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ON THE DUFFIN-KEMMER PROPAGATOR IN SCALAR ELECTRODYNAMICS

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The first approximation of the Schwinger-Dyson equation for spin-0 particle propagator in the Duffin-Kemmer formalism is derived for arbitrary space-time dimension. The Duffin-Kemmer propagator is presented in the simple and very effective form. It is shown that even in the simplest 2-dimensional scalar electrodynamics there is no finite solution.

1. *Introduction*

It is well known that the (pseudo) scalar particle is usually described by the Klein-Gordon (*KG*) field. This convention is also frequently used in scalar electrodynamics—the theory describing interaction between charged spin-0 particles and photons. On the other hand, in the late thirties, an alternative approach was introduced to described spin-0 fields which is known in literature as the Duffin[1]-Kemmer[2] (*DK*) formalism. In the *DK* formalism spin-0 particle is described by a 5-component field which is solution to the *DK* first-order differential equation of motion. An introductory review of scalar electrodynamics in terms of the *DK* wave functions is given in the book of Akhiezer and Berestetskii[3].

The question concerning the equivalence and advantage of one formalism to the other is not yet definitely solved. It is not difficult to conclude that the *DK* and the KG formulations of the free fields are completely equivalent. But, when we use these fields to describe scalar particle in an interaction, some differences may appear. About 10 years ago there was controversy on the equivalence in the use of the 5-component DK and 1-component KG wave functions in the presence of symmetry breaking, i. e. the interacting fields describing two spin-0 particles with different masses, like K_{l3} decay[4]. Starting with the fundamental dynamical principle, like local gauge invariance, the formalism which gives better agreement with experimental data is more appropriate and should have advantage in its use. When formalisms give the same theoretical results they are equivalent. Gauging the free Lagrangian of the charged scalar field one gets scalar electrodynamics with different interaction Lagrangians in the DK and the KG approaches but they are equivalent at all approximations of the ordinary S-matrix expansion and hence at the level of complete S-matrix.

It is also well known that the usual perturbation expansion of the S-matrix leads to the ultraviolet (UV) divergences. To reach finite (without UV divergences before applying the renormalisation procedure) spinor electrodynamics Johnson, Baker and Willey[5] have introduced modified perturbation expansion of the Schwinger-Dyson (SD) equations in such a way that each order of the new perturbation expansion contains an infinite sum of the standard Feynman diagrams. In such approach the UV divergences may be mutually annihilated giving finite result. They have shown that to reach complete finitness of spinor QED the appropriate value of the gauge parameter should be chosen, the bare electron mass should be zero and the fine-structure constant should be positive solution of the definite equation. The Johnson-Baker-Willey (JBW) approach gives a way of how to investigate dynamical mass generation in spinor QED and this problem is now rather well analysed[6]. Although the JBW programme for a finitness is very successful in spinor QED, it seems that there are serious problems to be directly applicable to the other quantum field models.

Finitness of scalar electrodynamics in the Klein-Gordon (KG) formalism along the lines of the JBW procedure was analysed bu Fry[7] who concluded that internally consistent finite scalar QED probably does not exist. The first non-trivial approximation of the SD equation for the GK propagator in scalar QED is investigated recently[8] and obtained results are respectable deserving further investigation. The aim of the present work is investigation of the first nontrivial approximation of the SD equation for the DK propagator. It is motivated by an idea on the inequivalence of the approximated SD equations (for spin-0 particle in scalar QED) in the DK and the KG formalisms. Namely, comparing these formalisms it is natural to expect that inequivalent set of Feynman diagrams in the first- and the higher-order approximations are contained. Hence, it is possible to obtain one result in the DK procedure which is absent in the KG procedure and vice versa. The present work completes a previously published one[9] with special attention to the general structure of the DK propagator as well as to the 2-dimensional case.

This paper is organized as follows. In § 2 we present some basic properties of the DK formalism for arbitrary space-time dimension. In § 3, scalar electrodynamics in the DK formalism and the SD equation for spin-0 particle are introduced. Section 4 is devoted to analysis of the UV divergences and investigation of the first approximation of the SD equation. Some more technical information on the SD formalism is given in Appendix.

