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Loxodrome in Conic and Azimuthal Projections

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Abstract: In the introductory part of the article, we recall the important role of the loxodrome in navigation and derive a formula for the length of the arc of this curve on the sphere between two points. The loxodrome is usually associated with the Mercator projection, because in this projection its image is a straight line. In this paper, we deal with the loxodrome in conic and azimuthal projections. The image of the loxodrome in such projections is not a straight line but a spiral curve. The determination of the distance measured along the loxodrome between two points in a normal aspect conic or azimuthal projection is presented. Formulas for the length of the arc of the loxodrome in each of these projections are derived using examples of conformal, equivalent and equidistant conic projections. Azimuthal projections are special cases of conic ones, so they do not require special derivations of formulas.

Keywords: conic projections, azimuthal projections, sphere, loxodrome, arc length

1 Introduction

A loxodrome or rhumb line on a sphere or a rotating ellipsoid is a spiral-shaped curve that intersects the meridians at the same angle, constantly approaching the pole but without reaching it. On a map made in the Mercator projection, it is shown as a straight line. The advantage of a loxodrome is that a ship sailing along it sails on an unchanged course. Problems related to loxodromes are solved graphically on a navigation chart or by calculation (Pomorski leksikon 1990). Grunert (1849) proposed that, unlike spherical and plane trigonometry, this part of trigonometry be called loxodrome trigonometry (Loxodromische Trigonometrie).

In this article we will deal with the loxodrome in conic and azimuthal projections. At the beginning, we notice that every map projection introduces certain distortions. As a measure of length distortion, the local linear scale factor c is used, which is defined for a sphere as follows (Snyder 1987):

$$c = \frac{ds'}{ds} \tag{1}$$

where ds is the differential of the arc on the surface being projected, and ds' is the corresponding differential of the arc in the projection plane. Let

$$x = x(\varphi,\lambda)\,,\ y = y(\varphi,\lambda)\,, \tag{2}$$

be the equations of map projections, that is, in our case, the formulas with which a sphere of radius R is mapped onto a plane. φ and λ are the latitude and longitude, $\varphi \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, $\lambda \in [-\pi,\pi]$, and x and y are coordinates of a

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Loksodroma u konusnim i azimutnim projekcijma

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Sažetak: U uvodnom dijelu članka podsjećamo na važnu ulogu loksodrome u navigaciji i izvodimo formulu za duljinu luka te krivulje na sferi između dviju točaka. Obično se loksodroma povezuje s Mercatorovom projekcijom, jer je u toj projekciji njezina slika ravna crta. U ovom se radu bavimo loksodromom u konusnim i azimutnim projekcijama. Slika loksodrome u takvim projekcijama nije ravna crta nego spiralna krivulja. Prikazano je određivanje udaljenosti mjerene uzduž loksodrome između dviju točaka u nekoj uspravnoj konusnoj ili azimutnoj projekciji. Na primjerima konformnih, ekvivalentnih i ekvidistantnih konusnih projekcija izvedene su formule za duljinu luka loksodrome u svakoj od tih projekcija. Azimutne projekcije su posebni slučajevi konusnih, tako da za njih nisu potrebni posebni izvodi formula.

Ključne riječi: konusne projekcije, azimutne projekcije, sfera, loksodroma, duljina luka

1. Uvod

Loksodroma na sferi ili rotacijskom elipsoidu je krivulja spiralnog oblika koja siječe meridijane pod istim kutom, stalno se približava polu ali ga ne može dostići. Na karti izrađenoj u Mercatorovoj projekciji prikazana je ravnom crtom. Dobra strana loksodrome je u tome što brod koji plovi po loksodromi, plovi nepromijenjenim kursom. Zadatci u vezi s loksodromom rješavaju se grafički na navigacijskoj karti ili računski (Pomorski leksikon 1990). Grunert (1849) je predložio da se za razliku od sferne i ravninske trigonometrije, taj dio trigonometrije naziva loksodromskom trigonometrijom.

U ovom članku bavit ćemo se loksodromom u konusnim i azimutnim projekcijama. Na početku uočimo da svaka kartografska projekcija uvodi izvjesne distorzije. Kao mjera distorzije duljina služi lokalni faktor mjerila duljina c koji za sferu definiramo ovako (Snyder 1987):

$$c=\frac{ds'}{ds}, \tag{1}$$

gdje je ds diferencijal luka na plohi koja se preslikava, a ds' odgovarajući diferencijal luka u ravnini projekcije. Neka su

$$x=x(\varphi,\lambda),\ \ y=y(\varphi,\lambda) \tag{2}$$

jednadžbe kartografske projekcije, odnosno u našem slučaju formule s pomoću kojih se sfera polumjera R preslikava u ravninu. φ i λ su geografska širina i dužina, $\varphi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, $\lambda\in[-\pi,\pi]$, a x i y su koordinate točke u pravokutnom (matematičkom, desno orijentiranom) point in a rectangular (mathematical, right-oriented) coordinate system in the plane. We will assume that a regular mapping is defined by (2), except perhaps on the edges of the interval, i.e. for $\varphi\in\left\{-\frac{\pi}{2},\frac{\pi}{2}\right\}$ and $\lambda\in\{-\pi,\pi\}$.

Usually the coefficients E, F and G are defined in this way:

$$E=\left(\frac{\partial x}{\partial\varphi}\right)^2+\left(\frac{\partial y}{\partial\varphi}\right)^2,\ F=\frac{\partial x}{\partial\varphi}\frac{\partial x}{\partial\lambda}+\frac{\partial y}{\partial\varphi}\frac{\partial y}{\partial\lambda},\ G=\left(\frac{\partial x}{\partial\lambda}\right)^2+\left(\frac{\partial y}{\partial\lambda}\right)^2 \tag{3}$$

so then the square of the differential of the arc in the projection plane can be written in the form

$$ds'^2=dx^2+dy^2=Ed\varphi^2+2Fd\varphi d\lambda+Gd\lambda^2. \tag{4}$$

The well-known formula for the square of the differential of the arc of a curve on a sphere is valid (Snyder 1987, Frančula 2004)

$$ds^2=R^2(d\varphi^2+\cos^2\varphi\, d\lambda^2), \tag{5}$$

where R is the radius of the sphere. Considering (1), (4) and (5) we can write

$$c^2=\frac{ds'^2}{ds^2}=\frac{Ed\varphi^2+2Fd\varphi d\lambda+Gd\lambda^2}{R^2(d\varphi^2+\cos^2\varphi\, d\lambda^2)}. \tag{6}$$

2. Loxodrome

Let a sphere of radius R and centered at the origin of the coordinate system be defined by the geographic parameterization

$$X=R\cos\varphi\cos\lambda,\ Y=R\cos\varphi\sin\lambda,\ Z=R\sin\varphi \tag{7}$$

where φ and λ are the latitude and longitude, $\varphi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, $\lambda\in[-\pi,\pi]$. An infinitesimal triangle on a sphere is shown in Figure 1. The arc length differential of a curve is ds, the meridian arc length differential is $Rd\varphi$, and the parallel arc length differential is $R\cos\varphi d\lambda$.

