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# HARMONIC OSCILLATOR QUARK MODELS FOR BARYONS AND THE MOMENTUM-DEPENDENT EFFECTS

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Baryonic form factors for nucleons and $1/2^+$ resonances have been investigated. General formulas for calculating the current form factors provide a common point of view for the comparison of various published results. There is also a comparison with the *MIT* bag model. Several inequivalent procedures for the calculation of magnetic moments are compared and discussed. It was important to find whether one set of model parameters can fix simultaneously charge, radius, magnetic moment and the axial-vector coupling constant. It is shown that this request favours the correlated and relativistically corrected harmonic oscillator model.

## 1. Introduction

All simple quark models[1-9], however successful in describing various static properties of hadrons fail in some respect in the description of the momentum dependent and/or relativistic hadronic features. With respect to quark dynamics models can be roughly divided into two groups.

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In the first group quarks move inside a central nuclear shell model like potential[5,7-9] or inside a bag[1,2] without experiencing any mutual correlation. If the central potential is of the harmonic oscillator type, we will talk about the harmonic oscillator shell model (*HOS*). In its simplest form such a model neither accounts for the momentum of the hadron as a whole (i. e. recoil) nor does it correctly describe the nearly relativistic internal quark motion inside the hadron at rest. The *MIT* bag model[1,2] offers a better relativistic description of the confined quarks but its essentially static character does not include any description of the recoil.

In the second group we include models with two forces acting among quarks. Especially interesting is the correlated harmonic oscillator (*CHO*) model[3-5] in which the center of mass motion of a hadron (i. e. recoil) can be treated exactly in the nonrelativistic limit. The full relativistic treatment would require tremendous sophistication of the model[10] which has not been so far fully carried out. However, it is possible to include relativistic corrections to all orders $p/m$ for the internal quark motion[5,6] as will be illustrated in the next section. One has then complete analogy with the *MIT* bag model results, with an important addition of a recoil effect.

In the discussion of the quark models we should terminologically distinguish between the relativistic corrections and the recoil corrections. The first term denotes the dynamics of the individual quarks inside the confinement (i. e. potential or bag). The second term means corrections caused by the motion of the hadrons as a whole. Such corrections could (and should) also in principle be relativistic. However, all models under discussion are nonrelativistic at the hadronic (i. e. baryonic) level[10,16].

It is well known that only a properly relativistic internal quark motion can explain the experimental axial-vector nucleon coupling constant $g_A$[11,12]. It turns out that such diagonal matrix elements of the axial-vector current are softly model dependent. A different situation emerges when one considers off-diagonal matrix elements involving a baryon and a resonant state.*) In that case there is a noticeable difference between the *CHO* model and the models[1,2,8,9] in which quark perform their motions with respect to the fixed center of the coordinates.

The same distinction one could in principle expect when calculating magnetic moments.**)

In this paper we have adopted a uniform approach by which all quantities in question are connected to the appropriate form factors. These form factors are always found by calculating the matrix element of the particular current (i. e. vector, axial-vector or scalar density) between the baryon states. As is shown in the following section, this approach is quite natural for the *CHO* model. For the *HOS* and *MIT* bag models one has to construct suitable wave packets and to resort to the boosting of the baryon states[10,13,14]. By that way one can easily reproduce older results[1,2,12] which were obtained by introducing suitable operators constructed on the basis of the classical analogy***). Moreover such a consistent approach can disentangle the problem caused by the fact that two different ope-

* Such matrix elements play a role in the study of nonleptonic decays[17].

** We will not consider here pionic (mesonic) corrections which appear in some variations of quark models[15].

*** See for example formula (3.5) below.

rators were used[1,6,9] (formula (3.5) and (4.4) below) when calculating magnetic moments.

It would be extremely useful, in view of the preonic or subconstituent models, to be able to decide whether a quark model can at the same time fit all experimental quantities (such as $g_A$, $\mu(p)$, $\langle r^2 \rangle_p$, etc.). Our quark models describe quarks as point-like particles so the failure in the above task might indicate the existence of the anomalous quark magnetic moment. In view of the relative crudity of all quark models such questions are too ambitious. At least one model, the *CHO* model, is just about able to accommodate all quantities within 10% accuracy with a suitable choice of the parameters.*) However, we do hope that the following studies can clear many aspects of the quark model dynamics. This can indicate which model (or models) are suitable for the description of a particular physical situation.

## 2. *Correlated harmonic oscillator quark model*

As the full relativistic treatment of the two-body potential is a complex and thorny problem[10,16], we are content to calculate relativistic corrections, to any order $p/m$, for the nonrelativistic *CHO* model[3−6]. These corrections are large enough to talk about a reformulated *CHO* (*RCHO*) model.**)

One has to calculate matrix elements of two quark operators of the type

$$J_\mu^{ij}(\vec{k}_1, \vec{k}_2) = \overline{\psi}^i(\vec{k}_1)\,\Gamma_\mu\,\psi^j(\vec{k}_2), \tag{2.1}$$

$i, j$ = flavor, color, spin. The quark field $\psi$ of flavor $f$ and color $c$ can be decomposed as follows

$$\psi_f^c(\vec{k}) = \left(\frac{m}{E}\right)_f^{1/2} \sum_r \left[u_{f,r}^c(\vec{k})\, b_{f,r}^c(\vec{k}) + v_{f,r}^c(\vec{k})\, d_{f,r}^{c+}(\vec{k})\right]. \tag{2.2}$$

The spinors are defined by

$$u_{f,r}^c(\vec{k}) = \left(\frac{E+m}{2m}\right)_f^{1/2} \begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\vec{k}}{(E+M)_f} \end{pmatrix} \chi_r^c, \tag{2.3}$$

$$v_{f,r}^c(\vec{k}) = \left(\frac{E+m}{2m}\right)_f^{1/2} \begin{pmatrix} \dfrac{\vec{\sigma}\vec{k}}{(E+m)_f} \\ 1 \end{pmatrix} \tilde{\chi}_r^c, \quad \tilde{\chi}_r^c = \mathrm{i}\,\sigma_2\,\chi_r^c.$$

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* Not quite the same as parameters determined by baryon spectroscopy[3,4,19,20].

** In Ref. 5 a version of that model was labeled *RHO*.

The operator (2.1) is sandwiched between baryon states. The proton state, for example, is

$$|P(\vec{k}, \vec{p}_\varrho, \vec{p}_\lambda)^\uparrow\rangle = N_P \frac{1}{6} \sum_{\substack{\text{Per.}\,\vec{p}_i \\ (i=1,2,3)}} G_P(p_\varrho, p_\lambda) \times$$

$$\times \frac{\varepsilon^{abc}}{\sqrt{18}} [(b^{a+}_{u\uparrow}(\vec{p}_1)\, b^{b+}_{d\downarrow}(\vec{p}_2) - b^{a+}_{u\downarrow}(\vec{p}_1)\, b^{b+}_{d\uparrow}(\vec{p}_2))\, b^{c+}_{u\uparrow}(p_3)]\,|\,0\rangle, \tag{2.4a}$$

where

$$\begin{aligned} \vec{p}_1 &= \frac{1}{3}\vec{k} + \frac{1}{\sqrt{2}}\vec{p}_\varrho + \frac{1}{\sqrt{6}}\vec{p}_\lambda, \\ \vec{p}_2 &= \frac{1}{3}\vec{k} - \frac{1}{\sqrt{2}}\vec{p}_\varrho + \frac{1}{\sqrt{6}}\vec{p}_\lambda, \\ \vec{p}_3 &= \frac{1}{3}\vec{k} - \sqrt{\frac{2}{3}}\,\vec{p}_\lambda. \end{aligned} \tag{2.4b}$$

The norm $N_P$ characterizes the *RCHO* model and it will be defined later. The function $G_P$ is a solution of the *CHO* model (in momentum space) for the octet baryons (nucleons)

$$G_P(p_\varrho, p_\lambda) = (\sqrt{\pi}\alpha)^{-3} \exp\left[\frac{-1}{2\alpha^2}(p_\varrho^2 + p_\lambda^2)\right]. \tag{2.4c}$$

For the first $1/2^+$ resonance N (1405) one has:

$$G_R(p_\varrho, p_\lambda) = \frac{1}{\sqrt{3}}\left[3 - \frac{1}{\alpha^2}(p_\varrho^2 + p_\lambda^2\right] G_P(p_\varrho, p_\lambda). \tag{2.4d}$$

The spin-flavor-color structure is the same as for the proton (2.4a).