2. Some properties of the formalism for arbitrary space-time dimension

In this section we develop the DK formalism for arbitrary space-time dimension (d). We introduce some useful properties and make connection between the DK and the KG formalism.

As it is known, the KG equation in d dimensions is

$$(\Box - m_0^2)\, \varphi (x) = 0 \tag{1}$$

where $\Box = -\partial^\mu \partial_\mu$, ($\mu = 0, 1, 2, \ldots, d - 1$), $g^{\mu\nu} = 0$ ($\mu \neq \nu$), $g^{\mu\nu} = 1$ ($\mu = \nu = 0$), $g^{\mu\nu} = -1$ ($\mu = \nu \neq 0$). Introducing a column wave function

$$\chi (x) = \begin{pmatrix} \chi_\mu(x) \\ \chi_d (x) \end{pmatrix}, \quad \chi_\mu (x) = \frac{\mathrm{i}}{\sqrt{m_0}}\, \partial_\mu \varphi (x), \quad \chi_d (x) = \sqrt{m_0}\, \varphi (x), \tag{2}$$

we can rewrite the KG equation in the form of the first-order differential matrix equation

$$(\mathrm{i}\, \beta^\mu \partial_\mu - m_0)\, \chi (x) = 0 \tag{3}$$

where β^μ are $(d + 1) \times (d + 1)$ matrices

$$\beta^\mu = \varepsilon^{\mu d} + g^{\mu\mu}\, \varepsilon^{d\mu} \tag{4}$$

which satisfy the following relations

$$\beta^\mu \beta^\nu \beta^\varrho + \beta^\varrho \beta^\nu \beta^\mu = g^{\mu\nu} \beta^\varrho + g^{\varrho\nu} \beta^\mu \tag{5}$$

(for ε-matrices see Appendix). Note that in (4) and some subsequent expressions there is no sum over repeated index μ. Equation (3) is known as the DK equation when corresponding β^μ matrices obey relation (5). Expression (4) is a real representation of β^μ matrices.

Taking $\mu = \nu \neq \varrho$ we can derive from (4)

$$\eta^\mu \beta^\varrho + \beta^\varrho \eta^\mu = 0$$

where $\eta^\mu = 2\beta^\mu \beta^\mu - g^{\mu\mu}$. It is easy to check that

$$\eta^\mu\, \eta^\mu = 1$$

$$\eta^\mu \eta^\nu = \eta^\nu \eta^\mu$$

$$\beta^{\mu +} = \tilde{\beta}^\mu = \eta^0 \beta^\mu \eta^0$$

where $+$ and $\sim$ denote adjoint and transpose operations, respectively, From the last equation and equation (3), it follows that the DK conjugate field $\bar{\chi} (x) = \chi^+ (x)\, \eta^0$ satisfy equation

$$-\mathrm{i}\, \partial_\mu \bar{\chi} (x)\, \beta^\mu - m_0 \bar{\chi} (x) = 0. \tag{6}$$

According to the above described procedure one can write the DK equation for the case when the particle mass is equal to zero ($m_0 = 0$)

$$(\mathrm{i}\,\beta^\mu \partial_\mu - \mu\beta)\,\chi(x) = 0 \tag{7}$$

where μ is an arbitrary parameter different from zero and β is a projective matrix to d dimensional space of component field $\chi(x)$ (see Appendix). Instead of (3) and (7) one can write only one equation

$$(\mathrm{i}\,\beta^\mu \partial_\mu - M\beta - m_0\overline{\beta})\,\chi(x) = 0 \tag{8}$$

where $\overline{\beta} = 1 - \beta = \varepsilon^{dd}$ and

$$M = \begin{cases} m_0, & m_0 \neq 0 \\ \mu \neq 0, & m_0 = 0 \end{cases}. \tag{9}$$

$$\chi_\mu = \frac{\mathrm{i}}{\sqrt{M}}\,\partial_\mu\varphi, \quad \chi_d = \sqrt{M}\,\varphi.$$

Lagrangian for the free DK field is

$$\mathscr{L}_{DK}(x) = \overline{\chi}(x)\,(\mathrm{i}\,\beta^\mu\partial_\mu - M\beta - m_0\overline{\beta})\,\chi(x). \tag{10}$$