For any curve on a sphere we can write (see Figure 1):

$$\tan\alpha=\frac{R\cos\varphi d\lambda}{Rd\varphi} \tag{8}$$

and from there

$$d\lambda=\tan\alpha\frac{d\varphi}{\cos\varphi}. \tag{9}$$

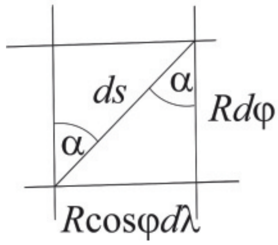


Fig. 1 Infinitesimal triangle on a sphere of radius R, ds is the differential of the arc of a curve on the sphere, $Rd\varphi$ is the differential of the arc of the meridian, and $R\cos\varphi d\lambda$ is the differential of the arc of the parallel.

Slika 1. Infinitezimalni trokut na sferi radijusa R, ds je diferencijal luka neke krivulje na sferi, $Rd\varphi$ je diferencijal luka meridijana, a $R\cos\varphi d\lambda$ je diferencijal luka paralele.

If we assume that α = *const.*, i.e. that it is a loxodrome, by integrating expression (8) we will obtain the equation of the loxodrome in the form

$$\lambda=a\ln\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)+\beta, \tag{10}$$

where $a=\tan\alpha$ and β are constants. For simplicity and brevity, we will introduce the isometric latitude, or the function $q=q(\varphi)$, as follows (Wiki 2024)

$$q(\varphi)=\ln\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)=\frac{1}{2}\ln\frac{1+\sin\varphi}{1-\sin\varphi}=\tanh^{-1}(\sin\varphi) \tag{11}$$

and from there

$$dq=\frac{d\varphi}{\cos\varphi}. \tag{12}$$

It is not difficult to show that these relations hold:

$$\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)=e^q,\ \cos\varphi=\frac{1}{\cosh q},$$

$$\sin\varphi=\tanh q,\ \tan\varphi=\sinh q. \tag{13}$$

Now we can write the equation of the loxodrome (10) in the form

$$\lambda=aq(\varphi)+\beta. \tag{14}$$

Furthermore, from (13) and (14) for $a\neq 0$ it follows

$$\sin\varphi=\tanh q(\varphi)=\tanh\frac{\lambda-\beta}{a}, \tag{15}$$

and then

koordinatom sustavu u ravnini. Pretpostavit ćemo da je s (2) definirano regularno preslikavanje, osim možda na rubovima intervala, tj. za $\varphi \in \left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$ i $\lambda \in \{-\pi, \pi\}$.

Obično se koeficijenti *E*, *F* i *G* definiraju na ovaj način:

$$E = \left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2, \; F = \frac{\partial x}{\partial \varphi}\frac{\partial x}{\partial \lambda} + \frac{\partial y}{\partial \varphi}\frac{\partial y}{\partial \lambda}, \; G = \left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2 \quad (3)$$

pa se onda kvadrat diferencijala luka u ravnini projekcije može napisati u obliku

$$ds'^2 = dx^2 + dy^2 = Ed\varphi^2 + 2Fd\varphi d\lambda + Gd\lambda^2. \quad (4)$$

Za kvadrat diferencijala luka krivulje na sferi vrijedi poznata formula (Snyder 1987, Frančula 2004)

$$ds^2 = R^2(d\varphi^2 + \cos^2\varphi \, d\lambda^2), \quad (5)$$

gdje je *R* radijus sfere. Uzevši u obzir (1), (4) i (5) možemo napisati

$$c^2 = \frac{ds'^2}{ds^2} = \frac{Ed\varphi^2 + 2Fd\varphi d\lambda + Gd\lambda^2}{R^2(d\varphi^2 + \cos^2\varphi \, d\lambda^2)} \; . \quad (6)$$

2. Loksodroma

Neka je sfera radijusa *R* sa središtem u ishodištu koordinatnog sustava definirana geografskom parametrizacijom

$$X = R\cos\varphi\cos\lambda, \; Y = R\cos\varphi\sin\lambda, \; Z = R\sin\varphi, \quad (7)$$

gdje su φ i λ geografska širina i dužina, $\varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\lambda \in [-\pi, \pi]$. Infinitezimalni trokut na sferi prikazan je na slici 1. Diferencijal duljine luka neke krivulje je *ds*, diferencijal duljine luka meridijana je $Rd\varphi$, a diferencijal duljine luka paralele je $R\cos\varphi d\lambda$.

Za bilo koju krivulju na sferi možemo napisati (vidi sliku 1):

$$\tan\alpha = \frac{R\cos\varphi d\lambda}{Rd\varphi} \quad (8)$$

i odatle

$$d\lambda = \tan\alpha \frac{d\varphi}{\cos\varphi} \; . \quad (9)$$

Pretpostavimo li da je $\alpha = const.$, tj. da je riječ o loksodromi, integriranjem izraza (8) dobit ćemo jednadžbu loksodrome u obliku

$$\lambda = a \ln \tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) + \beta, \quad (10)$$

gdje su $a = \tan\alpha$ i β konstante. Radi jednostavnijeg i kraćeg pisanja uvest ćemo izometrijsku širinu, odnosno funkciju $q = q(\varphi)$ na sljedeći način (Wiki 2024)

$$q(\varphi) = \ln\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) = \frac{1}{2}\ln\frac{1+\sin\varphi}{1-\sin\varphi} = \tanh^{-1}(\sin\varphi) \quad (11)$$

i odatle

$$dq = \frac{d\varphi}{\cos\varphi}. \quad (12)$$

Nije teško pokazati da vrijede ove relacije:

$$\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) = e^q, \; \cos\varphi = \frac{1}{\cosh q},$$
$$\sin\varphi = \tanh q, \; \tan\varphi = \sinh q. \quad (13)$$

Sad možemo jednadžbu loksodrome (10) napisati u obliku

$$\lambda = aq(\varphi) + \beta \quad (14)$$

Nadalje, iz (13) i (14) za $a \neq 0$ slijedi

$$\sin\varphi = \tanh q(\varphi) = \tanh\frac{\lambda - \beta}{a}, \quad (15)$$

i zatim

$$\varphi = \sin^{-1}\left(\tanh\frac{\lambda-\beta}{a}\right) = 2\tan^{-1}\left(e^{\frac{\lambda-\beta}{a}}\right) - \frac{\pi}{2}. \quad (16)$$

Ako loksodroma prolazi točkama s koordinatama (φ_1, λ_1) i (φ_2, λ_2) onda je

$$a = \tan\alpha = \frac{\lambda_2 - \lambda_1}{q(\varphi_2) - q(\varphi_1)}, \quad (17)$$

$$\beta = \lambda_1 - aq(\varphi_1) = \lambda_2 - aq(\varphi_2) = \frac{\lambda_2 q(\varphi_1) - \lambda_1 q(\varphi_2)}{q(\varphi_2) - q(\varphi_1)}. \quad (18)$$

