YU ISSN 0015—3206
FZKAAA 17 (1) 89 (1985)

DERIVATION OF MUON RANGE SPECTRUM UNDER KOLAR GOLD MINES

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Received 1 September 1984

UDC 539.12

Original scientific paper

Theoretical calculation of the vertical intensity of atmospheric muons as a function of depth under Kolar rock has been done. The recently estimated muon energy spectrum has been used in this analysis. The derived muon range spectrum is in accord with the recent experimental Kolar Gold Field depth-intensity results surveyed by Krishnaswamy et al. The procedure of Miyake, Narasimham and Ramanamurthy has been used for the correction of muon intensity at great depth due to range fluctuations.

1. Introduction

The direct measurements by magnetic spectrograph give muon spectrum up to 20 TeV (Matsuno et al., *MUTRON* Group[1]). The indirect methods are based on (i) underground observation by *ECC* and (ii) horizontal air shower *HAS* measurements. The Indo-Japan-Collaboration[2] in Kolar Gold Field deep under

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ground experiments gave precise results on the muon range spectrum at great depths. The latest proton decay search and neutrino flux measurements in deep underground resurrected our interest that the experimental range spectrum should have some correlation to primary nucleon spectrum via muon energy spectrum at ground level (through some conventional model dependent calculations). In an earlier investigation Miyake et al.[3] have neglected the loss of muon energy due to nuclear interaction of muons in Kolar rocks. The calculation of range spectrum from the energy spectrum of muons has two-fold importance, viz. (i) validity of the usual range-energy relation with experimental data interpretation, (ii) whether the calculated muon range spectrum from the primary suorce spectrum via different model calculations favours the experimental data? The energetic particles penetrating in rock gradually loose energy via several types of interactions until they come to rest due to decay or nuclear interactions in rocks. The energy loss of muons due to the interactions in electromagnetic field, results in excitation and ionization which can be well estimated by earlier conventional dynamical calculations. But the energy loss of muons due to nuclear interactions through virtual photons using different particle interaction models is still controversial but not negligible as has been found from the recent survey after Minorikawa et al.[4], Dau et al.[5] and Roychoudhury et al.[6]. We have found that the muon energy loss rate due to nuclear interactions in rock using Generalised Vector Dominance Model after Devenisch and Schildknecht[7] is considerable.

In the present paper we have used our derived muon energy spectrum[8] from the recently estimated primary nucleon spectrum[9,10] to calculate the range spectrum of muons due to decays of various measons in Kolar Gold Mines after accounting the energy loss of muons due to various interactions. The *KGF* rock density is 3.03 g/cm^3 which is higher than the usual rock density 2.65 g/cm^3 determined by Barrett et al.[11] for standard rock. Using the proposed range-energy relation for *KGF* rock the depth-intensity spectrum has been calculated from our muon energy spectrum at sea level[8]. The calculated result has been corrected for range fluctuations due to catastrophic process by following the procedure of Miyake et al.[3] and the result has been compared with the *KGF* data surveyed by Krishnaswamy et al.[2].

2. *Nuclear physics and kinematics*

In a recent investigation[10] we have derived the primary nucleon spectrum from the balloon flight measured data on proton and helium fluxes of Japanese American Emulsion Experiments[12] and nuclei intensity data compiled by Abulova et al.[13] which follows the form

$$N(E)\,\mathrm{d}E = 2.36 \times E^{-2.7}\,\mathrm{d}E/(\mathrm{cm}^2\,\mathrm{s\,sr\,GeV/nucleon})^{-1}, \qquad (1)$$