The corresponding propagator for scalar particle, which we shall call the DK propagator,

$$\langle 0\,|T\,\{\chi(x)\,\overline{\chi}(0)\}|\,0\rangle \tag{11}$$

is not strictly equal to a Green's function $T_0(x)$,

$$(\mathrm{i}\,\beta^\mu\partial_\mu - M\beta - m_0\overline{\beta})\,T_0(x) = -\delta(x), \tag{12}$$

but in an effective approach they may be taken as equal[3]. Hence, we work in this effective approach where the bare DK propagator is

$$T_0(k) = (M\beta + m_0\overline{\beta} - \hat{k} - \mathrm{i}\varepsilon)^{-1} = \frac{\hat{k}(\hat{k} + M) + m_0^2\beta + M^2\overline{\beta} - k^2}{M(m_0^2 - k^2 - \mathrm{i}\varepsilon)}. \tag{13}$$

It is very useful to introduce the constant DK field U, which has all components equal to 0 except the last component which is equal to 1, i. e.

$$U = \begin{pmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{pmatrix} \qquad \overline{U} = (00\,\ldots\,1). \tag{14}$$

It can be shown that the DK propagator is connected with the KG propagator as follows

$$\frac{1}{M} \bar{U} T_0 (k) U = \Delta (k) = (m_o^2 - k^2)^{-1}. \tag{15}$$

3. The DK Abelian gauge theory and the corresponding SD equation for scalar particle

We are now going to introduce scalar electrodynamics in the DK formalism. For this reason we shall consider the DK field like matter field and introduce its interaction with electromagnetic field following modern gauge approach. Lagrangian (10) is invariant in respect to the global gauge invariant group $U(1)$

$$\chi (x) \to \chi' (x) = e^{ie_o\Theta} \chi (x), \quad \bar{\chi} (x) \to \bar{\chi}' (x) = \bar{\chi} (x) e^{-ie_o\Theta}$$

where Θ is x independent parameter. An interaction with gauge vector field introduces demanding theory which should be invariant with respect to the local gauge transformations

$$\chi (x) \to \chi' (x) = e^{ie_o\Theta(x)} \chi (x), \quad \bar{\chi} (x) \to \bar{\chi}' (x) = \bar{\chi} (x) e^{-ie_o\Theta(x)}$$

$$A'_\mu (x) = A_\mu (x) + \partial_\mu \Theta (x). \tag{16}$$

Practically, it means that in the free DK Lagrangian $\mathscr{L}_{DK}$ we have to write covariant derivatives

$$D_\mu = \partial_\mu - ie_o A_\mu (x) \tag{17}$$

instead of ∂_μ and add gauge invariant Lagrangian for the free electromagnetic field

$$\mathscr{L}_A = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{18}$$

Total Lagrangian

$$\mathscr{L} = \bar{\chi} (i\beta^\mu \partial_\mu - M\beta - m_o \bar{\beta}) \chi + e_o \bar{\chi} \beta^\mu \chi A_\mu - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{19}$$

has the same form as it is in spinor electrodynamics. However there are two important differences which we should have in mind: (1) the DK wave function is a boson field and the Dirac wave function is a fermion field (in functional integration the DK field is an even and the Dirac spinor field is an odd element of the Grassmann algebra); 2) algebra of β^μ matrices (4) is different from well known algebra of the Dirac γ^μ matrices.

The corresponding quantum DK theory (perturbation expansion, Feynman rules, Green's and vertex function, SD equations, ...) can be formulated using functional integration having the same form as for a spinor QED. So all expressions are of the same form but in direct calculations we have to take into consideration features 1) and 2). According to this statement the SD equation for the DK propagator in scalar QED is[10]

$$T^{-1}(p) = T_o^{-1}(p) - \frac{ie_o^2}{(2\pi)^d} \int d^d q \, D_{\mu\nu}(p-q)\, \beta^\mu T(q)\, \Gamma^\nu(p,q) =$$
$$= T_o^{-1}(p) + \Sigma(p) \tag{20}$$

where T is the DK propagator, $D_{\mu\nu}$ is the electromagnetic propagator, Γ^ν is the vertex function and Σ is the self-energy of the scalar particle. Note also that (20) is the unrenormalised SD equation for the complete DK propagator in an energy-momentum space with arbitrary dimension d. We shall investigate the SD equation (20) according to the approach developed for finite spinor QED[5,6].