Za duljinu loksodrome na sferi imat ćemo prema (5) i (9)

$$ds^2 = R^2(d\varphi^2 + \cos^2\varphi \, d\lambda^2) = R^2(1+a^2)d\varphi^2, \quad (19)$$

$$ds = R\sqrt{1+a^2}d\varphi \quad (20)$$

$$s = R\sqrt{1+a^2}\int_{\varphi_1}^{\varphi_2} d\varphi = R\sqrt{1+a^2}(\varphi_2 - \varphi_1). \quad (21)$$

U nastavku ćemo pretpostaviti da je $a \neq 0$. Za $a = 0$, loksodroma je kružnica, paralela, pa su odgovarajuće formule vrlo jednostavne.

$$\varphi = \sin^{-1}\left(\tanh\frac{\lambda-\beta}{a}\right) = 2\tan^{-1}\left(e^{\frac{\lambda-\beta}{a}}\right) - \frac{\pi}{2}. \quad (16)$$

If the loxodrome passes through the points with coordinates (φ_1, λ_1) and (φ_2, λ_2) then

$$a = \tan\alpha = \frac{\lambda_2 - \lambda_1}{q(\varphi_2) - q(\varphi_1)}, \quad (17)$$

$$\beta = \lambda_1 - aq(\varphi_1) = \lambda_2 - aq(\varphi_2) = \frac{\lambda_2 q(\varphi_1) - \lambda_1 q(\varphi_2)}{q(\varphi_2) - q(\varphi_1)}. \quad (18)$$

For the length of a loxodrome on a sphere, we have according to (5) and (9)

$$ds^2 = R^2(d\varphi^2 + \cos^2\varphi \, d\lambda^2) = R^2(1+a^2)d\varphi^2, \quad (19)$$

$$ds = R\sqrt{1+a^2}\, d\varphi \quad (20)$$

$$s = R\sqrt{1+a^2}\int_{\varphi_1}^{\varphi_2} d\varphi = R\sqrt{1+a^2}(\varphi_2 - \varphi_1). \quad (21)$$

In the following we will assume that $a \neq 0$. For $a = 0$, the loxodrome is a circle, a parallel of latitude, so the corresponding formulas are very simple.

3. Conic and azimuthal projections and loxodromes

For normal aspect conic projections (Snyder 1987, Frančula 2004) the following holds:

$$x = \rho \sin\delta, \; y = q - \rho \cos\delta, \; \rho = \rho(\varphi), \; \delta = n\lambda \quad (22)$$

where φ and λ are latitude and longitude, $\varphi \in [-\pi/2, \pi/2]$, $\lambda \in [-\pi, \pi]$, δ the angle at which the meridians intersect in the projection, ρ the radii of the parallels in the projection, n $(0 < n < 1)$ the proportionality constant. Based on (22) we can calculate

$$\begin{aligned} dx &= \sin\delta\, d\rho + \rho\cos\delta\, d\delta, \\ dy &= -\cos\delta\, d\rho + \rho\sin\delta\, d\delta. \end{aligned} \quad (23)$$

Because it is

$$d\delta = n d\lambda, \quad (24)$$

then (23) can be written in the form

$$\begin{aligned} dx &= \sin\delta\, d\rho + n\rho\cos\delta\, d\lambda, \\ dy &= -\cos\delta\, d\rho + n\rho\sin\delta\, d\lambda. \end{aligned} \quad (25)$$

Finally, for every normal aspect conic projection, it follows

$$ds'^2 = d\rho^2 + n^2\rho^2 d\lambda^2 = \left[\left(\frac{d\rho}{d\varphi}\right)^2 + n^2\rho^2\left(\frac{d\lambda}{d\varphi}\right)^2\right] d\varphi^2. \quad (26)$$

For a loxodrome, according to (9)

$$\frac{d\lambda}{d\varphi} = \frac{a}{\cos\varphi}, \quad (27)$$

so (26) turns into

$$ds'^2 = \left[\left(\frac{d\rho}{d\varphi}\right)^2 + \frac{n^2\rho^2 a^2}{\cos^2\varphi}\right] d\varphi^2. \quad (28)$$

Let us mark

$$h = -\frac{d\rho}{R d\varphi}, \quad k = \frac{n\rho}{\cos\varphi}, \quad (29)$$

which are the local linear scale factors in the direction of the meridians and parallels, respectively. We can write a formula for the differential of the arc of the image of a loxodrome in a normal aspect conic projection in the form

$$ds' = R\sqrt{h^2 + a^2k^2}\, d\varphi, \quad (30)$$

and the local linear scale factor of the loxodrome will be

$$c = \frac{ds'}{ds} = \sqrt{\frac{h^2 + a^2k^2}{1+a^2}}. \quad (31)$$

Azimuthal projections are the limiting case of conic projections when $\delta = \lambda$. This means that all formulas for azimuthal projections can be obtained from the corresponding formulas derived for conic projections, we just need to put $n = 1$.

4. Loxodromes in conformal conic and azimuthal projection of the sphere

The equations of the normal aspect conformal conic projection of the sphere are

$$\delta = n\lambda, \; \rho = K\tan^n\left(\frac{\pi}{4} - \frac{\varphi}{2}\right), \quad (32)$$

where $0 < n < 1$ and K are constants (Snyder 1987, Frančula 2004). If we insert into (32) the expression for λ from (14) taking into account

$$\tan\left(\frac{\pi}{4} - \frac{\varphi}{2}\right) = \frac{1}{\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)} = e^{-q}, \quad (33)$$

we will obtain the equation of the image of the loxodrome in that projection in the form

$$\delta = n[aq(\varphi) + \beta], \; \rho = Ke^{-nq(\varphi)}. \quad (34)$$

3. Konusne i azimutne projekcije i loksodroma

Za uspravne konusne projekcije (Snyder 1987, Frančula 2004)

$$x = \rho \sin\delta,\ y = q - \rho \cos\delta,\ \rho = \rho(\varphi),\ \delta = n\lambda \quad (22)$$

gdje su φ i λ geografska širina i dužina, $\varphi \in [-\pi/2, \pi/2]$, $\lambda \in [-\pi, \pi]$, δ kut pod kojim se sijeku meridijani u projekciji, ρ polumjeri paralela u projekciji, n ($0 < n < 1$) konstanta proporcionalnosti. Na temelju (22) možemo izračunati

$$\begin{aligned} dx &= \sin\delta\, d\rho + \rho\cos\delta\, d\delta, \\ dy &= -\cos\delta\, d\rho + \rho\sin\delta\, d\delta \end{aligned} \quad (23)$$