where E is the nucleon energy in GeV/nucleon. Using this primary spectrum as hadron source the production spectra of pions and kaons have been calculated. The scaling hypothesis of Feynman[14] has been assumed to be approximately valid at super high energies viz. low-p_T region and has been applied here for the calculation of fractional hadronic energy moments for pion and kaon production from the accelerator data on the inclusive reactions viz., $pp \to \Pi^{\pm} X$ and $pp \to \to K^{\pm} X$. It is known from $p\overline{p}$ collider *CERN* experiments performed by Alpgard et al.[15] and Arnison et al.[16] for studying hadron production at lab energy around 155 TeV that Feynman scaling is weakly violated in the central region. Rushbrook[17] has also pointed out that in the very forward region $x_F \gtrsim 0.1$, Feynman scaling does work within the experimental errors of 10% in comparison with the cross sections at *ISR* pp (at 2 TeV) and *CERN SPS* $p\overline{p}$ collider (155 TeV) energy region, respectively. The meson atmospheric diffusion equations after Bugaev et al.[18] and Erlykin et al.[19] have been applied to meson spectra at production and the estimated integral sea level muon spectrum[8] follows the form

$$\sum_i M_i\,(>E) = 3.65 \times E^{-2.6}/(\mathrm{cm}^2\ \mathrm{s\ sr})^{-1}. \tag{2}$$

The muon energy spectrum at the ground level can be converted to depth-intensity spectrum through a range-energy relation in rock.

The interaction of muon with the atomic electrons, via their electromagnetic field creates ionization and excitation of the electrons. The rate of energy loss of muons under Kolar Gold Field rock for $Z/A = 0.493$, $Z^2/A = 6.3$ due to ionization and excitation after Sternheimer[20] follows

$$-\,\mathrm{d}e/\mathrm{d}h|_{coll.} = 1.84 + 0.076 \ln (E'_{max}/m_\mu c^2)/(\mathrm{MeV\ g^{-1}\ cm^2}), \tag{3}$$

where E_{max} is the maximum transferrable energy in MeV by a muon to an electron and E is the total energy of the muon.

$$E'_{max} = E^2/[E_\mu + (m\ c^2)^2/2m_e c^2], \tag{4}$$

and the muon and electron rest energies are represented by $m_\mu c^2$ and $m_e c^2$, respectively. The kinematical form of the muon energy loss coefficient in Kolar rock due to pair production and bremsstrahlung have been simplified by[3] and their sum follows the form

$$b_{PP} + b_B = 3.9 \cdot 10^{-6} \times E/(\mathrm{MeV\ g^{-1}\ cm^2}), \tag{5}$$

where E is the muon energy expressed in MeV.

The inelastic interaction of muon with a nucleus is regarded as the interaction between the virtual photon cloud surrounding the muon and the nucleus. Miyake et al.[3] and Menon and Ramanamurthy[21] have accepted the procedure of Fowler and Wolfendale[22] which is based on the classical *W-W* spectrum of virtual photons after Heitler[23] and the photo-nuclear cross section of value 10^{-2} fm²/nucleon have been used by them. Their adopted relation for the muon energy loss in rocks due to nuclear interactions follow

$$-\,\mathrm{d}E/\mathrm{d}h|_{nucl.} = \frac{2a\,\sigma_{h\nu}\,N\,E}{\pi} = 0.28 \cdot 10^{-6} \cdot E/(\mathrm{MeVg^{-1}cm^2}), \tag{6}$$

where $\sigma_{h\nu} = 10^{-2}$ fm^2/nucleon, $\alpha = 1/137$, $N = 6.02 \times 10^{23}$, E in MeV. The upper limit of muon energy loss due to nuclear interaction in rock found by Miyake et al.[3] is $0.9 \times 10^{-6} E$ (MeVg^{-1}cm^2). But the recent development of the particle interaction models, the rate of muon energy loss in rocks due to nuclear interactions expected from the Generalised Vector Dominance Model[7] and ξ-scaling model (Georgi and Politzer[24]) show much higher values than that estimated by Fowler and Wolfendale[22].