Higher $1/2^+$ excited states as N (1705), $\Delta$ (1875), ..., have different spin--flavor-color-spatial structure, but the calculational procedure is based on the same general formalism outlined in this paper and not necessarily more complicated. More about the properties of those states is described in Ref. 21.

In the above formula we have neglected the difference among quark masses. This leads to considerable simplification with possible error being less than 10%[18)*)]. A general matrix element is

$$M^{ij}_\mu(B_1(\vec{k}), B_2(\vec{k}')) = \int d^3 k_1 d^3 k_2\, d^3 p_\varrho\, d^3 p_\lambda\, d^3 p'_\varrho\, d^3 p'_\lambda \times$$

$$\langle B_1(\vec{k}, \vec{p}_\varrho, \vec{p}_\lambda)\,|\bar{\psi}^i(\vec{k}_1)\,\Gamma_\mu\,\psi^j(\vec{k}_2)|\,B_2(\vec{k}', \vec{p}'_\varrho, \vec{p}'_\lambda)\rangle. \tag{2.5}$$

* With flavor dependent masses formulas (2.4) would contain several different correlation function $G(p_\varrho, p_\lambda)$[18)].

The quark creation and annihilation operators appearing in (2.2) and in baryon states (2.4a) can be contracted in various ways. As long as quark masses are equal everything is so symmetric that only one of the possible contributions ought to be calculated. Nevertheless, it might be useful for future use to outline a general procedure. For a typical combination**)

$$(M^{p-n}_{m,n})^1 = \langle P \,|b^{u+}_{rm}(\vec{k}_1)\, b^d_{rn}(\vec{k}_2)|\, N\rangle \tag{2.6a}$$

possible contractions are illustrated in Fig. 1. There are 3! permutations of impulses in any baryon state, but only 18 different diagrams. Further constraints are imposed by spin and flavor (i. e. SU (6)) structure of the baryonic wave functions. Thus contractions give us,

$$(M^{p-n}_{m,n})^1 = \frac{1}{2}\,[\delta_{m\uparrow}\,\delta_{n\uparrow}\,(2\delta^1_1 + \delta^1_2 + \delta^1_3) - \delta_{m\downarrow}\,\delta_{n\downarrow}\,\delta^1_4] \tag{2.6b}$$

where

$$\begin{aligned}
\delta^1_1 &= \delta^{(3)}(\vec{k}_1 - \vec{p}_3)\,\delta^{(3)}(\vec{k}_2 - \vec{p}'_1)\,\delta^{(3)}(\vec{p}_1 - \vec{p}'_2)\,\delta^{(3)}(\vec{p}_2 - \vec{p}'_3),\\
\delta^1_2 &= \delta^{(3)}(\vec{k}_1 - \vec{p}_3)\,\delta^{(3)}(\vec{k}_2 - \vec{p}'_3)\,\delta^{(3)}(\vec{p}_1 - \vec{p}'_2)\,\delta^{(3)}(\vec{p}_2 - \vec{p}'_1),\\
\delta^1_3 &= \delta^{(3)}(\vec{k}_1 - \vec{p}_1)\,\delta^{(3)}(\vec{k}_2 - \vec{p}'_1)\,\delta^{(3)}(\vec{p}_2 - \vec{p}'_3)\,\delta^{(3)}(\vec{p}_3 - \vec{p}'_2),\\
\delta^1_4 &= \delta^{(3)}(\vec{k}_1 - \vec{p}_1)\,\delta^{(3)}(\vec{k}_2 - \vec{p}'_3)\,\delta^{(3)}(\vec{p}_2 - \vec{p}'_1)\,\delta^{(3)}(\vec{p}_3 - \vec{p}'_2),\\
\vec{k}_2 - \vec{k}_1 &= \vec{K}' - \vec{K}.
\end{aligned} \tag{2.6c}$$

Here $\delta^1_1$, $\delta^1_2$, $\delta^1_3$ and $\delta^1_4$ correspond to diagrams 13, 18, 2 and 5 in Fig. 1, respectively. The symmetry with respect to quark permutations reduces this to

$$\begin{aligned}
M_\mu &= N_{B_1} N_{B_2} \sum_{m,n} (M^{B_1B_2}_{m,n})^0 \int \mathrm{d}^3 p_\varrho\, \mathrm{d}^3 p_\lambda\, G_{B_1}(p_\varrho, p_\lambda)\, G_{B_2}(p_\varrho, p_\lambda) \times\\
&\times \frac{1}{3} \sum_{i=1}^{3} \left(\frac{m_u m_d}{E_u E'_d}\right)^{1/2}_i \bar{u}_m(\vec{p}_i)\, \Gamma_\mu\, u_n(\vec{p}'_i) \times\\
&\times \exp\left[\frac{-1}{a^2}\left(\vec{p}_i - \frac{1}{3}\vec{K}\right)\left(\vec{K}' - \vec{K}\right)\right],\\
E^2_{i,q} &= \vec{p}^{\,2}_{i,q} + m^2_q, \quad q = u, d,\\
\vec{p}'_i &= \vec{p}_i + \vec{K}' - \vec{K}.
\end{aligned} \tag{2.7a}$$

** Here we extracted out the dependence of quark creation operators of the corresponding matrix element, and for simplicity omitted integrals over $\vec{k}_1$ and $\vec{k}_2$.

The factor $(M)^0$ is obtained from $M^1$ (2.6) if all $\delta_i^1 \equiv 1$. It can be calculated for any given flavor from the corresponding $M^1$ by omitting all dependence on the impulses $\vec{p}_i$.

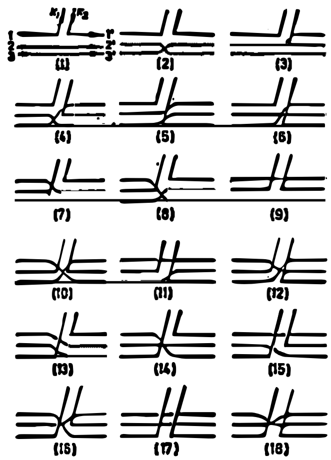

Fig. 1. This figure shows various ways in which current quarks (impulses $k_1$ and $k_2$) can be contracted with the valence quarks (impulses $p_1, \ldots, p_3'$ are indicated by symbols 1, ..., 3').

The formula (2.7a) is of a very general nature, as it contains recoil (nonrelativistic) corrections and the relativistic corrections due to the internal quark motion for any bilinear operator in quark field. Any particular operator is defined by the appropriate choice of the matrix $\Gamma_\mu$. Thus for example $\Gamma^\mu = \gamma^\mu$ leads to the vector current matrix element whose form factor structure is defined by the formula (2.10) below. The choice of the Breit frame of reference is the most natural one for the *RCHO* model which is nonrelativistic at the baryonic level [13,14].

If all quark masses are equal one can define $\hat{p}_\varrho$ and $\hat{p}_\lambda$ by exchanging $\vec{p}_1 \leftrightarrow \vec{p}_3$ in the set (2.4b) so that

$$G_B(p_\varrho, p_\lambda) \equiv G_B(\hat{p}_\varrho, \hat{p}_\lambda).$$

Analogous relations are valid for either exchanges of the quark impulses. Therefore one can write in the zero momentum transfer limit ($\vec{K} = \vec{K}'$)

$$M^\mu_{\alpha\beta} = N_{B_1} N_{B_2} \sum_{m,n} (M^{B_1 B_2}_{mn})^0 \int d^3 p_\varrho \, d^3 p_\lambda \, G_{B_1}(p_\varrho, p_\lambda) \, G_{B_2}(p_\varrho, p_\lambda) \times$$

$$\times \left(\frac{m_\alpha m_\beta}{E_3^\alpha E_3^\beta}\right)^{1/2} \bar{u}^\alpha_m(\vec{p}_3) \, \Gamma^\mu u^\beta_n(\vec{p}_3), \qquad (2.7b)$$

$\alpha$, $\beta$ flavours.*)

* This formalism can be easily applied for strange baryons leading to $g_A$ and $\mu(P)$ values comparable with the published ones[6,14] and in reasonable agreement with experiment. Excited baryons (resonances) can also be described, just specifying $B_1 \equiv R$ and $M^0_{m,n}$ for the proper resonant state $R$.