4. *The first-order approximation of the SD equation*

We can now consider the UV divergences for scalar QED in the d-dimensional DK formalism. For this reason one introduces degree of divergence $\omega_d(G)$ for each one-particle irreducible Feynman graph G with n vertices, I internal lines and E external lines. From the Feynman rules and the propagators behaviour

$$D^0_{\mu\nu}(k) \sim k^{-2}, \quad T_0(k) \sim P_2(k)\, k^{-2}, \quad (k \to \infty) \tag{21}$$

where $P_2(k) = \hat{k}^2 - k^2 + M\hat{k} + m_o^2\beta + M^2\bar{\beta}$, it follows the expression for power counting

$$\omega_d(G) = I_A(d-2) + I_\chi(d-2) + C - d(n-1) \tag{22}$$

where C is contribution from a polinom $P_2(k)$. Using relation

$$(\hat{k}^2 - k^2) \prod_{i=1}^{2r+1} \beta^{\mu_i} (\hat{k}^2 - k^2) = 0, \tag{23}$$

which is easy to check with (A.7), we conclude that $C = n$. Taking into consideration topology relations

$$n = E_A + 2I_A, \qquad 2n = E_\chi + 2I_\chi \tag{24}$$

we get

$$\omega_d(G) = \frac{d-4}{2} n + d - \frac{d-2}{2}(E_A + E_\chi). \tag{25}$$

Note that expression for the degree of divergence $\omega_d(\Theta)$ in the DK formalism differs from that in spinor QED. For $d = 4$ $\omega_d(G)$ does not depend on number of vertices and for $d = 2$ it does not depend on the external lines.

For the DK self-energy $\Sigma(p)$ we have ($E_A = 0$, $E_\chi = 2$)

$$\omega_d(\Sigma) = \frac{d-4}{2} n + 2 = \begin{cases} 2, & d = 4 \\ 2 - n, & d = 2. \end{cases} \tag{26}$$

From (26) it follows that the DK propagator in 4-dimensional case is quadratically divergent and in 2-dimensional scalar QED is logaritmically divergent with only one-loop divergence ($n = 2$) .

We shall introduce now the first approximation of the SD equation (20) according to the procedure[6] already used in spinor QED. Making approximations

$$\Gamma^\nu(p, q) \to \beta^\nu$$

$$D_{\mu\nu}(k) \to D^0_{\mu\nu}(k) = -\frac{1}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) - \frac{G}{k^2}\frac{k_\mu k_\nu}{k^2} \tag{27}$$

we obtain the wanted first-order approximation of the SD equation

$$T^{-1}(p) = M\beta + m_0\overline{\beta} - \hat{p} - \frac{\mathrm{i}\, e_0^2}{(2\pi)^d}\int \mathrm{d}^d q\, D^0_{\mu\nu}(p-q)\,\beta^\mu\, T(q)\,\beta^\nu. \tag{28}$$

The DK propagator $T(p)$ and its inverse $T^{-1}(p)$ can be expanded in terms of 5 linearly independent matrices $s_i(p)$ as follows

$$T(p) = \sum_{i=1}^{5} S_i(-p^2)\, s_i(p), \quad T^{-1}(p) = \sum_{i=1}^{5} T_i(-p^2)\, s_i(p) \tag{29}$$

where

$$s_1 = \beta, \quad s_2 = \overline{\beta}, \quad s_3 = \beta\hat{p}, \quad s_4 = \overline{\beta}\hat{p}, \quad s_5 = \beta\hat{p}\hat{p}. \tag{30}$$