Budući da je

$$d\delta = n d\lambda, \quad (24)$$

to se (23) može napisati u obliku

$$\begin{aligned} dx &= \sin\delta\, d\rho + n\rho\cos\delta\, d\lambda, \\ dy &= -\cos\delta\, d\rho + n\rho\sin\delta\, d\lambda. \end{aligned} \quad (25)$$

Konačno, za svaku uspravnu konusnu projekciju slijedi

$$ds'^2 = d\rho^2 + n^2\rho^2 d\lambda^2 = \left[\left(\frac{d\rho}{d\varphi}\right)^2 + n^2\rho^2\left(\frac{d\lambda}{d\varphi}\right)^2\right] d\varphi^2. \quad (26)$$

Za loksodromu je prema (9)

$$\frac{d\lambda}{d\varphi} = \frac{a}{\cos\varphi}, \quad (27)$$

pa (26) prelazi u

$$ds'^2 = \left[\left(\frac{d\rho}{d\varphi}\right)^2 + \frac{n^2\rho^2 a^2}{\cos^2\varphi}\right] d\varphi^2. \quad (28)$$

Označimo li još

$$h = -\frac{d\rho}{R d\varphi}, \quad k = \frac{n\rho}{\cos\varphi}, \quad (29)$$

što su faktori lokalnih mjerila duljina u smjeru meridijana, odnosno paralela, možemo napisati formulu za diferencijal luka slike loksodrome u nekoj uspravnoj konusnoj projekciji u obliku

$$ds' = R\sqrt{h^2 + a^2 k^2}\, d\varphi, \quad (30)$$

a faktor lokalnog mjerila duljina loksodrome bit će

$$c = \frac{ds'}{ds} = \sqrt{\frac{h^2 + a^2 k^2}{1 + a^2}}. \quad (31)$$

Azimutne projekcije su granični slučaj konusnih projekcija kad je $\delta = \lambda$. To znači da sve formule za azimutne projekcije možemo dobiti iz odgovarajućih formula izvedenih za konusne projekcije, samo treba u njih staviti $n = 1$.

4. Loksodroma u konformnoj konusnoj i azimutnoj projekciji sfere

Jednadžbe uspravne konformne konusne projekcije sfere glase

$$\delta = n\lambda, \quad \rho = K\tan^n\left(\frac{\pi}{4} - \frac{\varphi}{2}\right) \quad (32)$$

gdje su $0 < n < 1$ i K konstante (Snyder 1987, Frančula 2004). Uvrstimo li u (32) izraz za λ iz (14) uzevši u obzir

$$\tan\left(\frac{\pi}{4} - \frac{\varphi}{2}\right) = \frac{1}{\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)} = e^{-q}, \quad (33)$$

dobit ćemo jednadžbu slike loksodrome u toj projekciji u obliku

$$\delta = n[aq(\varphi) + \beta], \quad \rho = Ke^{-nq(\varphi)}. \quad (34)$$

Budući da su K, n, a, β konstante, to (34) predstavlja eksponencijalnu vezu između ρ i δ pa će grafički prikaz relacije (34) biti logaritamska spirala u ravnini projekcije. Drugim riječima, slika loksodrome na karti izrađenoj u uspravnoj konformnoj konusnoj projekciji je logaritamska zavojnica.

Izračunajmo faktor lokalnog mjerila duljina u bilo kojoj točki spirale (34). Budući da je riječ o konformnoj projekciji, možemo napisati

$$c = h = k = \frac{n\rho}{R\cos\varphi} = \frac{nKe^{-nq(\varphi)}}{R\cos\varphi} \quad (35)$$

i zatim prema (30)

$$ds' = nK\sqrt{1+a^2}\frac{e^{-nq(\varphi)}}{\cos\varphi} d\varphi = nK\sqrt{1+a^2}\, e^{-nq(\varphi)} dq, \quad (36)$$

pa je

$$s' = nK\sqrt{1+a^2}\int_{q_1}^{q_2} e^{-nq(\varphi)} dq = -K\sqrt{1+a^2}\, e^{-nq(\varphi)}\Big|_{q_1}^{q_2} \quad (37)$$

i prema tome duljina luka loksodrome između točaka kojima odgovaraju geografske širine u ravnini konfromne konusne projekcije

$$s' = -\sqrt{1+a^2}\left[\rho(q_2) - \rho(q_1)\right] = \sqrt{1+a^2}\left[\rho(\varphi_1) - \rho(\varphi_2)\right] \quad (38)$$

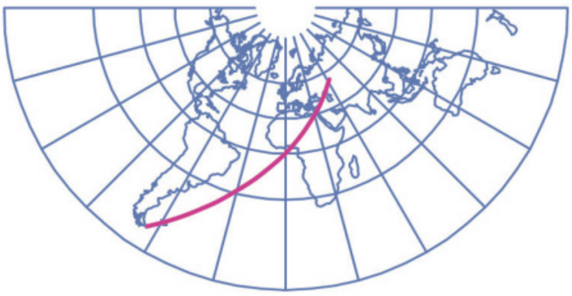


Fig. 2 Part of the world map ($\varphi > -60°$N) in a normal aspect conic conformal projection with $n = 0.5$. The curve connecting the points with coordinates $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W and $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E is the image of the loxodrome in that projection.

Slika 2. Dio karte svijeta ($\varphi > -60°$) u uspravnoj konusnoj konformnoj projekciji uz $n = 0.5$. Krivulja koja spaja točke s koordinatama $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W i $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E slika je loksodrome u toj projekciji.

Since *K, n, a, β* are constants, (34) represents an exponential relationship between ρ and δ, so the graphical representation of relation (34) will be a logarithmic spiral in the projection plane. In other words, the image of the loxodrome on a map made in a normal aspect conformal conic projection is a logarithmic spiral.

Let us calculate the local linear scale factor at any point of the spiral (34). Since this is a conformal projection, we can write

$$c = h = k = \frac{n\rho}{R\cos\varphi} = \frac{nKe^{-nq(\varphi)}}{R\cos\varphi} \tag{35}$$

and then according to (30)

$$ds' = nK\sqrt{1+a^2}\,\frac{e^{-nq\,\varphi}}{\cos\varphi}d\varphi = nK\sqrt{1+a^2}\,e^{-nq(\varphi)}dq, \tag{36}$$

so it is

$$s' = nK\sqrt{1+a^2}\int_{q_1}^{q_2} e^{-nq(\varphi)}dq = -K\sqrt{1+a^2}\,e^{-nq(\varphi)}\Big|_{q_1}^{q_2} \tag{37}$$

and therefore the length of the arc of the loxodrome between the points corresponding to the latitudes in the plane of the conformal conic projection

$$s' = -\sqrt{1+a^2}\left[\rho(q_2)-\rho(q_1)\right] = \sqrt{1+a^2}\left[\rho(\varphi_1)-\rho(\varphi_2)\right]. \tag{38}$$

If we want the equator to be a standard parallel, then it must be $Kn = R$. The length of the arc of this loxodrome on the sphere is given by (21), and the local linear scale factor is given by (35).

