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Enhancing Algebraic Teaching through Imaginary Dialogues: Exploring Pre-service Teachers' Instructional Explanations

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Abstract

Effective instructional explanations are crucial for students' learning, and teachers' pedagogical content knowledge plays a significant role in enhancing the quality of these explanations. This study aimed to investigate how prospective teachers construct their instructional explanations to address common algebraic misconceptions, employing imaginary dialogues as a methodological tool. The study involved 28 pre-service teachers, whose instructional explanations were examined through semi-structured imaginary mathematical dialogues. Qualitative analysis revealed that while some pre-service teachers effectively connected student curiosity with content, many struggled to provide comprehensive explanations and adequately address students' misconceptions and learning difficulties. This paper highlights the importance of aligning instructional explanations with students' everyday experiences and the need for pre-service teachers to strengthen their specialized content knowledge. It also suggests integrating microteaching practices, such as developing and applying imaginary dialogues, in algebra teaching course to enhance instructional skills and encourage self-reflection.

Key words: algebraic teaching; pre-service teachers; instructional explanations; imaginary dialogues

Introduction

Significant research in the field of teaching and teacher education has focused on teachers' professional knowledge, emphasizing the critical role of Content Knowledge (CK) and Pedagogical Content Knowledge (PCK) (Campbell et al., 2014; Copur-Gencturk & Li, 2023; Depaepe et al., 2013; Ponte & Chapman, 2016; Şahin et al., 2016). CK pertains to a comprehensive understanding of the subject matter that teachers are responsible for imparting, serving as a foundation for effective teaching practices and the development of PCK. Originally conceptualized by Shulman (1986), PCK embodies the integration of subject matter knowledge with general pedagogical principles. This unique blend is essential for transforming teachers' content expertise into formats that are both accessible and engaging for students, thus facilitating deeper learning. Shulman (1987) described PCK as "that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 8). The framework of PCK encompasses three essential components: the knowledge of student understanding, the knowledge of instructional strategies, and the knowledge of curriculum. Effective teaching transcends mere content mastery; it requires the ability to render content accessible through diverse pedagogical approaches, including analogies, examples, explanations, simulations, and educational materials (Ball et al., 2008; Blömeke & Delaney, 2012; Mohr-Schroeder et al., 2017). This aspect is especially valuable in classroom settings, where teachers need skills to discuss ideas, gauge student comprehension, explain concepts clearly, and create environments conducive to learning (Ball & Forzani, 2009). Instructional explanations, as one component of PCK's instructional strategies, are significant because they must align with students' cognitive levels to foster understanding (Ma, 2020). Despite its importance, research indicates that pre-service teachers often face challenges in providing clear instructional explanations that clarify the principles underlying mathematical concepts. These difficulties suggest a need for targeted development in teacher education programs (Karakus, 2018; Thanheiser, 2009; Toluk-Ucar, 2009).

Algebra, as an essential part of the school mathematics curriculum, emphasizes the use of mathematical symbols and their manipulation. It equips students with the skills to employ variables and algebraic expressions to represent and analyze mathematical relationships. This foundational knowledge enables students to solve practical problems such as calculating interest rates, determining best-value purchases, analyzing growth patterns, and making predictions from data. The exploration of patterns develops abstract thinking and algebraic reasoning skills, which are essential for understanding equations and functions in daily life (NCTM, 2000). Despite its significance, students often struggle with its concepts, facing challenges such as distinguishing between variables and constants, interpreting algebraic notation, and understanding the concept of equivalence. These challenges can lead to misconceptions, influenced by both cognitive development and instructional approaches (Stewart, 2013). Teachers must understand student thought processes, algebraic reasoning, and common misconceptions to address these difficulties effectively (Guler & Celik, 2021). Instructional explanations serve as crucial bridges between students' existing knowledge and new algebraic concepts, offering structured guidance to overcome misconceptions (Charalambous et al., 2011). Effective algebraic explanations clarify reasoning processes, connect abstract concepts to concrete contexts, and address typical challenges in recognizing and applying algebraic rules, simplifying expressions, and solving equations (Leinhardt, 2001, 2010). These explanations often incorporate visual representations, contextual examples and hands-on demonstrations to enhance student comprehension (Lachner et al., 2019).

Pre-service teachers' content-specific knowledge has a substantial impact on the quality of their instructional explanations (Charalambous et al., 2011; Leinhardt, 2001). The clarity and coherence of these instructional explanations reflects teachers' depth of conceptual knowledge and their capacity to make abstract content comprehensible to students (Leinhardt, 2010). Despite the foundational role of algebra and instructional explanations in mathematics education, research on pre-service teachers' PCK remain limited. Thus, it is crucial to evaluate how prospective teachers construct their explanations in response to common algebraic misconceptions and difficulties within the PCK framework, as these strategies directly influence the quality of mathematics teaching and ultimately student learning outcomes. This study explores how middle school pre-service mathematics teachers construct instructional explanations to address algebraic mis-conceptions, utilizing imaginary dialogues as a tool to structure their explanations within the PCK framework.