This is the most general expression for the equal quark masses. The formula (2.7b) stays valid even where one exchanges $\vec{p}_3$ by $\vec{p}_1$ or $\vec{p}_2$. The choice of $\vec{p}_3$ leads to the simplest integrations. One can use (2.7a) in order to extract form factors (for example $G_M(\vec{Q}^2)$, $\vec{K}' = -\vec{K} = -\vec{Q}/2$; see (2.10) below) and then one can switch to (2.7b) in order to calculate their zero momentum transfer value (for example $G_M(0) = e^{-1}\mu(P)$, formula (2.15)).

As the *CHO* model is nonrelativistic, the usual procedure would be to use a nonrelativistic approximation for the combination $\bar{u}\Gamma^\mu u$ which would lead to too large a value for $g_A$. Besides this does wipe out the magnetic moment form factor in the matrix element of the electromagnetic current-see expression (2.10) below. In the strictly nonrelativistic approach the magnetic moment is calculated by an ad hoc introduction of the Pauli matrix $\vec{\sigma}$. In order to remedy these deficiencies, at least to some extent, one has to retain all components of the spinors $u$. This fixes the normalization factor $N_B$ from (2.4), as all baryon states have to be normalized consistently with the relativistic corrections. This means that the vector coupling constant should be one, i. e.

$$\int d^3k_1\, d^3k_2\, \langle P(\vec{K}=0)\,|\bar{\psi}_u(\vec{K}_1)\,\gamma_0\,\psi_d(\vec{K}_2)|\,N(\vec{K}'=0)\rangle =$$

$$= \int d^3p_\varrho\, d^3p_\lambda\, G_P^2(p_\varrho, p_\lambda) = N_P^2 = g_V = 1. \tag{2.8}$$

This result is due to a particular normalization of the quark fields (2.4).

*Nucleon axial-vector form factor*

The formula (2.7) gives for the nucleon axial-vector coupling constant:

$$g_A^{p-n} = \frac{5}{3}\left(U^2 - \frac{1}{3}V^2\right), \tag{2.9a}$$

$$\left.\begin{matrix} U^2 \\ V^2 \end{matrix}\right\} = \int d^3p_\varrho\, d^3p_\lambda\, G_P^2(p_\varrho, p_\lambda)\begin{cases} \dfrac{E_3+m}{2E_3}, \\ \dfrac{E_3-m}{2E_3}, \end{cases} \tag{2.9b}$$

$$E_3^2 = \frac{2}{3}\vec{p}_\lambda^{\,2} + m^2.$$

The quantities $U^2$ and $V^2$ are introduced for further comparison with the *MIT* bag model. In the nonrelativistic limit: $E_3 \to m \Rightarrow U^2 \to 1$ and $V^2 \to 0$. The expression (2.9) is in complete agreement with Ref. 6. Note that it is trivial to see that $g_V = U^2 + V^2 = 1$. The numerical calculations are performed for 7 sets of parameters listed in Table 1. The values of $U^2$, $V^2$, $g_S$, $g_A$ and the r. h. s. of the Goldberger-Trieman relation $g_{\pi NN} f_\pi = \sqrt{2}\, m_N\, g_A$ are shown in Table 2.

TABLE 1

| Set of param. No. | $m_u$/GeV | $a$/GeV | Ref. |
|---|---|---|---|
| 1 | 0.33 | 0.32 | 17 |
| 2 | 0.33 | 0.25 | 6 |
| 3 | 0.22 | 0.25 | 6; (a) |
| 4 | 0.22 | 0.27 | (a) |
| 5 | 0.22 | 0.32 | (a) |
| 6 | 0.22 | 0.38 | (b) |
| 7 | 0.38 | 0.57 | (b) |

(a) These parameters fit excellently charge radius, magnetic moments and the $g_A/g_V$ ratio for nucleons.

(b) These two possibilities of parameter choice give us the value of the r. h. s. of the Goldberger--Trieman relation, for the N (1405) resonance, which lies inside the experimentally allowed range of values (see Table 5 and Fig. 6).

Sets of parameters used for the calculations performed in the *RCHO* model.

TABLE 2

| Set No. | $U^2$ | $V^2$ | $g_S$ | $g_A$ | r. s. h. of G.-T. relation |
|---|---|---|---|---|---|
| 1 | 0.87 | 0.12 | 0.75 | 1.38 | 1.84 |
| 2 | 0.91 | 0.09 | 0.82 | 1.47 | 1.95 |
| 3 | 0.85 | 0.15 | 0.70 | 1.33 | 1.77 |
| 4 | 0.84 | 0.16 | 0.68 | 1.31 | 1.73 |
| 5 | 0.813 | 0.187 | 0.62 | 1.25 | 1.66 |
| 6, 7 | 0.78 | 0.21 | 0.57 | 1.18 | 1.57 |
| Exp | | (a) | | 1.25 | 1.66 GeV |

(a) No experimental evidence.

The $U^2$, $V^2$ $g_S$, $g_A$ and Goldberger-Trieman relation $g_{\pi NN} f_\pi = \sqrt{2}\, m_N\, g_A$ for the p-n, calculated for the *RCHO* model.

Very illustrative is Fig. 2, which shows the $g_A$ dependence on the $u$-quark mass $m$ and on the harmonic oscillator strength parameter $a$. It is clear that for both masses ($m = 0.22$ and 0.33 GeV) and for both $a$' s ($a = 0.32$, 0.48 GeV), which are not that different one from the other, one can easily fit $g_A$.

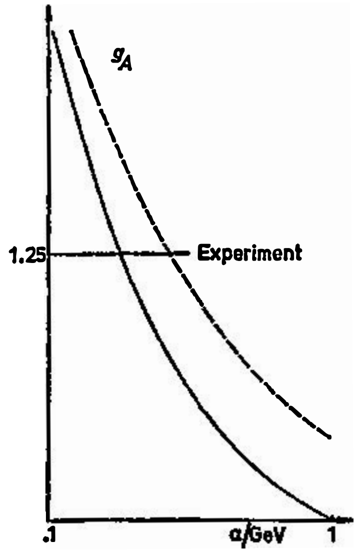


Fig. 2. The axial-vector coupling for the p-n transition as a function of the harmonic oscillator strength parameter $\alpha$. The solid line corresponds to the $g_A$ calculated for $m = 0.22$ GeV, the dashed line for $m = 0.33$ GeV. Note that in every other picture the solid line corresponds to the $m = 0.22$ case and the dashed line to the $m = 0.33$ case.

*Baryon magnetic moments*

The electromagnetic current matrix element can be decomposed in the Breit frame of reference as follows:

$$\langle \frac{\vec{Q}}{2} \left| \begin{Bmatrix} \mathcal{J}_0 \\ \vec{\mathcal{J}} \end{Bmatrix} \right| -\frac{\vec{Q}}{2} \rangle = \chi^+ \begin{Bmatrix} \frac{M}{E} G_E(\vec{Q}^2) \\ \mathrm{i}\frac{\vec{\sigma}\times\vec{Q}}{2E} G_M(\vec{Q}^2) \end{Bmatrix} \chi. \tag{2.10}$$

Here $M$, $E$ and $\vec{Q}$ refer to the baryon mass, energy and momentum transfer, respectively. In order to find $G_M(0)$ from our model one has to isolate the term multiplied by $\vec{Q}$ before the limit $\vec{Q} \to 0$ is taken. Therefore one has to start from the current density

$$\overline{\psi}(p)\,\vec{\gamma}\,\psi(p') = \sqrt{\frac{(E+m)(E'+m)}{4EE'}} \times \tag{2.11}$$

$$\times\, \chi^\dagger \left\{ \frac{\vec{p}}{E+m} + \frac{\vec{p}'}{E'+m} + \frac{\mathrm{i}\,\vec{\sigma}\times\vec{p}}{E+m} - \frac{\mathrm{i}\,\vec{\sigma}\times\vec{p}'}{E'+m} \right\} \chi.$$

It is streightforward to see that only the fourth term does contribute to the magnetic moment term which has to be proportional to $\vec{\sigma} \times \vec{Q}$. An existing contribution from the third term is cancelled by the equal contribution having an opposite sign, which comes from the fourth term. Both these cancelling contributions are due to the low — $\vec{Q}$ expansion of the normalization factor in front of (2.11). Careful expansion of the fourh term leads to

$$\frac{\vec{\sigma} \times \vec{p}'}{E' + m} = - \frac{\vec{\sigma} \times \vec{Q}}{E + m} + \frac{(\vec{\sigma} \times \vec{p})(\vec{p} \cdot \vec{Q})}{E(E + m)} + \dots \qquad (2.12)$$

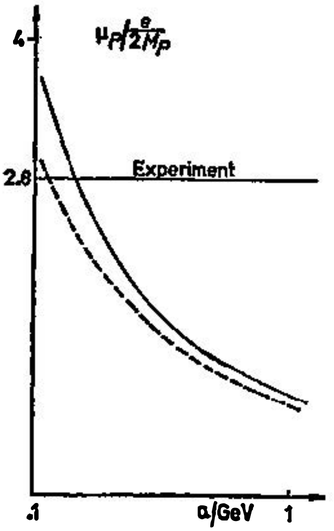


Fig. 3. The proton magnetic moment as a function of α.