Substituting (27), (29) and (30) into (28) and then multiplying such equation by all matrices s_i from (30) and taking traces, we obtain, after rather large calculations, system of 5 integral equations. After the Wick rotation, integration over spherical angles in Euclidean momentum space and use of relations (A 13) we get

$$1.\ dT_1(x) - xT_5(x) = dM + g_d\int_0^\infty y^{\frac{d-2}{2}} I_2^0 (G - 1 + d)\, S_2(y)\, \mathrm{d}y$$

$$2.\ T_2(x) = m_0 + g_d \int_0^\infty y^{\frac{d-2}{2}} \Big\{ (G - 1 + d)\, I_2^0 S_1(y) -$$

$$- \left[y I_2^0 + \frac{G-1}{4}\left((x-y)^2 I_4^0 + 2(y-x) I_2^0 + I_0^0\right)\right] S_5(y) \Big\}\, \mathrm{d}y$$

$$
\begin{aligned}
&3.\ xT_4(x) = -x + g_d \int_0^\infty y^{\frac{d-2}{2}} \Big\{ -\frac{1}{2} I_0^0 + \frac{x+y}{2} I_2^0 + \\
&+ \frac{G-1}{4} [(x-y)^2 I_4^0 - I_0^0] \Big\} S_3(y)\, \mathrm{d}y \\
&4.\ xT_3(x) = -x + g_d \int_0^\infty y^{\frac{d-2}{2}} \Big\{ -\frac{1}{2} I_0^0 + \frac{x+y}{2} I_2^0 + \\
&+ \frac{G-1}{4} [(x-y)^2 I_4^0 - I_0^0] \Big\} S_4(y)\, \mathrm{d}y \\
&5.\ xT_1(x) - x^2 T_5(x) = xM + g_d \int_0^\infty y^{\frac{d-2}{2}} \Big\{ x I_2^0 + \frac{G-1}{4} [(x-y)^2 I_4^0 + \\
&+ 2(x-y) I_2^0 + I_0^0] \Big\} S_2(y)\, \mathrm{d}y
\end{aligned}
\tag{31}
$$

where $x = -p^2 = \vec{p}^{\,2} + p_4^2$, $y = -q^2 = \vec{q}^{\,2} + q_4^2$, $g_d = \frac{e_0^2}{2(2\pi)^d}$ and

$$
I_m^n = \int \frac{(pq)^n \, \mathrm{d}^d \Omega_q}{(p-q)^m} \qquad \begin{aligned} n &= 0, 1, 2, \ldots \\ m &= 0, 2, 4, \ldots \end{aligned}
\tag{32}
$$

In the further investigation we shall turn to the 2-dimensional case because it has the lowest divergence and the 4-dimensional case is already partially investigated[9]. For $d = 2$ we have

$$
\begin{aligned}
I_0^0 &= 2\pi \qquad I_2^0 = \frac{2\pi}{x-y} [\Theta(x-y) - \Theta(y-x)] \\
I_4^0 &= \frac{2\pi(x+y)}{(x-y)^3} [\Theta(x-y) - \Theta(y-x)]
\end{aligned}
\tag{33}
$$

where $\Theta(x) = 1$ $(x > 0)$ and $\Theta(x) = 0$ $(x < 0)$. We find it interesting to investigate the existence of possible finite solutions. It can be done analysing asymptotic behaviour of all integrals in (31). For this reason we take

$$
I_2^0 (y \gg x) = \frac{2\pi}{y}, \qquad I_4^0 (y \gg x) = \frac{2\pi}{y^2}
\tag{34}
$$

and the bare value of the propagator functions $S_i(y)$. After some calculations we obtain the UV logaritmic divergence only in the integral of $T_2(x)$. Since gauge parameter G in front of this integral disappears, we cannot cancel this divergence. It follows that the first non-trivial approximation of the DK propagator in $d = 2$

scalar electrodynamics cannot be finite. Since this is the problem with only one--loop divergence it probably cannot be finite for a higher-order approximations of the SD equation.

Note that the system of equations (31) for $d = 2$ scalar QED can be simplyfied taking $G = -1$. Substituting (33) into (31) and performing relevant calculations we obtain

$$\begin{aligned}
&1.\ T_1(x) = M + \frac{\varepsilon}{x}\int_0^x S_2(y)\,\mathrm{d}y \\
&2.\ T_2(x) = m_0 + \varepsilon \int_x^\infty S_5(y)\,\mathrm{d}y \\
&3.\ T_3(x) = T_3^0(x) = -1 \\
&4.\ T_4(x) = T_4^0(x) = -1 \\
&5.\ T_5(x) = \frac{2}{x}(T_1(x) - M)
\end{aligned} \tag{35}$$

where $\varepsilon = \frac{e_0^2}{4\pi}$.