Figure 2 shows a part of the world map ($\varphi > -60°$N) in a normal aspect conic conformal projection with $n = 0.5$. The curve connecting the points with coordinates $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W and $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E is the image of the loxodrome in that projection.

Figure 3 shows a part of the world map ($\varphi > -60°$N) in a normal aspect azimuthal conformal projection, i.e. stereographic projection ($n = 1$). The curve connecting the points with coordinates $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W and $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E is the image of the loxodrome in that projection.

5. Loxodromes in equal-area conic and azimuthal projections of a sphere

The equations of the normal aspect equal-area conic projection of a sphere of radius R are

$$\delta = n\lambda,\quad \rho = R\sqrt{K - \frac{2}{n}\sin\varphi}, \tag{39}$$

where $0 < n < 1$ and K are constants (Snyder 1987, Frančula 2004). If we substitute in (39) the expression for λ from (10) or (14), we can obtain the parametric equations of the loxodrome image in that projection in the form

$$\delta = n\left[a\ln\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)+\beta\right],\quad \rho = R\sqrt{K-\frac{2}{n}\sin\varphi} \tag{40}$$

or

$$\delta = n\left[a\,q(\varphi)+\beta\right],\quad \rho = R\sqrt{K-\frac{2}{n}\tanh q}. \tag{41}$$

Since this is an equal-area conic projection, it is

$$k = \frac{n\rho}{R\cos\varphi} = \frac{1}{h}, \tag{42}$$

so the differential of the arc of the image of the loxodrome in the plane of the equal-area conic projection according to (30) is equal to

$$ds' = R\sqrt{\left(\frac{R\cos\varphi}{n\rho}\right)^2 + a^2\left(\frac{n\rho}{R\cos\varphi}\right)^2}\,d\varphi. \tag{43}$$

If we want the north pole to be mapped to a point, i.e. with the condition that $\rho = 0$ for $\varphi = \pi/2$, then the equation of the equal-area conic projection for $\rho = \rho(\varphi)$ simplifies to

$$\rho = R\sqrt{\frac{2}{n}(1-\sin\varphi)} = \frac{2R}{\sqrt{n}}\sin\left(\frac{\pi}{4}-\frac{\varphi}{2}\right). \tag{44}$$

Then it is

$$k = \frac{\sqrt{n}}{\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)} = \frac{1}{h}, \tag{45}$$

Ako želimo da ekvator bude standardna paralela, onda treba biti $Kn = R$. Duljina luka te loksodrome na sferi dana je s (21), a faktor lokalnog mjerila duljina s (35).

Na slici 2 prikazan je dio karte svijeta ($\varphi > -60°$) u uspravnoj konusnoj konformnoj projekciji uz $n = 0.5$. Krivulja koja spaja točke s koordinatama $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W i $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E slika je loksodrome u toj projekciji.

Na slici 3 prikazan je dio karte svijeta ($\varphi > -60°$) u uspravnoj azimutnoj konformnoj projekciji, odnosno stereografskoj ($n = 1$). Krivulja koja spaja točke s koordinatama $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W i $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E slika je loksodrome u toj projekciji.

Slika 3. Dio karte svijeta ($\varphi > -60°$) u uspravnoj azimutnoj konformnoj, odnosno stereografskoj projekciji ($n = 1$). Krivulja koja spaja točke s koordinatama $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W i $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E slika je loksodrome u toj projekciji.

Fig. 3 Part of the world map ($\varphi > -60°$N) in a normal aspect azimuthal conformal projection ($n = 1$). The curve connecting the points with coordinates $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W and $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E is the image of the loxodrome in that projection.

5. Loksodroma u ekvivalentnoj konusnoj i azimutnoj projekciji sfere

Jednadžbe uspravne ekvivalentne konusne projekcije sfere radijusa R glase

$$\delta = n\lambda, \quad \rho = R\sqrt{K - \frac{2}{n}\sin\varphi}, \tag{39}$$

gdje su $0 < n < 1$ i C konstante (Snyder 1987, Frančula 2004). Uvrstimo li u (39) izraz za λ iz (10) ili (14) možemo dobiti parametarske jednadžbe slike loksodrome u toj projekciji u obliku

$$\delta = n\left[a \ln\tan\left(\frac{\pi}{4} - \frac{\varphi}{2}\right) + \beta\right], \quad \rho = R\sqrt{K - \frac{2}{n}\sin\varphi} \tag{40}$$

ili

$$\delta = n[a\,q(\varphi) + \beta], \quad \rho = R\sqrt{K - \frac{2}{n}\tanh q}. \tag{41}$$

Budući da je riječ o ekvivalentnoj konusnoj projekciji, to je

$$k = \frac{n\rho}{R\cos\varphi} = \frac{1}{h}. \tag{42}$$

pa je diferencijal luka slike loksodrome u ravnini ekvivalentne konusne projekcije prema (30) jednak

$$ds' = R\sqrt{\left(\frac{R\cos\varphi}{n\rho}\right)^2 + a^2\left(\frac{n\rho}{R\cos\varphi}\right)^2}\,d\varphi. \tag{43}$$

Ako želimo da se sjeverni pol preslika u točku, tj. uz uvjet da je $\rho = 0$ za $\varphi = \pi/2$, onda se jednadžba ekvivalentne konusne projekcije za $\rho = \rho(\varphi)$ pojednostavnjuje i glasi

$$\rho = R\sqrt{\frac{2}{n}(1 - \sin\varphi)} = \frac{2R}{\sqrt{n}}\sin\left(\frac{\pi}{4} - \frac{\varphi}{2}\right). \tag{44}$$

Tada je

$$k = \frac{\sqrt{n}}{\cos\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)} = \frac{1}{h}, \tag{45}$$

i

$$ds' = R\sqrt{\frac{\cos^2\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)}{n} + a^2\frac{n}{\cos^2\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)}}\,d\varphi =$$

$$= R\sqrt{\frac{1 + \sin\varphi}{2n} + a^2\frac{2n}{1 + \sin\varphi}}\,d\varphi. \tag{46}$$

Integriranje diferencijala (43) ili (46) može se obaviti nekom od metoda numeričke integracije. Duljina luka loksodrome na sferi dana je s (21), a faktor lokalnog mjerila duljina u točkama loksodrome s (47):

$$c = \frac{ds'}{ds} = \sqrt{\frac{h^2 + \frac{a^2}{h^2}}{1 + a^2}}. \tag{47}$$

Fig. 4 A map of the world in a normal aspect equal-area conic projection ($n = 0.5$). The curve connecting the points with coordinates φ_1 = 55°00' S, λ_1 = 65°00' W and φ_2 = 55°00' N, λ_2 = 65°00' E is the image of the loxodrome in that projection.
Slika 4. Karta svijeta u uspravnoj ekvivalentnoj konusnoj projekciji ($n = 0.5$). Krivulja koja spaja točke s koordinatama φ_1 = 55°00' S, λ_1 = 65°00' W i φ_2 = 55°00' N, λ_2 = 65°00' E slika je loksodrome u toj projekciji.