When (2.12) is introduced into the expresion (2.7), the spherically symmetric integration over $d^3p_\lambda$ awerrages the second term in (2.12). Multiplying (2.7) by $e$ (electric charge), one immediately finds

$$\mu (P) = e \int d^3p_\lambda \, d^3p_\varrho \, G_P^2 (p_\varrho, p_\lambda) \frac{2E_3 + m}{6E_3^2}. \qquad (2.13a)$$

Expressed in units of nuclear magneton ($\mu_P = e/2 M_p$, $M_p$ is proton mass) Eq. (2.13a) reads

$$\mu (P) = \frac{e}{2M_p} \int d^3 p_\varrho \, d^3 p_\lambda \, G_P^2 (p_\varrho, p_\lambda) \frac{M_p (2E_3 + m)}{3E_3^2}. \qquad (2.13b)$$

The magnetic moment for the N (1405) resonance can be calculated from formulas (2.13a,b) by replacing $G_p$ with the corresponding wave function $G_R$ (2.4d). The values of the proton and N (1405) magnetic moments are listed in Table 3. The dependence on parameters is very illustrative.

TABLE 3

| Set No. | Proton | N (1405) |
|---|---|---|
| 1 | 1.98 | 1.79 |
| 2 | 2.20 | 2.02 |
| 3 | 2.74 | 2.46 |
| 4 | 2.63 | 2.35 |
| 5 | 2.36 | 2.11 |
| 6 | 2.10 | 1.88 |
| 7 | 1.40 | 1.25 |
| Exp | 2.79 ($e/2M_p$) | (a) |

(a) No experimental evidence.

The proton and N (1405) magnetic moments (in $e/2m_p$ units) for the *RCHO* model.

*Charge radius*

At the hadronic level the *RCHO* model is nonrelativistic as can be seen by calculating the charge radius:

$$\begin{aligned}\langle r^2\rangle_B &= \left[-6\frac{\partial}{\partial\vec{Q}^2}G_E(\vec{Q}^2)\right]^B_{\vec{Q}^2=0}\\ &= \langle r^2\rangle^0_B + \langle r^2\rangle^1_B + \dots\end{aligned} \tag{2.16}$$

Specifying $\Gamma^\mu = \gamma^0$ and taking all quark masses equal, one can write according to formulas (2.7a) and (2.10):

$$G_E(\vec{Q}^2) = \int d^3 p_\varrho\, d^3 p_\lambda\, G_B^2(p_\varrho, p_\lambda)\, \varrho(p_\lambda, \vec{Q}^2),$$

$$\varrho(p_\varrho, p_\lambda) = \varrho_0(p_\lambda, \vec{Q}^2)\, \varrho_1(p_\lambda, \vec{Q}^2),$$

$$\varrho_0(p_\lambda, \vec{Q}^2) = \exp\left[-\frac{\vec{Q}}{\alpha^2}\left(\vec{p}_3 + \frac{1}{6}\vec{Q}\right)\right], \tag{2.17a}$$

$$\varrho_1(p_\lambda, \vec{Q}^2) = \frac{(E_3+m)(E_3'+m) + \vec{p}_3\cdot\vec{p}_3'}{2\sqrt{E_3 E_3'(E_3+m)(E_3'+m)}},$$

$$\vec{p}_3' = \vec{p}_3 + \vec{Q},$$

$$\vec{p}_3 = -\frac{1}{6}\vec{Q} - \sqrt{\frac{2}{3}}\vec{p}_\lambda,$$

$$\vec{Q} = \vec{K} - \vec{K} = -2\vec{K} = 2\vec{K}'. \tag{2.17b}$$

The expansion in the powers of $\vec{Q}^2$ gives

$$\varrho_0 (p_\lambda, \vec{Q}^2) = \left(\frac{-1}{6\alpha^2} + \frac{\vec{p}_3^2}{6\alpha^4}\right) \vec{Q}^2 + \dots \tag{2.18a}$$

$$\varrho_1 (p_\lambda, Q^2) = - \frac{4E_3^3 + E_3 m^2 + m^3}{24E_3^4 (E_3 + m)} \vec{Q}^2 + \dots \tag{2.18b}$$

The term (2.18a) is responsible for the nonrelativistic expression for the charge radii*)

$$\langle r^2\rangle_P^0 = - 6 \int \mathrm{d}^3 p_\varrho \, \mathrm{d}^3 p_\lambda \, G_P^2 (p_\varrho, p_\lambda) \left[\frac{\partial}{\partial \vec{Q}^2} \varrho_0 (p_\lambda, \vec{Q}^2)\right]_{\vec{Q}^2=0} \varrho_1 (\vec{p}_\lambda, 0)$$
$$= \alpha^{-2} \tag{2.19}$$

The term (2.18b) leads to the relativistic correction $\langle r^2\rangle_P^1$ already noted by Ref. 6. The spurious term comes from the normalization factor appearing in (2.10):

$$- 6 \frac{\partial}{\partial \vec{Q}^2} \frac{1}{M} \left(\frac{1}{4} \vec{Q}^2 + M^2\right)^{1/2} \Big|_{\vec{Q}^2=0} = - \frac{3}{4M^2}. \tag{2.20}$$

t reflects the already stated fact that the *RCHO* model is not, really, relativistically nvariant. The recoil in that model is actually treated completely nonrelativistically and the relativistic corrections apply to the relative motions of the quarks only. It makes eminent physical sense to use $E = M$ in (2.10) thus disposing of the term (2.20).

The charge radius of the first excited baryon state (N (1405) 1/2$^+$) follows easily as the relativistic correction (2.18b) is always of the same form:

$$\langle r^2\rangle_R^0 = \frac{5}{3} \alpha^{-2},$$

$$\langle r^2\rangle_R^1 = \int \mathrm{d}^3 p_\varrho \, \mathrm{d}^3 p_\lambda \, G_R^2 (p_\varrho, p_\lambda) \frac{4E_3^3 + E_3 m^2 + m^3}{4E_3^4 (E_3 + m)}. \tag{2.21}$$

The numerical values can be found in Table 4, where we particularly indicated the importance of the term $\langle r^2\rangle^1$. That term, for proton, is shown as a function of $m$ and $\alpha$ in Fig. 4.

* One has

$$\varrho_0 (\vec{Q}^2 = 0) = \varrho_1 (\vec{Q}^2 = 0) = 1,$$

$$\langle \vec{r}^2\rangle_P^0 = \langle \frac{2}{3} (\vec{r}_1^2 + \vec{r}_2^2) - \frac{1}{3} \vec{r}_3^2\rangle_P = \langle \frac{2}{3} \vec{\varrho}^2\rangle_P = \alpha^{-2}.$$

However, this is valid only when all quarks are in the $s$-states. With the $p$--state quark, say in $1/2^-$ resonance[19], the relativistic correction $\langle r^2 \rangle^1$ changes.

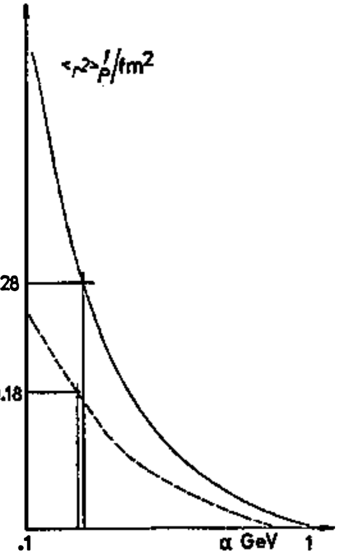


Fig. 4. The $\langle r^2 \rangle^1_p$ term as a function of $\alpha$. The vertical and horizontal lines indicate two possible cases for $\alpha$ which produce excellent agreement with experiment ($\langle r^2 \rangle_P = \alpha^{-2} + \langle r^2 \rangle^1_p$).

