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LETTER TO THE EDITOR

NONLINEAR ELECTROSTATIC ION-CYCLOTRON WAVES

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The nonlinear effects on electrostatic ion-cyclotron waves travelling perpendicular to the applied magnetic field in a cold-ion and hot-electron plasma are investigated. The method of strained parameters is used to develop a special solution which represents periodic waves with the nonlinear effects showing up as an amplitude dependent frequency shift and a multiperiodicity.

One of the issues in nonlinear plasma phenomena is whether such plasma waves as are predicted to be purely periodic by the linear theory remain so when their amplitude becomes finite and the concomitant nonlinear effects become important. The nonlinear effects on the electron cyclotron standing waves parallel to the applied magnetic field were investigated by Tidman and Stainer[1] using the Krylov-Bogoliubov-Mitropolski method (Bogoliubov and Mitropolski[2]), and they showed that purely periodic electron cyclotron modes exist even when their amplitude becomes finite, with the effects of the latter showing up as amplitude-dependent shift in the frequency. A treatment fo finite-amplitude electrostatic ion-cyclotron modes propagating perpendicular to the applied magnetic field wherein the underlying motion is that of the ions, with the electrons remaining in thermodynamic equilibrium and distributed according to the Boltzmann formula was given by Chaturvedi[3] who showed that some special stationary solutions having nonsinusoi-

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dal saw-tooth-like shapes exist. The purpose of this paper is to use the method of strained parameters (Nayfeh[4]) to treat the latter problem and to develop another special solution which represents purely periodic waves with the nonlinear effects showing up as an amplitude-dependent frequency shift and multiperiodicity.

Consider a plasma consisting of cold ions and hot electrons and subjected to a uniform magnetic field $\vec{B} = B\,\hat{i}_\beta$. Since we consider in the following low-frequency waves in such a plasma, the electrons will remain in thermodynamic equilibrium at a constant temperature T_e in these wave motions with a number density given by the Boltzmann distribution

$$n_e = N_0\, e^{\frac{e\Phi}{KT_e}} \tag{1}$$

where Φ is the electrostatic potential associated with the wave-motions, and N_0 the number density of electrons (or ions) in the unperturbed state. The ion motion across the field lines produce a charge imbalance which is rapidly destroyed by the fast electron motion along the field lines. Thus, one has the wave propagation almost perpendicular to $\vec{B}_0$, and only a small component parallel to the magnetic field of the wave-vector is necessary to obtain (1). The motion of the ions in the one-dimensional electrostatic wave motions occurring perpendicular to the applied magnetic field are governed by the following equations (Chaturvedi[3])

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u) = 0 \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -c_s^2\frac{\partial \psi}{\partial x} + \Omega_i v \tag{3}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} = -\Omega_i v \tag{4}$$

where, n_i is the number density of the ions, u and v the velocity components in the x, y-directions of the ion fluid, and

$$\Omega_i = \frac{eB}{m_i c}, \quad \psi = \frac{e\Phi}{KT_e}, \quad c_s^2 = \frac{KT_e}{m_i}$$

m_i being the mass of an ion, e its electric charge, c the speed of light, and c_s the ion-acoustic speed.

For travelling waves, the various quantities depend on x and t only in the combination $\xi = kx - \omega t$. In general, the wave amplitude can also vary slowly in x and t, but we shall consider in the following a special type of solutions for which the wave amplitude is constant. Using also the quasi-neutrality condition one derives from Eqs. (1)—(4):

$$(ku-\omega)^3 \frac{\mathrm{d}}{\mathrm{d}\xi}\left[(ku-\omega)\frac{\mathrm{d}u}{\mathrm{d}\xi}\right] + \Omega_i^2 (ku-\omega)^2 u -$$

$$- \omega k^2 c_s^2 \left[(\omega - ku)\frac{\mathrm{d}^2 u}{\mathrm{d}\xi^2} + 2k\left(\frac{\mathrm{d}u}{\mathrm{d}\xi}\right)^2\right] = 0. \tag{5}$$

Introduce a small parameter $\varepsilon \ll 1$, which characterises the magnitude of a typical perturbation, and seek solutions to Eq. (5) in the form

$$u(\xi;\varepsilon) = \sum_{n=1}^{\infty} \varepsilon^n u_n(\xi)$$

$$\omega(k;\varepsilon) = \sum_{n=0}^{\infty} \varepsilon^n \omega_n(k). \tag{6}$$

Using (6), Eq. (5) gives

$$\omega_0^2(\omega_0^2 - k^2 c_s^2)\frac{\mathrm{d}^2 u_1}{\mathrm{d}\xi^2} + \Omega_i^2 \omega_0^2 u_1 = 0 \tag{7}$$

$$\omega_0^2(\omega_0^2 - k^2 c_s^2)\frac{\mathrm{d}^2 u_2}{\mathrm{d}\xi^2} + \Omega_i^2 \omega_0^2 u_2 = \omega_0^3\left[k\left(\frac{\mathrm{d}u_1}{\mathrm{d}\xi}\right)^2 + (ku_1 - \omega_1)\frac{\mathrm{d}^2 u_1}{\mathrm{d}\xi^2}\right] -$$

$$- \omega_0 \frac{\mathrm{d}^2 u_1}{\mathrm{d}\xi^2}(3\omega_0^2 \omega_1 - 3k\omega_0^2 u_1) - \Omega_i^2 (2\omega_0\omega_1 - 2\omega_0 k u_1) u_1 +$$