and

$$ds' = R\sqrt{\frac{\cos^2\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)}{n}+a^2\frac{n}{\cos^2\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)}}d\varphi =$$

$$= R\sqrt{\frac{1+\sin\varphi}{2n}+a^2\frac{2n}{1+\sin\varphi}}d\varphi . \quad (46)$$

The integration of differentials (43) or (46) can be done by some of the numerical integration methods. The arc length of a loxodrome on a sphere is given by (21), and the local linear scale factor at points of the loxodrome is given by (47):

$$c = \frac{ds'}{ds} = \sqrt{\frac{h^2+\frac{a^2}{h^2}}{1+a^2}} . \quad (47)$$

Figure 4 shows a map of the world in a normal aspect equal-area conic projection (n = 0.5). The curve connecting the points with coordinates φ_1 = 55°00' S, λ_1 = 65°00' W and φ_2 = 55°00' N, λ_2 = 65°00' E is the image of the loxodrome in that projection.

Figure 5 shows a map of the world in a normal aspect equal-area azimuzhal projection (n = 1). The curve connecting the points with coordinates φ_1 = 55°00' S, λ_1 = 65°00' W and φ_2 = 55°00' N, λ_2 = 65°00' E is the image of the loxodrome in that projection.

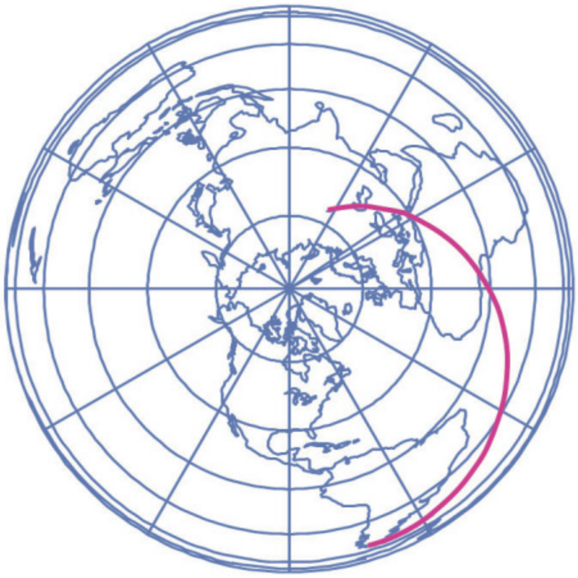


Fig. 5 A map of the world in a normal aspect equal-area azimuthal projection ($n = 1$). The curve connecting the points with coordinates φ_1 = 55°00' S, λ_1 = 65°00' W and φ_2 = 55°00' N, λ_2 = 65°00' E is the image of the loxodrome in that projection.
Slika 5. Karta svijeta u uspravnoj ekvivalentnoj azimutnoj projekciji ($n = 1$). Krivulja koja spaja točke s koordinatama φ_1 = 55°00' S, λ_1 = 65°00' W i φ_2 = 55°00' N, λ_2 = 65°00' E slika je loksodrome u toj projekciji.

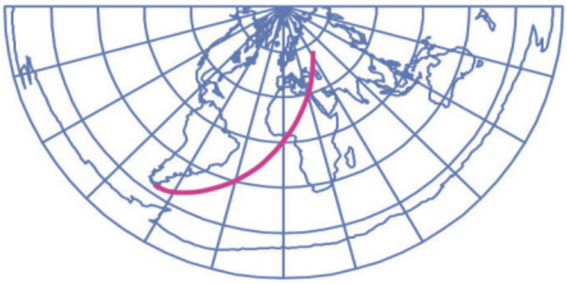
6. Loxodromes in equidistant along meridians conic and azimuthal projections of a sphere

The equations of the normal aspect equidistant along meridians conic projection of a sphere are

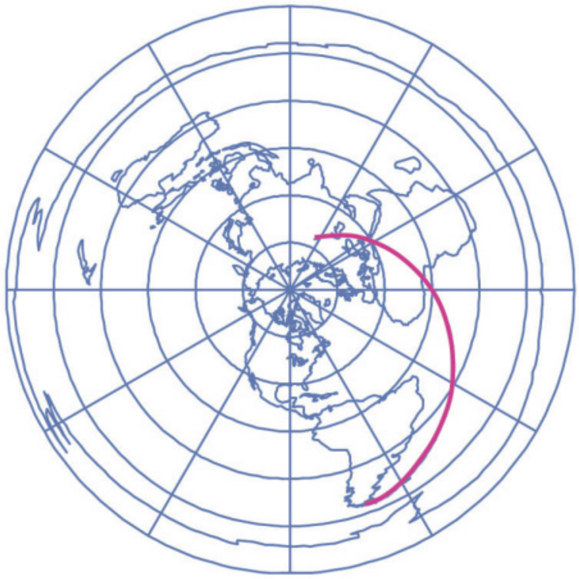
$$\delta = n\lambda,\ \rho = K - R\varphi, \quad (48)$$

where $0 < n < 1$ and K are constants (Snyder 1987, Frančula 2004). If we substitute in (48) the expression for λ from (10), we can obtain the parametric equations of the loxodrome image in that projection in the form

$$\delta = n[a\ln\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)+\beta\ ],\ \rho = K - R\varphi. \quad (49)$$



Slika 6. Karta svijeta u uspravnoj konusnoj projekciji ekvidistantnoj uzduž meridijana uz n = 0.5. Krivulja koja spaja točke s koordinatama φ_1 = 55°00' S, λ_1 = 65°00' W i φ_2 = 55°00' N, λ_2 = 65°00' E slika je loksodrome u toj projekciji.
Fig. 6 A map of the world in a normal aspect conic projection equidistant along the meridians with n = 0.5. The curve connecting the points with coordinates φ_1 = 55°00' S, λ_1 = 65°00' W and φ_2 = 55°00' N, λ_2 = 65°00' E is the image of the loxodrome in that projection.



Slika 7. Karta svijeta u uspravnoj azimutnoj projekciji ekvidistantnoj uduž meridijana (n = 1). Krivulja koja spaja točke s koordinatama φ_1 = 55°00' S, λ_1 = 65°00' W i φ_2 = 55°00' N, λ_2 = 65°00' E slika je loksodrome u toj projekciji.
Fig. 7 A map of the world in a normal aspect azimuthal projection equidistant along the meridians (n = 1). The curve connecting the points with coordinates φ_1 = 55°00' S, λ_1 = 65°00' W and φ_2 = 55°00' N, λ_2 = 65°00' E is the image of the loxodrome in that projection.

Na slici 4 prikazana je karta svijeta u uspravnoj ekvivalentnoj konusnoj projekciji (n = 0.5). Krivulja koja spaja točke s koordinatama φ_1 = 55°00' S, λ_1 = 65°00' W i φ_2 = 55°00' N, λ_2 = 65°00' E slika je loksodrome u toj projekciji.