TABLE 4

| Set No. | Proton $\langle r^2 \rangle^0 +$ | $\langle r^2 \rangle^1 =$ | $\langle r^2 \rangle_P$ | N (1405) $\langle r^2 \rangle^0 +$ | $\langle r^2 \rangle^1 =$ | $\langle r^2 \rangle_P$ |
|---|---|---|---|---|---|---|
| 1 | 0.38 | 0.15 | 0.53 | 0.63 | 0.13 | 0.76 |
| 2 | 0.62 | 0.18 | 0.80 | 1.03 | 0.16 | 1.19 |
| 3 | 0.62 | 0.30 | 0.92 | 1.03 | 0.26 | 1.29 |
| 4 | 0.53 | 0.28 | 0.81 | 0.88 | 0.24 | 1.12 |
| 5 | 0.38 | 0.24 | 0.62 | 0.63 | 0.21 | 0.84 |
| 6 | 0.27 | 0.21 | 0.48 | 0.35 | 0.18 | 0.63 |
| 7 | 0.12 | 0.09 | 0.21 | 0.20 | 0.8 0 | 0.28 |
| Exp. | | 0.81 fm² | | | (a) | |

(a) No experimental evidence.

The charge radii of the proton and the N (1405) resonance for the *RCHO* model (units: fm²).

## *Off-diagonal matrix elements*

### (*i*) *Axial-vector form factor*

Off-diagonal matrix elements clearly illustrate the differences between quark models and the limitations of the semiempirical model. Such matrix elements are also needed in the description of the $p$-wave nonleptonic hyperon decays[17] and in the study of the parity violating nucleon-pion scattering[21].

The matrix element of the axial-vector current between octet baryons and the $1/2^+$ resonance state N (1405)[3] for example, determines the axial-vector coupling constant as follows

$$g_A^* = \frac{5}{3}\left(UU^* - \frac{1}{3}\, VV^*\right) \equiv g_A^{p-n^*},$$

$$\left.\begin{matrix} UU^* \\ \\ VV^* \end{matrix}\right\} = \int \mathrm{d}^3 p_\varrho\, \mathrm{d}^3 p_\lambda\, G_P(p_\varrho, p_\lambda)\, G_R(p_\varrho, p_\lambda) \left\{\begin{matrix} \dfrac{E_3 + m}{2E_3} \\ \\ \dfrac{E_3 - m}{2E_3} \end{matrix}\right. . \qquad (2.22)$$

This form is especially suitable for the comparison with the $MIT$ bag model and $HOS$ model results. In the strict nonrelativistic limit ($E_3 \to m$) both $UU^*$ and $VV^*$ vanish. In the second case this is an identity, while in the first case this is caused by the orthogonality of the model wave functions $G_P$ and $G_R$. In the literature one finds one set of the model parameters ($\alpha$, etc.) for the ground state and the other set for the first excited state ($\alpha'$, etc.). If $\alpha \neq \alpha'$ there is no orthogonality, so $g_A^* \neq 0$ even in the nonrelativistic limit. Obviously this leads to an inconsistency and these results should not be taken too seriously even when the full relativistic formula (2.22) is used. On the other hand, one can claim that one is dealing with a semiempirical model and that the model parameters or $\alpha'$ have been determined by

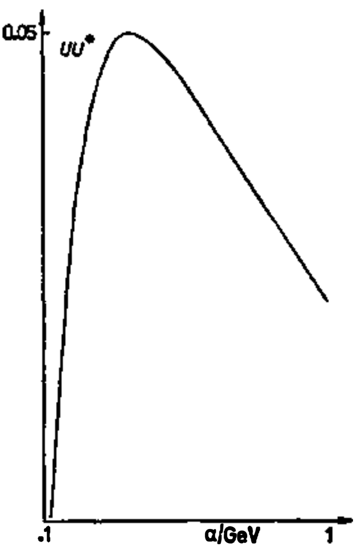


Fig. 5. The $UU^*$ matrix element calculated in the $RCHO$ model.

fitting experimental data. However, when non-diagonal overlap is concerned, one feels that the most resonable approach might be to use some average $\alpha$ for both $1/2^+$ and $(1/2^+)^*$ states. Some interesting results we present in Table 5 and illustrate in Figs. 5 and 6. Note that in the physical range of $\alpha$-values $(0.1 < \alpha < 1)$ the $g_A^*$ has the same maximum for both values of $m$ (see Fig. 6).

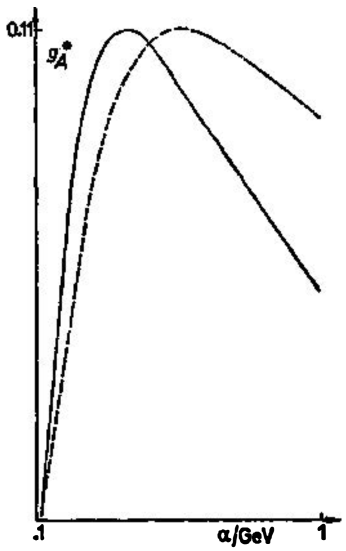


Fig. 6. The axial-vector coupling for the $p$—$n^*$ transition as a function of $\alpha$ and $m$.

TABLE 5

| Set No. | $UU^* = -VV^*$ | $g_S^*$ | $g_A^*$ | r. h. s. of G.-T. relation |
|---|---|---|---|---|
| 1 | 0.042 | 0.084 | 0.093 | 0.154 |
| 2 | 0.035 | 0.070 | 0.078 | 0.129 |
| 3 | 0.045 | 0.090 | 0.100 | 0.166 |
| 4 | 0.047 | 0.093 | 0.104 | 0.172 |
| 5 | 0.0485 | 0.097 | 0.108 | 0.179 |
| 6, 7 | 0.049 | 0.098 | 0.109 | 0.181 |
| Exp. | | (a) | | $0.42\,^{+0.22}_{-0.26}$ GeV |

(a) No experimental evidence.

The $UU^*$, $g_S^*$, $v_A^*$ and the r. h. s. of the G.-T. relation (2.23) for the N (1405) resonance calculated in the *RCHO* model.

It is interesting to check the validity of the Goldberger-Trieman relation for those states. For that reason in Table 5 the value of the r. h. s. of Eq. (2.23)

$$g^*_{RB\pi} f_\pi = \frac{1}{\sqrt{2}} (m_R + m_B)\, g^*_A \tag{2.23}$$

is shown. We have used the usual value $f_\pi = 128$ MeV, while $g^*_{RB\pi}$ was extracted from the particle data by assuming the simple coupling $g^*_{RB\pi}\, \overline{\psi}\, \gamma_5\, \psi \pi$. This procedure does not give the sign of (2.23). However it is remarkable that its magnitude agrees with the theoretical $g^*_A$ within a factor of 2.

In our calculational sheme the nondiagonal contributions ($g^*_S, g^*_A$) involving the decouplet $1/2^+$ resonances ($\Delta$ (1875), $\Delta$ (1925)) [3] vanish as there is no overlap with the octet nucleons, i. e. $\langle \underline{10} | \mathcal{J}^8_\mu | \underline{8} \rangle \equiv 0$.

*(ii) Scalar and vector densities*

For the sake of completeness we list some results for the matrix elements of scalar

$$\overline{\psi}\,(k_1)\, \psi\,(k_2), \tag{2.24}$$

and vector densities

$$\overline{\psi}\,(k_1)\, \gamma_0\, \psi\,(k_2). \tag{2.25}$$

The first are needed in the study of the parity-violating nucleon-pion scattering[21] while the calculation of the second provides a useful consistency check. In the notations defined by (2.9) and (2.22) one has for the scalar and for the vector form factor, respectively

$$g_S = U^2 - V^2, \tag{2.26a}$$

$$g^*_S = UU^* - VV^*, \tag{2.26b}$$

$$g_V = U^2 + V^2 = 1, \tag{2.26c}$$

$$v^*_V = UU^* + VV^* = 0. \tag{2.26d}$$

The last result follows only if the model wave functions are propertly orthonormalized, as was discussed above. The results can be found in Table 5.