Theoretical framework

Pedagogical content knowledge

Shulman (1986) identified three critical types of knowledge necessary for effective teaching: CK, PCK, and curriculum knowledge. CK refers to a deep understanding of the subject matter, including its underlying principles, structures, and the ability to validate concepts and facts in diverse contexts. CK is crucial as it establishes a strong disciplinary foundation, equipping teachers with the depth of knowledge required to address students' questions accurately and to correct misunderstandings. Though it does not directly relate to teaching methods, CK forms the basis upon which pedagogical strategies are built. PCK, as defined by Shulman (1986), extends beyond CK by integrating it with pedagogical skills specifically tailored for teaching a subject effectively. Unlike general content knowledge, PCK enables teachers to transform complex mathematical ideas into accessible learning experiences, adapting their explanations to align with students' cognitive levels and prior understanding (Ball, 1990; Hill et al., 2008). PCK involves interpreting subject matter and selecting appropriate teaching approaches that encourage student engagement and comprehension (Baki, 2020; Grossman, 1990). Central to PCK are components such as knowledge of diverse representations, analogies, examples, and explanations, combined with an awareness of common student misconceptions and strategies to address them (Ball, 1990; Shulman, 1986). A significant focus within this framework is the understanding of students' thought processes, including their errors and misconceptions, which informs the development of effective instructional practices. For pre-service and in-service teachers, their capability to provide effective instructional explanations is a crucial dimension of PCK (Karakuş, 2018). These explanations demonstrate teachers' ability to transform complex mathematical concepts into understandable forms, address common student misconceptions, select appropriate examples and representations, and scaffold mathematical understanding. Research has demonstrated that teachers with stronger PCK provide higher quality instructional explanations (Charalambous et al., 2011; Ma, 2020). These explanations serve as a bridge between teachers' mathematical knowledge and effective classroom practice, making them a critical indicator of teaching quality and student learning outcomes. Each mathematical topic (e.g. algebra) requires a unique set of PCK. For effective teaching, it is crucial for teachers to understand the nature of the subject they are teaching, as well as common student misconceptions and difficulties related to that topic.

Algebra is often considered an important topic in school mathematics, as it introduces students to concepts that are essential for higher-level mathematics, such as the equals sign, variables, formal notation and equations (Usiskin, 1997; Witzel et al., 2003). These core elements underpin various areas in mathematics, supporting the development of algebraic reasoning and problem-solving skills (Matthews & Fuchs, 2018). However, many students face challenges in understanding algebra's systematic structure and applying it in problem-solving contexts (Kieran, 1992). For example, common learning difficulties include interpreting the equal sign as a relational symbol, rather than merely an operator, and accurately translating verbal statements into algebraic expressions. Introducing algebra at an early age presents additional complexities, such as fostering early algebraic reasoning and equipping teachers with the necessary skills and strategies to support student learning effectively. This transition from arithmetic to algebraic thinking-typically observed during middle school-varies depending on education systems and curriculum structures (Asquith et al., 2007). Despite ongoing efforts, students continue to find algebra challenging (Butuner & Guler, 2017), a difficulty further compounded by the often inadequate algebraic knowledge of pre-service teachers (Guler & Celik, 2021; Tanisli & Kose, 2013). Key challenges in teaching algebra can be categorized into content-specific difficulties, cognitive obstacles, instructional approaches, and the complexities involved in transitioning from arithmetic to algebra (Wang, 2015). Addressing these challenges requires well-planned instructional strategies and supportive learning environments. The effectiveness of instructional explanations provided by teachers is a crucial factor in helping students overcome these obstacles; however, research on the quality of instructional explanations offered by pre-service teachers in algebra is limited (Guler & Celik, 2016).