Na slici 5 prikazana je karta svijeta u uspravnoj ekvivalentnoj azimutnoj projekciji (n = 1). Krivulja koja spaja točke s koordinatama φ_1 = 55°00' S, λ_1 = 65°00' W i φ_2 = 55°00' N, λ_2 = 65°00' E slika je loksodrome u toj projekciji.

6. Loksodroma u konusnoj i azimutnoj projekciji sfere ekvidistantnoj uzduž meridijana

Jednadžbe uspravne konusne projekcije sfere ekvidistantne uzduž meridijana glase

$$\delta = n\lambda,\ \rho = K - R\varphi, \tag{48}$$

gdje su 0 < n < 1 i K konstante (Snyder 1987, Frančula 2004). Uvrstimo li u (48) izraz za λ iz (10) možemo dobiti parametarske jednadžbe slike loksodrome u toj projekciji u obliku

$$\delta = n[a \ln \tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)+\beta],\ \rho = K - R\varphi. \tag{49}$$

Budući da je riječ o konusnoj projekciji ekvidistantnoj uzduž meridijana, to je

$$h = 1,\quad k = \frac{n\rho}{R\cos\varphi} \tag{50}$$

pa je diferencijal luka slike loksodrome u ravnini te projekcije prema (30) jednak

$$ds' = R\sqrt{1+a^2\left(\frac{n\rho}{R\cos\varphi}\right)^2}\,d\varphi. \tag{51}$$

Ako želimo da se sjeverni pol preslika u točku, tj. uz uvjet da je $\rho = 0$ za $\varphi = \pi/2$, onda jednadžba za $\rho = \rho(\varphi)$ konusne projekcije ekividistantne uzduž meridijana glasi

$$\rho = R\left(\frac{\pi}{2}-\varphi\right) \tag{52}$$

Na slici 6 prikazana je karta svijeta u uspravnoj konusnoj projekciji ekvidistantnoj uzduž meridijana uz

Since it is a conic projection equidistant along the meridians, it is

$$h = 1, \quad k = \frac{n\rho}{R\cos\varphi}, \tag{50}$$

so the differential of the arc of the image of the loxodrome in the plane of that projection according to (30) is equal to

$$ds' = R\sqrt{1 + a^2\left(\frac{n\rho}{R\cos\varphi}\right)^2}\,d\varphi. \tag{51}$$

If we want the North Pole to be mapped to a point, i.e. with the condition that $\rho = 0$ for $\varphi = \pi/2$, then the equation for $\rho = \rho(\varphi)$ of the equidistant conic projection along the meridian is

$$\rho = R\left(\frac{\pi}{2} - \varphi\right). \tag{52}$$

Figure 6 shows a map of the world in a normal aspect conic projection equidistant along the meridians with $n = 0.5$. The curve connecting the points with coordinates $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W and $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E is the image of the loxodrome in that projection.

Then it is

$$ds' = R\sqrt{1 + \left(\frac{an\left(\frac{\pi}{2} - \varphi\right)}{\cos\varphi}\right)^2}\,d\varphi. \tag{53}$$

Integrating the differential (53) can be done using some of the numerical integration methods.

The length of the arc of a loxodrome on a sphere is given by (21), and the local linear scale factor at the points of the loxodrome is given by (54):

$$c = \frac{ds'}{ds} = \sqrt{\frac{1 + \left(\frac{an\left(\frac{\pi}{2} - \varphi\right)}{\cos\varphi}\right)^2}{1 + a^2}}. \tag{54}$$

Figure 7 shows a map of the world in a normal aspect azimuthal projection equidistant along the meridians ($n = 1$). The curve connecting the points with coordinates $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W and $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E is the image of the loxodrome in that projection.

7. Loxodrome in the equidistant along the parallels conic and azimuthal projection of a sphere

The equations of the normal aspect equidistant along parallels conic projection of a sphere are:

$$\delta = n\lambda, \quad \rho = \frac{R}{n}\cos\varphi, \tag{55}$$

where $0 < n < 1$ and R are constants. If we substitute the expression for λ from (10) into (55), we can obtain the parametric equations of the image of the loxodrome in this projection in the form

$$\delta = n\left[a\ln\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) + \beta\right], \quad \rho = \frac{R}{n}\cos\varphi. \tag{56}$$

Figure 8 shows the map of the world in a normal aspect conic projection equidistant along the parallels with $n = 0.5$. The curve connecting the points with the coordinates $\varphi_1 = 0°00'$ S, $\lambda_1 = 165°00'$ W and $\varphi_2 = 55°00'$ N, $\lambda_2 = 165°00'$ E is the image of the loxodrome in that projection.

Since it is a conic projection equidistant along the parallels, it is

$$k = 1, \tag{57}$$

$$\frac{d\rho}{d\varphi} = -\frac{R}{n}\sin\varphi, \quad h = \frac{\sin\varphi}{n}, \tag{58}$$

so the differential of the arc of the image of the loxodrome in the plane of that projection according to (30) is equal to

$$ds' = R\sqrt{\left(\frac{\sin\varphi}{n}\right)^2 + a^2}\,d\varphi. \tag{59}$$

Integrating the differential (59) can be done using some of the numerical integration methods.

The length of the arc of a loxodrome on a sphere is given by (21), and the local linear scale factor at the points of the loxodrome is given by (54):

$$c = \frac{ds'}{ds} = \sqrt{\frac{\left(\frac{\sin\varphi}{n}\right)^2 + a^2}{1 + a^2}}. \tag{60}$$

Figure 9 shows a map of the world in a normal aspect azimuthal projection equidistant along the parallels ($n = 1$). Usually this projection is called orthographic. The curve connecting the points with the coordinates $\varphi_1 = 0°00'$ S, $\lambda_1 = 165°00'$ W and $\varphi_2 = 55°00'$ N, $\lambda_2 = 165°00'$ E is the image of the loxodrome in that projection.

8. Conclusion

The advantage of a loxodrome is that a ship sailing on it sails on a fixed course. On a map made in the Mercator projection, the loxodrome is shown as a straight line. For this reason, the loxodrome is regularly linked and explored in connection with the

Slika 8. Karta sjeverne hemisfere u uspravnoj konusnoj projekciji ekvidistantnoj uduž paralela uz $n = 0.5$. Krivulja koja povezuje točke s koordinatama $\varphi_1 = 0°00'$ S, $\lambda_1 = 165°00'$ W i $\varphi_2 = 55°00'$ N, $\lambda_2 = 165°00'$ E slika je loksodrome u toj projekciji.

Fig. 8 Map of the North hemisphere in a normal aspect conic projection equidistant along the parallels with $n = 0.5$. The curve connecting the points with the coordinates $\varphi_1 = 0°00'$ S, $\lambda_1 = 165°00'$ W and $\varphi_2 = 55°00'$ N, $\lambda_2 = 165°00'$ E is the image of the loxodrome in that projection.