In (35) we have two integral equations instead of five equations in (31). We can rewrite it in a form of a non-linear first-order differential boundary value problem, but because our main interest is the investigation of finite solutions we shall stop here.

5. *Conclusion*

We have investigated the first approximation of the SD equation for the DK propagator. Our investigation is performed in close procedure to that in spinor QED. This problem is more complicated in scalar QED than in spinor QED. It is a consequence of a more complicated algebra of β^μ compared to γ^μ matrices, so that we have five basic functions in the DK propagator (instead of two in spinor QED). On the other hand, the DK propagator is more UV divergent than the Dirac one.

We have shown in an explicit form that there are no finite solutions in the first--order approximation of the 2-dimensional DK propagator. This conclusion should also be valid for higher-order approximations because the UV divergence is present in the one-loop diagram which cannot be removed by an appropriate choice of the gauge parameter G. From preliminary investigation[9] of $d = 4$ case it follows that the approximated DK propagator cannot be made free of the UV divergences by any choice of the parameters in the theory. The 3-dimensional case is not explicitely investigated here because it is a more complicated problem (I_m^n are more complicated) and we expect the same conclusion about finitness, as for $d = 2, 4$ cases.

Scalar electrodynamics including self-interaction term of $\lambda\varphi^4$ type should be more appropriate to get finitness. For the KG propagator it is shown[8] that this inclusion leads to the finite solution (for $d = 2, 4$). We hope that the DK propagator, which will include scalar particle self-interaction, will also be finite in the first approximation.

Appendix

In this Appendix we shall give some new properties of the DK formalism which are important for a better understanding of the performed calculations in the present work.

Let us define $(d + 1)$-dimensional *vectors*

$$e^A = \begin{pmatrix} \delta^A_\mu \\ \delta^A_d \end{pmatrix} = \begin{pmatrix} \delta^A_0 \\ \cdot \\ \cdot \\ \cdot \\ \delta^A_{d-1} \\ \delta^A_d \end{pmatrix}, \qquad (A = 0, 1, \ldots, d) \tag{A1}$$

whith only one element different from 0 and $(d + 1) \times (d + 1)$ matrices

$$\varepsilon^{AB} = e^A \tilde{e}^B \tag{A2}$$

where $\sim$ denotes transposing. Matrix ε^{AB} also has only one element different from 0

$$(\varepsilon^{AB})_{CD} = \delta^A_C \delta^B_D. \tag{A3}$$

The following properties have place

$$\varepsilon^{AB} e^C = \delta^{BC} e^A, \quad \tilde{e}^A \varepsilon^{BC} = \delta^{AB} \tilde{e}^C, \quad \varepsilon^{AB} \varepsilon^{CD} = \delta^{BC} \varepsilon^{AD}. \tag{A4}$$

It is useful to introduce new symbols

$$\begin{aligned} U &= e^d & \overline{U} &= \tilde{e}^d \\ \overline{\beta} &= U\overline{U} = \varepsilon^{dd}, & \beta &= \sum_\mu \varepsilon^{\mu\mu} \end{aligned} \tag{A5}$$

because of their frequent use. We have the properties for $U\,(\overline{U})$

$$\beta^\mu U = e^\mu, \qquad \overline{U}\beta^\mu = g^{\mu\nu}\tilde{e}^\nu \tag{A6}$$

and the properties for $\beta\,(\overline{\beta})$

$$\beta^2 = \beta, \quad \overline{\beta}^2 = \overline{\beta}, \quad \beta + \overline{\beta} = 1, \quad \beta\overline{\beta} = \overline{\beta}\beta = 0,$$

$$\beta\beta^{\mu} = \beta^{\mu}\overline{\beta}, \quad \overline{\beta}\beta^{\mu} = \beta^{\mu}\beta, \quad \overline{\beta}\beta^{\mu}\beta^{\nu} = g^{\mu\nu}\overline{\beta},$$
$$\mathrm{Tr}\,(\beta\hat{p}) = \mathrm{Tr}\,(\overline{\beta}\hat{p}) = 0, \qquad \mathrm{Tr}\,(\beta\hat{p}\hat{q}) = (pq)$$
$$\hat{p}\hat{p}\hat{p} = p^2\hat{p} \qquad (\hat{p} = p_{\mu}\beta^{\mu}). \tag{A7}$$