In the present investigation we have used the value of the coefficient of photonuclear interactions after Minorikawa et al.[4], which is based on the vector dominance model of Devenisch and Schildknecht[7]. Thus the muon energy loss rate due to nuclear interaction of high energy muons in rock by considering as a photoproduction interaction of a virtual photon of the electromagnetic field of the muon with a nucleon which follows the form

$$b_N = -\,\mathrm{d}E/\mathrm{d}h|_{nucl.} = 0.425 \cdot 10^{-6} \cdot E/(\mathrm{MeVg^{-1}cm^2}). \tag{8}$$

The total energy loss rate of muon dE in a thickness dh of Kolar rock follows the form

$$-\mathrm{d}E/\mathrm{d}h|_{total} = 1.84 + 0.076 \ln (E'_{max}/m_u c^2) + 4.325 \cdot 10^{-6} \times E/(\mathrm{MeVg^{-1}cm^2}). \tag{7}$$

The usual solution of the above equation follows for

$$a' = 1.84 + 0.076 \ln (E^2/e\, m_\mu c^2 (E + e\, A)/(\mathrm{MeVg^{-1}cm^2}),$$

$$A = 11.3\ \mathrm{GeV}, \quad e = 2.718 \quad \text{and} \quad b = 4.325 \cdot 10^{-6}/(\mathrm{gcm^{-2}}),$$

the form

$$E = a' (\exp [b \cdot h] - 1)/b. \tag{8}$$

At great depths viz. above 4000 hgcm^{-2} the range-energy relation is not unique due to fluctuations in the number of encounters in bremsstrahlung and nuclear interactions and the large energy transfers are responsible for such encounters. The integral muon spectrum on the surface is represented by the expression (2) and the average range-energy relation (8) give the muon flux at depth h

$$M_{av} (> h) = 3.65 \left[\frac{a'}{b} (\exp [b \cdot h] - 1)\right]^{-2.6}, \tag{8}$$

where a' and b can be found from the expressions in (8). Miyake et al.[3] considered that the losses of muon energy due to bremsstrahlung only in the consideration among the fluctuating problem due to catastrophic collissions of muons through the rocks. They expressed $M_{fl} (>E)$ as the expected muon flux at depth h after accounting the range fluctuations by introducing the correction factor R_{fl}

$$R_{fl} = M_{av} (>h)/M_{fl} (>h). \tag{10}$$

This correction factor is a function of the integral energy spectral exponent γ_μ. Thus knowing R_{fl} from the method used by Miyake et al.[3] and $M_{av}(>h)$ from relation (9) one can estimate the actual muon flux at depth h viz. $M_{fl}(>h)$ which corresponds the experimental data.

3. *Results and discussions*

The recently estimated primary cosmic ray spectrum[10] has been used as the source of hadrons near the top of the atmosphere and with the adoption of the procedures stated in Ref. 8 the derived integral muon energy spectrum has been found to follow the power law fit of the form $A\,E^{-\gamma_\mu}$. The muon-range-energy relation for Kolar Gold Field rocks proposed in the present analysis was computed from relation (8) and has been displayed in the Fig. 1 along with the

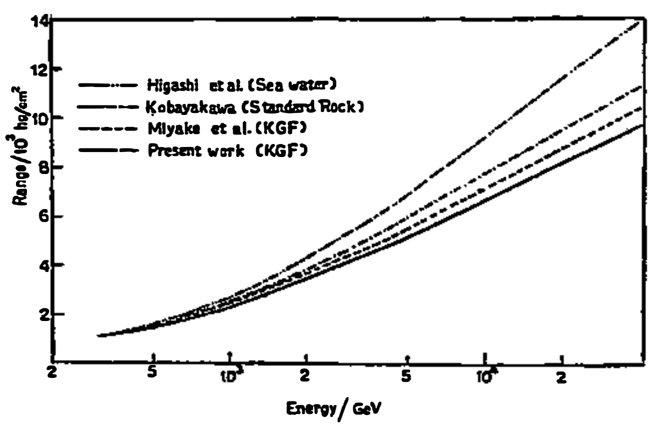

Fig. 1. Depth-energy curves for mouns in different media: ————————, present work for Kolar Gold Field rocks, - - - - - -, Miyake et al.[3] for Kolar Gold Field rocks, - · · - · · · · -, Kobayakawa[25] for standard rocks, - ·· - ·· - ·· - ··, Highsi et al.[26] for sea water.

