The last two terms in this expression tend towards zero at least as ε. To estimate the first term, let us mention first that the function $x \mapsto x^2\, s(x)$ has the same singularities at $x = \pm 1$ as the function s. Therefore its Fourier transform behaves in the same way for large p as the transform of the function s. Hence $|\hat{\hat{s}}(p)| = O(|p|^{-5/4})$. This implies

$$|\bar{\hat{\bar{u}}}_1(\varepsilon, 0) - \bar{\hat{\bar{u}}}_1(0)| < C_3\,\varepsilon.$$

We obtained

$$\left| \frac{R_N(u_N(\varepsilon))}{R_N(u_N)} - 1 \right| < \varepsilon\, C_4.$$

The asymptotic expression (3.3) follows if $\varepsilon(N)$ tends towards zero at least as $1/N$.

Proof of (3.4). We have

$$\frac{\mathscr{J}_N(u_N(\varepsilon))}{R_N(u_N(\varepsilon))} = 1 - \frac{1}{8} \cdot \frac{N I_2(u_N(\varepsilon))}{R_N(u_N(\varepsilon))} + \frac{1}{32} \cdot \frac{I_3(u_N(\varepsilon))}{R_N(u_N(\varepsilon))}.$$

Because of (3.3), there exists a positive number C_1 such that $R_N(u_N(\varepsilon)) > C_1^{-1} N$ (for N large enough). Then (3.8) gives

$$a(N) = \frac{N I_2(u_N(\varepsilon))}{R_N(u_N(\varepsilon))} < C_1\, N^{-1} \int |p|\, |\hat{u}_1(\varepsilon, p)|^2\, dp.$$

Let us estimate the integral uniformly in ε. From (3.7) and (2.4) we have

$$|\hat{u}_1(\varepsilon, p)| < C_2 \int \frac{1}{1 + |q|^\sigma} \cdot \frac{1}{1 + |p - q|^\sigma}\, dq =$$

$$C_2 \int \frac{1}{1 + \left|\dfrac{p-q}{2}\right|^\sigma} \cdot \frac{1}{1 + \left|\dfrac{p+q}{2}\right|^\sigma}\, dq,$$

where $\sigma = 1 + 1/4$. Now it is easy to obtain an upper bound on $\hat{u}_1$:

$$(1 + |p|)^\sigma\, |\hat{u}_1(\varepsilon, p)| < C_3.$$

Hence $a(N) < C_1\, C_3\, N^{-1} \int |p|\,(1 + |p|)^{-5/2}\, dp = O(1/N)$.

Again using (3.3), (3.8) and (2.4) we have

$$b(N) = \frac{I_3(u_N(\varepsilon))}{R_N(u_N(\varepsilon))} < 4 N^{-2}\, C_1 \int p^2\, |\hat{s}(p)|^2|\, \hat{\varphi}(\varepsilon p)|^2\, dp <$$

$$4N^{-2}\, \varrho_1^2\, C_1 \int \frac{\hat{\varphi}(\varepsilon p)^2}{[1 + |p|]^{1/2}}\, dp < 8\varepsilon^{-1/2}\, N^{-2}\, \varrho_1^2\, C_1 \int\limits_0^2 \frac{dp}{\sqrt{\varepsilon + p}} < \frac{C_2}{\varepsilon\, N^2}.$$

Hence $b(N) = O(1/N)$ if $N\varepsilon$ is bounded from bellow. This condition on $\varepsilon(N)$ is compatible with the condition on $\varepsilon(N)$ implying (3.3). We conclude that the chosen regularization $u_N(\varepsilon)$ with $\varepsilon(N) = 1/N$ gives (3.3) and (3.4).

4. Conclusion

It would be interesting to see whether the asymptotic behaviour (1.4) holds for the symplectic case as the more important one. However, in the case of symplectic symmetry, the functional (1.2) can have undesired features. For a class of potentials, the minimization problem (1.3) can have no solution, as has been shown in Ref 4. In this case the kinetic term is essential in studying problem (1.3) irrespectively of its small order in $1/N$. If an approximation of the form

$$\int \varrho(x) [\int_0^1 \varrho(y) \chi(x-y)\, dy]^2\, dx$$

of the kinetic term is known, where χ depends on N, then the asymptotic behaviour (1.4) can be obtained even for symplectic case.

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LIMES ZA VELIKI N U MODELU S KOLEKTIVNOM VARIJABLOM

NEDŽAD LIMIĆ

Institut »Ruđer Bošković«, 41000 Zagreb

Uspoređeni su limesi za veliki N potpunog i reduciranog efektivnog potencijala u jednodimenzionalnom modelu teorije polja s kolektivnim varijablom i ortogonalnom simetrijom.