$n = 0.5$. Krivulja koja spaja točke s koordinatama $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W i $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E slika je loksodrome u toj projekciji.

Tada je

$$ds' = R\sqrt{1+\left(\frac{an\left(\frac{\pi}{2}-\varphi\right)}{\cos\varphi}\right)^2}\,d\varphi. \tag{53}$$

Integriranje diferencijala (53) može se obaviti nekom od metoda numeričke integracije.

Duljina luka loksodrome na sferi dana je s (21), a faktor lokalnog mjerila duljina u točkama loksodrome s (54):

$$c = \frac{ds'}{ds} = \sqrt{\frac{1+\left(\frac{an\left(\frac{\pi}{2}-\varphi\right)}{\cos\varphi}\right)^2}{1+a^2}}. \tag{54}$$

Na slici 7 prikazana je karta svijeta u uspravnoj azimutnoj projekciji ekvidistantnoj uzduž meridijana ($n = 1$). Krivulja koja spaja točke s koordinatama $\varphi_1 = 55°00'$ S, $\lambda_1 = 65°00'$ W i $\varphi_2 = 55°00'$ N, $\lambda_2 = 65°00'$ E slika je loksodrome u toj projekciji.

Slika 9. Karta sjeverne hemisfere u ortografskoj ili uspravnoj azimutnoj projekciji ekvidistantnoj uduž paralela ($n = 1$). Krivulja koja povezuje točke s koordinatama $\varphi_1 = 0°00'$ S, $\lambda_1 = 165°00'$ W i $\varphi_2 = 55°00'$ N, $\lambda_2 = 165°00'$ E slika je loksodrome u toj projekciji.

Fig. 9 Map of the North hemisphere in a normal aspect azimuthal projection equidistant along the parallels ($n = 1$). Usually this projection is called orthographic. The curve connecting the points with the coordinates $\varphi_1 = 0°00'$ S, $\lambda_1 = 165°00'$ W and $\varphi_2 = 55°00'$ N, $\lambda_2 = 165°00'$ E is the image of the loxodrome in that projection.

7. Loksodroma u konusnoj i azimutnoj projekciji sfere ekvidistantnoj uzduž paralela

Jednadžbe uspravne konusne projekcije sfere ekvidistantne uzduž paralela glase

$$\delta = n\lambda, \quad \rho = \frac{R}{n}\cos\varphi, \tag{55}$$

gdje su $0 < n < 1$ i R konstante. Uvrstimo li u (55) izraz za λ iz (10) možemo dobiti parametarske jednadžbe slike loksodrome u toj projekciji u obliku

$$\delta = n\left[a\ln\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)+\beta\right], \quad \rho = \frac{R}{n}\cos\varphi \tag{56}$$

Na slici 8 prikazana je karta svijeta u uspravnoj konusnoj projekciji ekvidistantnoj uzduž paralela uz $n = 0.5$.

Mercator projection of a sphere or a rotating ellipsoid, and all nautical charts have been made in the Mercator projection for centuries.

In this paper, we deal with the loxodrome in conic and azimuthal projections. It is shown that the image of the loxodrome in such projections is not a straight line but a spiral curve. Furthermore, the determination of the distance measured along the loxodrome between two points in a normal aspect conic or azimuthal projection is presented. Formulas for the length of the arc of the loxodrome in each of these projections are derived using examples of conformal, equivalent and equidistant conic projections. Azimuthal projections are special cases of conic projections, so no special derivations of formulas are required for them.

References / Literatura

Frančula, N. (2004) Kartografske projekcije, skripta, Geodetski fakultet Sveučilišta u Zagrebu

Grunert, J. A. (1849) Loxodromische Trigonometrie, Verlag von E. B. Schwickert, Leipzig

Pomorski leksikon (1990) loksodroma, Pomorski leksikon (1990), mrežno izdanje. Leksikografski zavod Miroslav Krleža, 2024. https://pomorski.lzmk.hr/clanak/loksodroma, Pristupljeno 15. 12. 2024.

Snyder J P (1987) Map Projections: A Working Manual, USGS Professional Paper 1395, Washington, D.C.

Wiki (2024) Integral funkcije sekans. <http://en.wikipedia.org/wiki/Integral_of_the_secant_function>, Pristupljeno 15. 12. 2024.

Krivulja koja povezuje točke s koordinatama φ_1 = 0°00' S, λ_1=165°00' W i φ_2 = 55°00' N, λ_2 = 165°00' E slika je loksodrome u toj projekciji.

Budući da je riječ o konusnoj projekciji ekvidistantnoj uzduž paralela, to je

$$k = 1, \tag{57}$$

$$\frac{d\rho}{d\varphi} = -\frac{R}{n}\sin\varphi, \quad h = \frac{\sin\varphi}{n}, \tag{58}$$

pa je diferencijal luka slike loksodrome u ravnini te projekcije prema (30) jednak

$$ds' = R\sqrt{\left(\frac{\sin\varphi}{n}\right)^2 + a^2}\, d\varphi. \tag{59}$$

Integriranje diferencijala (59) može se obaviti nekom od metoda numeričke integracije.

Duljina luka loksodrome na sferi dana je s (21), a faktor lokalnog mjerila duljina u točkama loksodrome s (54):

$$c = \frac{ds'}{ds} = \sqrt{\frac{\left(\frac{\sin\varphi}{n}\right)^2 + a^2}{1 + a^2}}. \tag{60}$$

Na slici 9 prikazana je karta svijeta u uspravnoj azimutnoj projekciji ekvidistantnoj uzduž paralela (n = 1). Obično se ta projekcija naziva ortografskom. Krivulja koja povezuje točke s koordinatama φ_1 = 0°00' S, λ_1 = 165°00' W i φ_2 = 55°00' N, λ_2 = 165°00' E slika je loksodrome u toj projekciji.

8. Zaključak

Dobra strana loksodrome je u tome što brod koji plovi po loksodromi, plovi nepromijenjenim kursom. Na karti izrađenoj u Mercatorovoj projekciji prikazana je ravnom crtom. Zbog toga se loksodroma redovito povezuje i istražuje u vezi s Mercatorovom projekcijom sfere ili rotacijskog elipsoida, a sve pomorske karte već se stoljećima izrađuju u Mercatorovoj projekciji.

U ovom se radu bavimo loksodromom u konusnim i azimutnim projekcijama. Pokazuje se da slika loksodrome u takvim projekcijama nije ravna crta nego spiralna krivulja. Nadalje, prikazano je određivanje udaljenosti mjerene uzduž loksodrome između dviju točaka u nekoj uspravnoj konusnoj ili azimutnoj projekciji. Na primjerima konformnih, ekvivalentnih i ekvidistantnih konusnih projekcija izvedene su formule za duljinu luka loksodrome u svakoj od tih projekcija. Azimutne projekcije su posebni slučajevi konusnih, tako da za njih nisu potrebni posebni izvodi formula.