## 3. *Comparison with MIT bag model*

The model wave functions for the ground state and the first radially excited state are of the form:

$$\psi_{MIT}\,(r) = \begin{pmatrix} \mathrm{i}\, u\,(pr) \\ \vec{\sigma}\, \vec{r}_0\, v\,(pr) \end{pmatrix}, \quad p = \frac{1}{R} \sqrt{\omega^2 - (m\, R)^2} \tag{3.1}$$

where the respective frequencies are $\omega_{-1,1} = 2.0428$ and $\omega_{-1,2} = 5.3960$ as follows from Refs. 1, 2. When $\omega_{-1,2}$ is used, we introduce the notation $u^*$ and $v^*$ instead of $u$ and $v$ for the components of the wave functions. Further useful notations are:

$$U^2_{MIT} = \int_0^R \mathrm{d}^3 r\, u^2(r); \quad (UU^*)_{MIT} = \int_0^R \mathrm{d}^3 r\, u(r)\, u^*(r),$$

$$V^2_{MIT} = \int_0^R \mathrm{d}^3 r\, v^2(r); \quad (VV^*)_{MIT} = \int_0^R \mathrm{d}^3 r\, v(r)\, v^*(r). \tag{3.2}$$

The axial-vector and vector coupling constants are given by formulas (2.9) and (2.16), respectively. The characteristic values for the $MIT$ bag model[1,2] are

$$\begin{aligned} U^2_{MIT} &= 0.74, \quad (UU^*)_{MIT} = -(VV^*)_{MIT} \\ V^2_{MIT} &= 0.26, \quad (UU^*)_{MIT} = 0.104. \end{aligned} \tag{3.3}$$

It is interesting to compare them with the values for the $RCHO$ model which are, for the various combinations of the model parameters, listed in Table 2. For both models the respective contributions have the same physical meaning. The $V$'s correspond to the so-called *small* components of the quark wave function, i. e. those that do vanish in the exact nonrelativistic limit. In the $RCHO$ model the $U^2/V^2$ ratio tends to be somewhat larger than in the $MIT$ bag model, thus leading to $g_A(RCHO) > g_A(MIT)$. The $RCHO$ $UU^*$'s*) tend to be smaller than the corresponding $MIT$ model quantities. For illustration see Table 5 and Figs. 5 and 6. As discussed above, in the exact nonrelativistic limit, the $CHO$ model ($RCHO \to CHO$) implies $UU^* \equiv 0$ ($VV^* \equiv 0$) on the basis of the orthogonality of the nonrelativistic quark model states. The nondiagonal axial-vector coupling constant is generally larger with the $MIT$ bag model than the $RCHO$ model predictions, $g_A^*(MIT) = 0.231$, while $g_A(MIT) = 1.09$ is smaller than the corresponding $RCHO$ value. Finally there are also $MIT$ bag model scalar densities $g_S(MIT) = 0.38$ and $g_S^*(MIT) = 0.204$. Analogously to $g_A(MIT)$ and $g_A^*(MIT)$, here also $g_S(MIT)$ is smaller, while $g_S^*(MIT)$ is larger than the corresponding $RCHO$ quantities.

It is not completely obvious how much this difference is due to the recoil corrections and how much it depends on the detailed dynamics of the model. It seems that the detailed dynamics, i. e. confining $HO$ potential instead of confining bag (or square-well[11]) is more important for the quantities which do survive in the $\vec{Q} \to 0$ limit, as is the case with $g_A$ and $g_S$. This is supported to some extent by the $HOS$ models, studied in the next section, which also lack any recoil effect.

* One has $g_V = U^2 + V^2 = 1$ and $g_V^* = UU^* + VV^* = 0$ as it should be with a proper normalization, independent of a particular quark model.

The magnetic moment which is associated with the recoil effect can be envisaged form the formula (2.10). A direct calculation in the models without recoil gives

$$\langle \frac{\vec{Q}}{2} = 0 \,|\vec{\mathcal{J}}(0)|\, -\frac{\vec{Q}}{2} = 0 \rangle \equiv 0, \tag{3.4}$$

so the magnetic moment has to be calculated by an ad hoc introduction of a suitable classically inspired operator, such as

$$\vec{r} \times \vec{\mathcal{J}}(\vec{r}). \tag{3.5}$$

A somewhat more general approach is possible by boosting quark model states. The details are discussed in Ref. 13. One finds

$$\langle \frac{\vec{Q}}{2} \,|\vec{\mathcal{J}}(0)|\, -\frac{\vec{Q}}{2} \rangle = \sum_i \eta_i \int_0^R \mathrm{d}^3\, e^{i\vec{Q}\vec{r}} \psi_i^\dagger(r) B^\dagger \left(\frac{\vec{Q}}{2}\right) \vec{\alpha} B \left(-\frac{\vec{Q}}{2}\right) \psi_i(r). \tag{3.6}$$

Here $B(p)$ is some boost operator while the summation runs over various quark flavors weighted by the factor $\eta_i$ according to their aboundances in a particular baryon. The standard value for the magnetic moment, corresponding to the operator (3.5), is obtained by neglecting boost $B$:

$$\langle 0 \,|\mathcal{J}(0)|\, 0 \rangle = \lim_{\vec{Q}\to 0} \sum_i \int_0^R \mathrm{d}^3 r\, e^{i\vec{Q}\vec{r}} \psi_i^\dagger(r)\, \vec{\alpha}\, \psi_i(r) =$$

$$= -i \chi^\dagger \frac{\vec{\sigma} \times \vec{Q}}{2M} \chi (4M) \lim_{\vec{Q}\to 0} \int_0^R \mathrm{d}^3 r\, u(r)\, v(r) \frac{j_1(Qr)}{Q} =$$

$$= i \chi^\dagger \frac{\vec{\sigma} \times \vec{Q}}{2M} \chi\, \mu_S(P), \tag{3.7}$$

where

$$\mu_S(P) = -\frac{4}{3} M \int_0^R \mathrm{d}^3 r\, r\, u(r)\, v(r).$$

The formulae (3.7) has entirely different structure from (2.13) found in the *RCHO* model. This is not surprising. In the *RCHO* model the recoil (i. e. motion of the center of the baryon mass) can be easily and naturally identified and separated in the model's dynamical scheme. This effect had to be put in by hand[13,14] in the *MIT* bag model assuming that spherical bags, which represent baryons, move with a velocity $\vec{Q}/2E$. The form of the expressions (3.7) is true for any static model and will be used in the discussion of the *HOS* models in the next section.*)

It is interesting to compare numerical values. With the parameters of Refs. 1, 2 one finds $\mu_S(P) = 1.90$, a value which is to small. The quoted value follows for the bag radius[1] $R = 5$ GeV$^{-1}$. A change to $R = 7.34$ GeV$^{-1}$ leads to $\mu_S(P) = 2.79\,(e/2M_p)$.

The expression for the proton charge radius can be derived from (3.7) by expanding the exponential and then averaging over the angles between $\vec{Q}$ and $\vec{r}$, i. e.

$$\int d^3 r\, e^{i\vec{Q}\vec{r}} \to -\frac{4\pi}{6} \vec{Q}^2 \int d^3 r\, r^2. \tag{3.8}$$

This results in the usual formula

$$\begin{aligned} \langle r^2 \rangle_P &= \langle r^2 U^2 \rangle_P + \langle r^2 V^2 \rangle_P \\ &= 4\pi \int_{\text{Bag}} r^4\, dr\, (u^2(r) + v^2(r)). \end{aligned} \tag{3.9}$$

The formula (3.9) is not significantly changed if one takes into account the boost $B$. As shown in Appendix, one can estimate such a contribution by adding the term

$$\frac{3}{4M^2} \tag{3.10}$$

to (3.9). Due to its small magnitude of 0.03 fm$^2$ this term is as unimportant as the similar term (2.20). This is very fortunate as both these terms constitute spurious relativistic correction to the essentially nonrelativistic baryon kinematics.

For better understanding of the problems with *MIT* bag model calculations, we listed in Table 6: $\mu(P)$ (3.7), $\langle r^2 U^2 \rangle_p + \langle r^2 V^2 \rangle_p = \langle r^2 \rangle_p$ and $g_A^{p-n}$ calculated for the there sets of parameters[1,26].