$$+ \omega_0 k^2 c_s^2 \left[(\omega_1 - ku_1)\frac{\mathrm{d}^2 u_1}{\mathrm{d}\xi^2} + 2k\left(\frac{\mathrm{d}u_1}{\mathrm{d}\xi}\right)^2\right] \tag{8}$$

$$\omega_0^2(\omega_0^2 - k^2 c_s^2)\frac{\mathrm{d}^2 u_3}{\mathrm{d}\xi^2} + \Omega_i^2 \omega_0^2 u_3 = -(3k\omega_0^2 u_1 - 3\omega_0^2\omega_1)\left[k\left(\frac{\mathrm{d}u_1}{\mathrm{d}\xi}\right)^2 +\right.$$

$$\left. + (ku_1 - \omega_1)\frac{\mathrm{d}^2 u_1}{\mathrm{d}\xi^2} - \omega_0 \frac{\mathrm{d}^2 u_2}{\mathrm{d}\xi^2}\right] + \omega_0 \frac{\mathrm{d}^2 u_1}{\mathrm{d}\xi^2}[-3k^2\omega_0 u_1^2 + 3k\omega_0^2 u_2 +$$

$$+ 6k\omega_0\omega_1 u_1 - 3\omega_0^2\omega_2 - 3\omega_0^2\omega_1^2] - \Omega_i^2 [k^2 u_1^3 + (2\omega_0\omega_2 + \omega_1^2) u_1 +$$

$$+ 2\omega_0\omega_1 u_2 - 4k\omega_0 u_1 u_2 - 2k\omega_1 u_1^2] + \omega_0 k^2 c_s^2 [(\omega_1 - ku_1)\frac{\mathrm{d}^2 u_2}{\mathrm{d}\xi^2} +$$

$$\left. + (\omega_2 - ku_2)\frac{\mathrm{d}^2 u_1}{\mathrm{d}\xi^2} + 4k\frac{\mathrm{d}u_1}{\mathrm{d}\xi}\frac{\mathrm{d}u_2}{\mathrm{d}\xi}\right] + \omega_1 k^2 c_s^2 \left[(\omega_1 - ku_1)\frac{\mathrm{d}^2 u_1}{\mathrm{d}\xi^2} +\right.$$

$$\left. + 2k\left(\frac{\mathrm{d}u_1}{\mathrm{d}\xi}\right)^2 + \omega_0 \frac{\mathrm{d}^2 u_2}{\mathrm{d}\xi^2}\right] + 2\omega_0\omega_2 k^2 c_s^2 \frac{\mathrm{d}^2 u_1}{\mathrm{d}\xi^2}. \tag{9}$$

In Eqs. (7)—(9), $\xi = kx - \omega_0 t$.

One obtains from Eq. (7), the familiar linear result,

$$u_1 = A \cos \xi$$

$$\omega_0^2 = \Omega_i^2 + k^2 c_s^2. \tag{10}$$

Using (10), the removal of secular terms on the right hand side of Eq. (8) requires

$$\omega_1 = 0 \tag{11}$$

and then the solution to Eqs. (8) is given by

$$u_2 = -\frac{k}{2\omega_0} A^2 + \left(\frac{\Omega_i^2/2 + k^2 c_s^2}{\omega_0 \Omega_i}\right) k A^2 \cos 2\xi. \tag{12}$$

Using (10)—(12), Eq. (9) becomes

$$\omega_0^2 (\omega_0^2 - k^2 c_s^2) \frac{d^2 u_3}{d\xi^2} + \Omega_i^2 \omega_0^2 u_3 = \Big[\Omega_i^2 (\Omega_i^2 + k^2 c_s^2)^{1/2} \omega_2 - (2k^2 c_s^2 +$$

$$+ \frac{k^6 c_s^4}{\Omega_i^2} - \frac{7}{2} k^2 \Omega_i^2) A^2\Big] A \cos \xi + \text{Higher harmonics}. \tag{13}$$

The condition of removal of secular terms on the right hand side of Eq. (13) then gives

$$\omega_2 = \left[\frac{2k^2 c_s^2 + \frac{k^6 c_s^4}{\Omega_i^2} - \frac{7}{4} k^2 \Omega_i^2}{\Omega_i^2 (\Omega_i^2 + k^2 c_s^2)^{1/2}}\right] A^2. \tag{14}$$

The frequency ω is given by

$$\omega = (\Omega_i^2 + k^2 c_s^2)^{1/2} \left[1 + \left\{\frac{2k^2 c_s^2 + k^6 c_s^4/\Omega_i^2 - \frac{7}{4} k^2 \Omega_i^2}{\Omega_i^2 (\Omega_i^2 + k^2 c_s^2)}\right\} \varepsilon^2 A^2 + \dots\right]. \tag{15}$$

Thus, the nonlinear effects show up as an amplitude-dependent frequency shift and multiperiodicity in the solution for the wave.

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NELINEARNI ELEKTROSTATIČKI IONSKO-CIKLOTRONSKI VALOVI

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Razmatran je utjecaj nelinearnih efekata na karakteristike elektrostatskog ionsko--ciklotronskog vala koji se prostire normalno na vanjsko magnetsko polje. Medij kroz koji se val prostire je plazma s toplim elektronima i hladnim ionima. Pokazano je da slabe nelinearnosti mijenjaju početnu frekvenciju vala tako da ona počinje ovisiti od amplitude samoga vala.