result of Miyake et al.[3] for Kolar Gold Field rocks, Kobayakawa[25] for standard rocks and Higashi et al.[26] for sea water. The present result lies below the results of Miyake et al.[3] due to their lower value of the energy loss coefficient as they have neglected the muon energy loss rate due to nuclear interactions in rocks. Using the range energy relation (8) the depth intensity spectrum from the relation (9) has been estimated. The total value of the energy loss coefficient $b = b_P + b_B + b_N = 4.325 \times 10^{-6}/(\mathrm{g}^{-1}\mathrm{cm}^2)$ has been considered in the present calculations. The derived depth-intensity spectrum has been plotted in Fig. 2 (broken curve) along with the Kolar Gold Field data surveyed by Krishnaswamy et al[2]. During this calculation we have assumed that the muon energy losses are assumed to be continuous. The chi-square test has been done for depth up to 6000 hgcm^{-2} and has been found to be $\chi^2 = 23.6$ for 24 data points.

Following the method of Miyake et al.[3] we have corrected the derived depth-intensity for the range fluctuations due to catastrophic collisions viz. for muon bremsstrahlung, for $\gamma_{\mu} = 2.6$ in the expression (10). The corrected depth-intensity spectrum has been plotted in Fig. 2 (full curve). It is evident from the plot

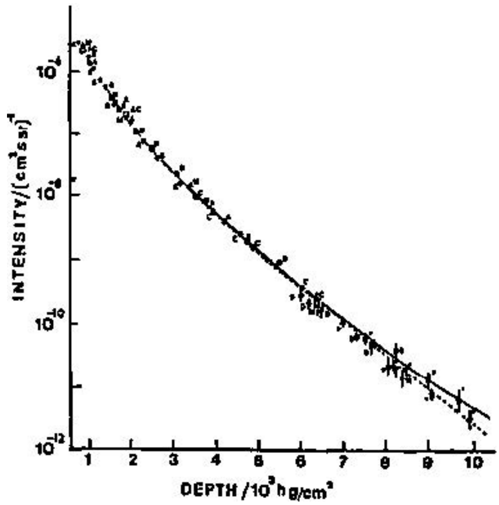

Fig. 2. Vertical integral muon intensity plotted as a function of depth under Kolar Gold Field Rock. Broken curve represents the depth-intensity curve without correction for range fluctuations and full curve represents the same with the corrections for range fluctuations. Experimental data after Krishnaswamy et al.[2]: A, B, C, D, E represent the depth-intensity data recorded at vertical rock depths 754, 1500, 3375, 6045 and 1500 $hgcm^{-2}$, respectively.

ν — the muon intensity data recorded during neutrino flux measurements at a depth of 7000 $kgcm^{-2}$ by Indo—Japan—British Collaborators during the year 1965—1969; M — Miyake et al.[3] AC — Acher et al.[27]

that the depth-intensity spectrum when corrected for range straggling supports fairly well the muon intensity data recorded at great depths during neutrino flux measurements by Indo—Japan—British collaborators[2].

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DOBIVANJE DOMENE MUONSKOG SPEKTRA IZ KOLAROVIH ZLATNIH RUDNIKA

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UDK 539.12

Originalni znanstveni rad

Proveden je teoretski račun intenziteta atmosferskih muona kao funkcije dubine njihovog prodiranja u Kolarovoj stijeni. U analizi je korišten već ranije procijenjeni energetski spektar muona. Dobiveno područje muonskog spektra slaže se s nedavnim eksperimentalnim rezultatima mjerenja u Kolarovoj stijeni. Postupak Miyakea, Narasimhama i Ramanamurthya korišten je da se odredi korekcija intenziteta muona kod velikih dubina.