TABLE 6

| Set of parameters | $\mu_p$ $(e/2M_p)$ | $\langle r^2 U^2 \rangle_p$ + | $\langle r^2 V^2 \rangle_p$ = | $\langle r^2 \rangle_p$ (fm$^2$) | $g_A^{p-n}$ (a) |
|---|---|---|---|---|---|
| Ref. 1,2 | 2.55 | 0.35 | 0.17 | 0.52 | 1.09 |
| Ref. $26_A$ | 2.55 | 0.42 | 0.21 | 0.63 | 1.09 |
| Ref. $26_S$ | 3.44 | 0.75 | 0.37 | 1.12 | 1.09 |
| Exp. | 2.79 | | | 0.81 | 1.25 |

(a) The equality of $g_A$ for these three sets of parameters is the consequence of the fact that integrals $U^2$ and $V^2$, for light quark masses equal to zero, do not depend on the radius $R$ and are equal to 0.74 and 0.26, respectively.

Magnetic moment, charge radii and $g_A$ for nucleons for the three sets of *MIT* bag parameters.

* A very similar discussion for a *HOS* model was presented some time ago in Ref. 22.

Note here that Table 6. explicitely show us, for the calculations of $\langle r^2 \rangle_p$ in bag model, the importance of the smaller component of the wave function. Also the ratio $\langle r^2\, U^2 \rangle_p / \langle r^2\, V^2 \rangle_p = 2.05$ is independent on radius $R$ for the massless quarks in a bag.

## 4. *Harmonic oscillator shell models*

Several *HOS* models have been studied in the past (for example Refs. 5—9) in various physical contexts. In all those models the harmonic oscillator potential is fixed around the origin of the coordinate system. Such a situation corresponds closely to the square-well potential „shell model'' in nuclear physic and also parallels the *MIT* bag model. Relativistic corrections can be introduced in the same way one employed in the case of the *RCHO* model[5]. A somewhat more ambitious approach is to start with the Dirac equation with an external harmonic oscillator potential[8]. Although an exact solution can be found for the ground state[8], it is more practical to work with approximate wave functions[8,9] whose general structure is then the same for all the various *HOS* models and completely analogous to the *MIT* bag model structure. As an example one finds from Ref. 8:

$$\psi_{f,m}(\vec{r}) = \begin{pmatrix} \mathrm{i}\,\hat{u} \\ \vec{\sigma}\,\vec{r}_0\,\hat{v} \end{pmatrix} = N_f \begin{pmatrix} \Phi_f(r) \\ \mathrm{i}\,\dfrac{\xi_f}{x_f}\,\vec{\sigma}\,\vec{r}_0\,\Phi_f(r) \end{pmatrix} \chi_m,$$

$$\Phi_f(r) = \left(\frac{\xi_f^2}{\pi}\right)^{3/4} \exp\left(-\frac{1}{2}\,\xi_f^2\, r^2\right), \tag{4.1a}$$

$$N_f = \frac{3\,(E_f + m_f)}{4E_f + 2m_f - V_0},$$

$$\xi_f = \left(\frac{2}{3\sqrt{\pi}}\,k\,x_f\right)^{1/3}, \quad x_f = E_f + m_f.$$

For future use we also quote two possible sets (there are more of these) of parameter values[8)]

| Set A | Set B |
|---|---|
| $m_u = 0.290\, M_P$ | $m_u = 0.176\, M_P$ |
| $V_0 = -\ 0.30\, M_P$ | $V_0 = -\ 0.238\, M_P$ |
| $E_u = 0.830\, M_P$ | $E_u = 0.502\, M_P.$ |

(4.1b)

The wave functions of Ref. 5 are of the form

$$\psi_{f,m}(\vec{r}) = \begin{pmatrix} \mathrm{i}\,\hat{u}(r) \\ \vec{\sigma}\,\vec{r}_0\,\hat{v}(r) \end{pmatrix},$$

$$\hat{u}(r) = N \left(\frac{\beta}{\sqrt{\pi}}\right)^{3/2} \exp\left(\frac{-1}{2}\,\beta^2 r^2\right), \tag{4.2a}$$

$$\hat{v}(r) = \frac{1}{2m}\frac{\partial}{\sigma r}\hat{u}(r),$$

$$N = \left(1 + \frac{3}{8}\left(\frac{\beta}{m}\right)^2\right)^{-1/2}.$$

The characteristic values of the parameters are represented by the following sets

| | (A) | (B) | (C) | |
|---|---|---|---|---|
| $m$ (GeV) | 0.31 | 0.22 | 0.29 | |
| $\beta$ (GeV) | 0.24 | 0.32 | 0.29 | (4.2b) |

The $g_A^{p-n}$, magnetic moment and charge radii calculated in the model of Ref. 5 by using formulas (2.9a), (3.2), (3.7) and (3.10), respectively are:

$$g_A^{p-n} = \frac{5}{3}\left(1 + 4\frac{2m^2 - \beta^2}{8m^2 + 3\beta^2}\right), \tag{4.2c}$$

$$\mu(P) = \frac{M_P}{m}\left(1 + \frac{3}{8}\frac{\beta^2}{m^2}\right)^{-1}, \tag{4.2d}$$

$$\langle r^2 \rangle_P = \frac{3}{2}\beta^{-2} + \frac{3}{8m^2 + 3\beta^2}. \tag{4.2e}$$

The numerics are listed in Table 7. The parameter dependence is very illustrative and shows us explicitly that simultaneous fixing of all the above quantities is impossible.

The model of Ref. 9 is based on the Dirac equation but its radial coordinate dependence is described by a nonrelativistic harmonic oscillator with an energy dependent spring constant. Its wave functions are

$$\psi(\vec{r}) = \begin{pmatrix} i\,\tilde{u}(r) \\ \vec{\sigma}\,\vec{r}_0\,\tilde{v}(r) \end{pmatrix}\chi,$$

$$\tilde{u}(r) = \exp\left(-\frac{\varkappa^2 r^2}{6f}\right),$$

$$\tilde{v}(r) = \frac{1}{3}\varkappa\,\tilde{u}(r), \tag{4.3a}$$

$$f^2 = \varkappa(E + m).$$

TABLE 7

| Ref. param. | $g_A^{p-n}$ | $\mu_S(P)$ | $\mu_\sigma(P)$ | $\langle r^2 \rangle_P$ |
|---|---|---|---|---|
| 8/A | 1.22 | 1.45 | / | 0.36 |
| 8/B | 1.18 | 2.30 | / | 0.87 |
| 5/A | 1.25 | 2.47 | 2.63 | 1.14 |
| 5/B | 0.68 | 2.38 | 2.11 | 0.74 |
| 5/C | 1.09 | 2.35 | 2.45 | 0.81 |
| 9/A | 0.93 | / | 2.33 | 1.10 |
| 9/B | 1.28 | / | 2.65 | 2.20 |
| 9/C | 0.93 | / | 3.11 | 2.20 |
| E;p. | 1.25 | | 2.79 $(e/2M_p)$ | 0.81 (fm$^2$) |

The nucleon (proton) values in the *HOS* models.

For the massless quarks in the ground state $E = \varkappa$. Several sets of parameters were discussed in Ref. 9:

$$\begin{array}{llll} & \text{(A)} & \text{(B)} & \text{(C)} \\ m\,(\text{GeV}) & 0 & 0.127 & 0 \\ \varkappa\,(\text{GeV}) & 0.313 & 0.248 & 0.235 \\ f & 1 & 0.75 & 1 \end{array} \qquad (4.3b)$$

For the last set of parameters (i. e. C), the volume energy of a bag was also introduced.

