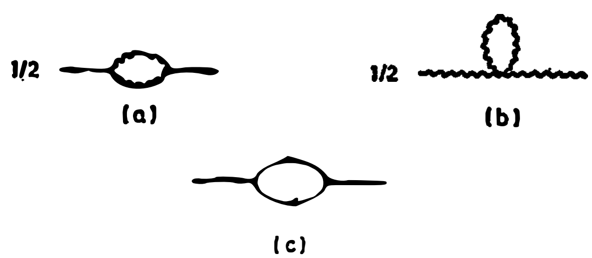

Fig. 1. $0(g)^2$ contributions to the A self-energy.

This propagator is identical in form to the propagator in the planar gauge[3].

In order to find Z_3 constant, we have to calculate $0(g^2)$ contributions to the A self-energy shown in Fig. 1. The fermion loop (c) is the same as in any other gauge (the fermion propagator does not contain any n), so for our consideration it does not matter.

In extracting the logarithmic divergence from the self-energy, we use the operations $\frac{1}{2} p_\alpha p_\beta \frac{\partial^2}{\partial p_\alpha \partial p_\beta}\Big|_{p=0}$.

The integrals manifestly proportional to n^2 cancel among themselves, giving the same result for the self-energy, i. e. the same Z_3 as in the axial gauge.

$$Z_3^{\mathrm{L.c.}} = 1 + \frac{g^2}{48\pi^2} \cdot 11\, C_{YM} \ln \Lambda^2, \tag{2.5}$$

Here L. c. denote the light cone and Λ the ultraviolet cut-off.

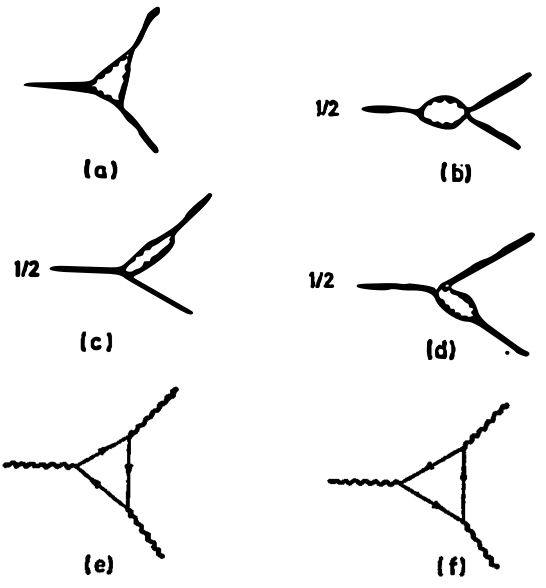

Fig. 2. $0(g^3)$ contributions to the AAA vertex.

The same sort of miracle happens with Z_1. In extracting the logarithmic divergence from the vertex graphs in Fig. 2, we use the operation $p_\alpha \frac{\partial}{\partial p_\alpha}\Big|_{p=0}$ and a special configuration of momenta $p = q$.

Thus, for the gluon field, the known axial-gauge Ward identity

$$Z_1 = Z_3 \tag{2.6}$$

holds in the light-cone gauge.

Let us mention that the corresponding results for the general planar gauge[3,4] are

$$Z_3^P = 1 + \frac{g^2}{48\pi^2}(11 - 6\alpha)\, C_{YM} \ln \Lambda^2, \tag{2.7}$$

$$Z_1^P = 1 + \frac{g^2}{48\pi^2}(11 - 9\alpha)\, C_{YM} \ln \Lambda^2, \tag{2.8}$$

giving the same result for the gauge-invariant β-function (we obtain the planar gauge by setting $\alpha = 1$).

$$Z_1^2 = 1 - \frac{g^2}{48\pi^2} \cdot 11\, C_{YM} \ln \Lambda^2. \tag{2.9}$$

3. *Renormalization constants for the fermion field*

Fermion self-energy to order g^2 involves only one diagram, shown in Fig. 3. Here we apply the operation $p_a \frac{\partial}{\partial p_a}\Big|_{p=0}$.

Fig. 3. $0(g^2)$ contribution to the $\psi\psi$ self-energy.

After an easy calculation, the result for the fermion-field renormalization constant is

$$Z_2^M(\psi) = 1 + \frac{g^2}{16\pi^2} C_2(R) \ln \Lambda^2. \tag{3.1}$$

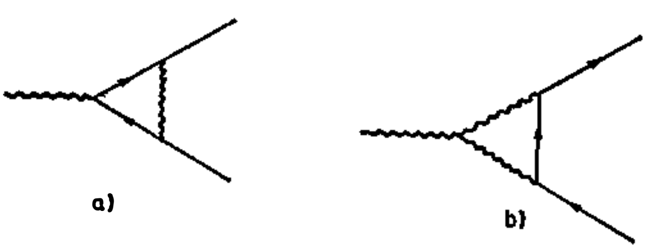

Fig. 4. $0(g^3)$ contribution to the $A\psi\psi$ vertex.

The evaluation of the $A\Psi\Psi$ vertex shown in Fig. 4 gives

$$Z_1^M(\psi) = 1 + \frac{g^2}{16\pi^2} C_2(R) \ln \Lambda^2, \tag{3.2}$$

i. e.

$$Z_2^M = Z_1^M. \tag{3.3}$$

In the planar gauge ($\alpha = 1$), the corresponding results are

$$Z_2^P(\psi) = 1 + \frac{g^2}{16\pi^2} C_2(R) \ln \Lambda^2, \tag{3.4}$$

$$Z_1^P(\psi) = 1 + \frac{g^2}{16\pi^2} |C_2(R) - C_2(G)| \ln \Lambda^2. \tag{3.5}$$

4. Conclusion

Althougs the light-cone and planar gauges share the same type of the propagator, the renormalization constants are very different.

There is no continuous transition from one to the other. The light-cone gauge satisfies the same Ward identities as the axial gauge. The β-function in the light-cone gauge is gauge invariant.

Acknowledgment

The author is grateful to Prof. J. C. Taylor for many valuable discussions.

References

1) S. Mandelstam, Nucl. Phys. B **213** (1983) 149;
2) W. Kainz, W. Kummer and M. Schweda, Nucl. Phys. B **79** (1974) 484;
3) Yu. L. Dokshitsev, D. I. D'yakonov and S. I. Troyan, Phys. Rep. **58** (1980) 269;
4) A. Andraši and J. C. Taylor, Nucl. Phys. B **192** (1981) 283.

WARDOVI IDENTITETI U BAŽDARNOM UVJETU SVJETLOSNOG KONUSA

ANĐELKA ANDRAŠI

Institut »Ruđer Bošković«, 41000 Zagreb

UDK 539.12

Originalni znanstveni rad

Provjeravaju se Wardovi identiteti u specijalnom nekovarijantnom baždarnom uvjetu definiranom sa $n^\mu A_\mu = 0$, gdje je n_μ fiksni vektor koji zadovoljava uvjet $n^\mu n_\mu = 0$. Premda se čini da je taj baždarni uvjet specijalan slučaj planarnog uvjeta u limesu, $n^2 \to 0$ ne može se kontinuirano prijeći iz planarnog u baždarni uvjet svjetlosnog konusa. Renormalizacione konstante se razlikuju od onih u planarnom uvjetu, međutim one daju iste omjere, tj. Wardove identitete kao u aksijalnom baždarnom uvjetu. Beta funkcija je baždarno invarijantna.