The matrices β and $\overline{\beta}$ are connected with β^{μ} matrices

$$\beta = \frac{1}{d-1}(d - \beta^{\mu}\beta_{\mu}), \quad \overline{\beta} = \frac{1}{d-1}(\beta^{\mu}\beta_{\mu} - 1). \tag{A8}$$

From the above properties one can derive relations for traces of multiplied β^{μ} matrices

$$\mathrm{Tr}\,(\beta^{\mu_1} \ldots \beta^{\mu_{2n+1}}) = 0$$
$$\mathrm{Tr}\,(\beta^{\mu_1} \ldots \beta^{\mu_{2n}}) = \prod_{i=1}^{n} g^{\mu_{2i-1}\mu_{2i}} + g^{\mu_{2n}\mu_1}\prod_{i=1}^{n-1} g^{\mu_{2i}\mu_{2i+1}}. \tag{A9}$$

To expand the DK propagator in the necessary form one has introduce basis which is a set of $(d+1) \times (d+1)$ matrices in the DK space and the scalars in the Lorentz space. Using the algebra (5) of the β^{μ} matrices we conclude that a complete set of linearly independent matrices contains 5 elements. We use a very simple and effective set $\{s_i(p)\}$

$$s_1(p) = \beta, \quad s_2(p) = \overline{\beta}, \quad s_3(p) = \beta\hat{p}, \quad s_4(p) = \overline{\beta}\hat{p}, \quad s_5(p) = \beta\hat{p}\hat{p}, \tag{A10}$$

(the corresponding set in the Dirac formalism has only 2 elements: $s_1 = 1$, $s_2 = p_{\mu}\gamma^{\mu}$). Now, we can write the DK propagator in a complete form

$$T(p) = \sum_{i=1}^{5} S_i(-p^2)\, s_i(p)$$

and its inverse propagator

$$T^{-1}(p) = \sum_{i=1}^{5} T_i(-p^2)\, s_i(p).$$

From the condition

$$T^{-1}(p) \cdot T(p) = T(p) \cdot T^{-1}(p) = 1 \tag{A11}$$

we can get relations between $S_i(x)$ and $T_j(x)$

$$S_1 = \frac{1}{T_1}, \quad S_2 = \frac{T_1 - xT_5}{\Delta}, \quad S_3 = \frac{-T_3}{\Delta}$$
$$S_4 = \frac{-T_4}{\Delta}, \quad S_5 = \frac{T_3T_4 - T_2T_5}{T_1\Delta} \tag{A12}$$

where

$$\Delta = T_1 T_2 - x T_2 T_5 + x T_3 T_4.$$

Let us also write some properties of the integral functions $I^n_m \equiv I^n_m(x, y)$ defined in (32)

$$I^n_m = -\frac{1}{2} I^{n-1}_{m-2} + \frac{x+y}{2} I^{n-1}_m \tag{A13}$$

$$(x-y)^2 I^0_4 + (d-3)(x+y) I^0_2 = (d-2) I^0_0 \tag{A14}$$

1	I^0_0	I^0_2
2	2π	$\frac{2\pi}{x-y}\,[\Theta(x-y) - \Theta(y-x)]$
3	4π	$\frac{\pi}{\sqrt{xy}} \ln \left(\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}\right)^2$
4	$2\pi^2$	$2\pi^2 \left[\frac{1}{x}\Theta(x-y) + \frac{1}{y}\Theta(y-x)\right]$ (A15)

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O DUFFIN-KEMMEROVOM PROPAGATORU U SKALARNOJ ELEKTRODINAMICI

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Izvedena je prva aproksimacija Schwinger-Dysonove jednačine za propagator čestice spina 0 u Duffin-Kemmerovom formalizmu sa proizvoljnim brojem prostorno-vremenskih dimenzija. Duffin-Kemmerov propagator je predstavljen u jednostavnoj i veoma efektivnoj formi. Pokazano je da ni u najjednostavnijoj 2-dimenzionoj skalarnoj elektrodinamici nema konačnog rešenja.