General terms of the theoretical expressions for the weak axial coupling $g_A$ and the proton magnetic moment $\mu_p$ are given by (2.9), (3.2) and (3.7) or (3.8). Here the *MIT* bag model wave functions $u$ and $v$ have to be replaced by the appropriate $u$ and $v$ defined by (4.1), (4.2) and (4.3), respectively. The expression for the proton charge radius is given by (3.10). In Ref. 9 the proton magnetic moment was estimated in a somewhat different way by using

$$\begin{aligned} \mu_\sigma(P) &= \sum_i \langle p \,|\int \mathrm{d}^3 r\, \overline{\psi}(r)\, \sigma_i\, \psi(r) |\, p\rangle \frac{e_i}{2E} = \\ &\frac{e}{2E}\int \mathrm{d}^3 r \left(\tilde{u}^2 + \frac{1}{3}\tilde{v}^2\right) = \\ &= \frac{M_P}{E}\frac{6+f^3}{6+3f^2}\left(\frac{e}{2M_P}\right). \end{aligned} \qquad (4.4)$$

The replacement of $f^3$ by $3/4\ \beta^2/m^2$ in Eq. (4.4) gives us immediately the $\sigma$-magnetic moment of the proton for the *HOS* model defined by Eq. (4.2d). Some interesting numerics are summarized in Table 7. The $m$, $\beta$ dependence of $\langle r^2 \rangle_p$, $\mu\ (P)$ and $g_A^{p-n}$ for the *HOS* model is illustrated in Fig. 7.

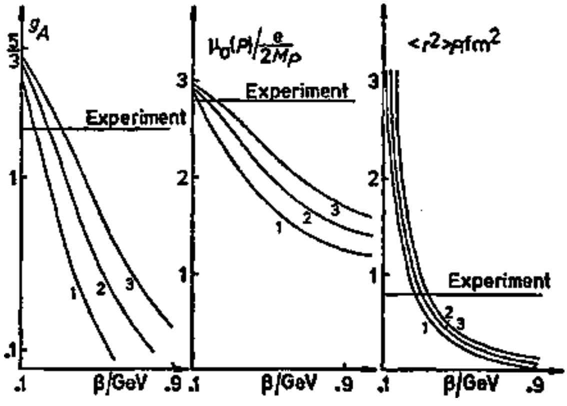


Fig. 7. The axial-vector coupling for the $p-n$ transition, magnetic moment $\mu_0(P)$ and the charge radius of the proton calculated in the *HOS* model (the formulas (4.2c,d) and (4.4)). The curves 1, 2, and 3 correspond to the $m = 0.22$, 0.31 and 0.40 GeV, respectively.

## 5. *Discussion*

It seems that the overall success of the harmonic oscillator quark models is quite good. They can reasonably well reproduce form factors which do not depend crucially on the recoil, such as $g_A$. In that respect they do not seem worse than the *MIT* bag model, and they might be actually slightly better. The situation with the non-diagonal form factors, such as $g_A^*$ is yet not quite clear. But even here harmonic oscillator models offer a practical advantage through the existence of the thorough and complete analyses[3,21] of the baryonic resonance. Their results can be easily improved by the configuration mixing [24].

It is difficult to say whether the relativistic internal quark motion is more adequately modeled by the *HO* potential or by the *MIT* bag. While the first approach produces better $g_A^*$ the second approach is certainly formally and physically more appealing.

There is no doubt that the *RCHO* model offers distinct advantages when dealing with recoil, or better momentum transfer, dependent quantities such as magnetic moment and charge radii. The *HOS* and *MIT* bag models have to introduce either ad hoc operators (see (3.5) and (3.9)) or to resort to somewhat artificial boosting. Unfortunately, the situation is not quite clear with the most interesting quantity, the magnetic moment. Special physical interest lies here in the fact that through this quantity eventually the internal structure of the quarks might show up.

Here the careful study of Table 8 is very instructional. In the framework of the *RCHO* model $g_A$, $\mu(P)$ and $\langle r^2\rangle^{1/2}$ can all be fitted with the same parameters with the accuracy of 15% or better. The parameters are somewhat different from the ones used in the baryon spectroscopy[3,4,20] and in the analysis of the nonleptonic decays[5,18]. But the differences are not too large. It certainly seems that nonleptonic decays could be fitted*) with the parameters used in Table 8. Thus it seems that electroweak processes can be understood in the *RCHO* model by using point-like quark (i.e. elementary quark).

* There are uncertainties with *QCD* corrections (enhancement) and with the dynamics which allow are 30% variation of $\alpha$[18].

TABLE 8

| $a^{(a)}$ | $\mu\,(e/2M_p)$ | $\langle r^2\rangle^{1/2}$ (fm) | $(g_A/g_V)^{p-n}$ |
|---|---|---|---|
| 0.25 | 2.74$^{(b)}$ | 0.96 | 1.33 |
| 0.27 | 2.63 | 0.90 | 1.31 |
| 0.32 | 2.36 | 0.79 | 1.25 |
| Exp. | 2.79 | 0.90 | 1.25 |

(a) $m_u^* = m_d = m = 0.22$ GeV

(b) Note that for $a = 0.24$ GeV we have found $\mu\,(e/2M_p) = 2.8$

The charge radii, magnetic moment and $g_A/g_V$ for nucleons calculated in the *RCHO* model.

Additional uncertainty exists about the harmonic oscillator constant which has been very nicely discussed in connection with the pionic form factor needed in the proton decay calculation[25]. One value (larger) of $a$ emerges from the fit to the baryon spectrum, and it is used in the description of the baryon resonances and of photon and pion couplings. Smaller $a$ is needed to reproduce the experimental charge radius of the proton. However, there is some indication shown in Table 4 that the relativistic correction (2.18b) might be extremely helpful[6] in this quandry.

Although *HOS* models are less flexibile when dealing with such problems, their predictions, as shown in Table 7, are not hopelessly wrong. Besides, being formally simple and more manageable, they are eminently more practical in some very important investigations[7].

Overall one is forced to conclude that quark models have come to stay and that there is no understanding of the baryonic physics without taking into account the quark structure. Thus produced theoretical predictions always have the correct order of magnitude and show the correct trend (i. e. relative magnitudes) when moving through the baryonic spectrum. Very educational in this respect are the Goldberger-Trieman relations for the nondiagonalized $g_A^*$ displayed in Table 5.

Our general conclusion is that the *RCHO* model at this stage of the quark model development has the advantage over other quark models when investigating momentum transfer dependent form factors.

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## *Appendix*

Integration over the angles in formula (3.6a) and in the formula which is obtained from (3.6a) by the replacement $\vec{\alpha} \to 1$ can be explicitly performed if one integrates over the sphere. A comparison with (2.10) gives

$$G_E(\vec{Q}^2) = 4\pi \int r^2 \,\mathrm{d}r\, (u^2(r) + v^2(r))\, j_0(Qr), \tag{1}$$

$$G_M(\vec{Q}^2) = 4\pi \int r^2 \,\mathrm{d}r \left(-\frac{4M}{Q} u(r)\, v(r)\right) j_1(Qr), \tag{2}$$

$$Q = \sqrt{\vec{Q}^2}.$$

Here $j_1(Qr)$ are spherical Bessel functions. It is easy to see that $G_M(0)$ leads to the expression (3.8). However, due to the boost the integration in (3.6a) is not over the sphere but over the rotational elipsoid which suffers Lorentz contraction in the direction of the motion. This effect can be approximately estimated by multiplying (1) and (2) by the contraction factor

$$F(\vec{Q}^2) = \sqrt{1 - \frac{\vec{Q}^2}{4E^2}}, \quad E^2 = \frac{1}{4}\vec{Q}^2 + M^2. \tag{3}$$

This factor does not make any difference to the value (3.10). The charge radius is

$$\langle r^2 \rangle_P = -6 \frac{\partial G_E}{\partial \vec{Q}^2}\Big|_{\vec{Q}^2=0} - 6\, G_E \frac{\partial F}{\partial \vec{Q}^2}\Big|_{\vec{Q}^2=0} \tag{4}$$

The first term in (4) gives the expression (3.9). The second term ($G_E(0) = 1$) leads to (3.10). This correction is very small as we are dealing with low momentum transfer, i. e. with the nonrelativistic situation.

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## MODEL HARMONIČKOG OSCILATORA I IMPULSNO OVISNI EFEKTI

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Istraživani su barionski form faktori za nukleone i 1/2* rezonance. Općenito formule za račun strujnih form faktora omogućuju usporedbu i zajednički pogled na različite ranije objavljene rezultate. Također je načinjena usporedba sa *MIT* modelom vreće. Prikazano je i uspoređeno nekoliko međusobno neekvivalentnih pravila za proračun magnetskih momenata. Osobito je bilo važno pokazati da postoji jedan model i jedan set parametara koji može simultano fiksirati radius naboja, magnetski moment i konstantu aksialnog vezanja za nukleone. Tabela 8 pokazuje da taj zahtjev najbolje zaodvoljava korelirani i relativistički popravljeni model harmoničkog oscilatora (*RCHO*).