Instructional explanations in mathematics teaching

Instructional explanations in mathematics teaching represent a specific pedagogical approach characterized by deliberate communicative strategies aimed at facilitating learners' understanding and acquisition of mathematical concepts, procedures, and reasoning (Leinhardt, 2010). These explanations constitute a distinct teaching method that involves systematic presentation and elaboration of mathematical ideas, different from other methods. The effectiveness of instructional explanations is determined by multiple interconnected factors: conceptual clarity and depth, mathematical accuracy and precision, responsiveness to students' prior knowledge and misconceptions, use of appropriate representations and examples, integration of mathematical connections and relationships, quality of teacher-student discourse, and alignment with learning objectives (Stein & Kucan, 2010). These explanations, provided either through direct instruction or interactive discussions, promote active learning and encourage student inquiry (Inoue, 2009). When aligned with curriculum goals, strong instructional explanations can significantly enhance both students' mathematical understanding and their meta-cognitive abilities (Leinhardt & Schwarz, 1997). Research indicates that effective and ineffective explanations differ primarily in their approach to mathematical understanding. Effective explanations build on students' existing knowledge, address common misconceptions, make explicit mathematical connections, use precise mathematical language, and engage students in meaningful mathematical discourse (Inoue, 2009). For instance, when teaching fractions, effective instructional explanations must address various conceptual interpretations (part-whole, ratio, operator, quotient, and measure), not just simplified real-world applications. This requires careful sequencing of ideas and explicit connections between different representations (Grossman & McDonald, 2008). Similarly, in algebra, effective explanations integrate symbolic, graphic, and contextual representations while maintaining high cognitive demand, whereas procedural explanations often result in superficial learning and limited transfer abilities (Livers & Elmore, 2018).

Recent studies have identified several approaches to enhance the effectiveness of instructional explanations. Self-explanation, where learners articulate their understanding of a concept, has been shown to deepen comprehension and retention (Wittwer & Renkl, 2008). Moreover, worked examples provide structured demonstrations of solution processes, serving as tools for understanding both procedures and underlying mathematical concepts (Renkl, 2014). These approaches are complemented by problem-solving tasks and exploratory discussions that allow students to discover mathematical relationships and develop solution strategies (Chi & Wylie, 2014). However, research on pre-service teachers' instructional explanations has revealed significant challenges. Studies indicate that explanations often rely on procedural knowledge rather than conceptual understanding (Baki, 2013; Karakuş, 2018; Kinach, 2002), potentially due to limited PCK and mathematical beliefs about teaching and learning (Kul & Celik, 2017; Philipp, 2007; Thompson, 1992; Toluk-Ucar, 2011). Specifically, pre-service teachers struggle to provide conceptually-rich explanations, frequently focusing on procedural steps without addressing the underlying mathematical relationships (Avcu, 2022; Karakuş, 2018; Toluk-Ucar, 2011). These difficulties extend to connecting multiple representations and addressing student misconceptions effectively. For instance, Young and Zientek (2011) found that pre-service teachers struggled to explain algebraic concepts beyond computational procedures, while Toluk-Ucar (2011) revealed their limited ability to address student misconceptions effectively.

Previous research has explored instructional explanations through various analytical frameworks. For instance, Charalambous et al. (2011) examined instructional explanations in terms of how teachers balance procedural and conceptual knowledge, emphasizing the importance of clarity, coherence, and contextual awareness in effective teaching. This framework highlights that instructional explanations are not only about conveying steps but also about providing students with the underlying reasoning behind mathematical concepts. Similarly, Thanheiser (2009) has analyzed instructional explanations by focusing on the interplay between content knowledge and pedagogical skill, suggesting that an understanding of student misconceptions and prior knowledge is essential for meaningful explanations. The complexity of assessing teachers' mathematical knowledge stems from the subject's multifaceted nature. Krauss et al. (2008) argue that analyzing specific aspects of teaching practice, such as instructional explanations, provides more meaningful insights than general evaluations of teaching competence. This targeted approach allows for a more nuanced understanding of how teachers develop and apply their mathematical knowledge in practice. Building on this theoretical foundation, our research investigates how pre-service mathematics teachers structure instructional explanations to address algebraic misconceptions, providing insight into their pedagogical reasoning and content knowledge development.

Imaginary dialogues

Instructional explanations occur through mathematical dialogues between teachers and students. The initiation, continuation, and conclusion of an explanation are guided by structured disciplinary dialogue (Leinhardt, 1993). Imaginary dialogues act as a tool to gain insights into students' learning challenges and perceptions (Wille, 2008). The foundation of imaginary dialogues can be traced back to written dialogues or student journals (Bean & Zulich, 1989; Lee, 2004). Clarke et al. (1993) categorized student-written journals into three types: narrative, summary and dialogue, noting that dialogues stand out by allowing students to identify challenges and articulate their thought processes. Wille (2008; 2017) described mathematical written dialogues as imaginary dialogues in the context of mathematics education literature.

Gössinger and Götz (2023) desribed imaginary dialogues as a form of mathematical writing, emphasizing the importance of encouraging students to extend a written and unfinished dialogue between two fictional individuals discussing a mathematical problem. Morever, the teacher's role in sustaining classroom dialogue (Webb, 2009; Dobber & van Oers, 2015) is crucial for pre-service teachers who will use these skills in their future careers. Studies on imaginary mathematical dialogues involving teachers and pre-service teachers support this focus. For example, Brodahl and Wathne (2018) investigated the experiences of pre-service teachers with imaginary mathematical dialogues and examined their perception of these dialogues in explaining students' mathematical thinking. Spangler and Hallman-Thrasher (2014) engaged pre-service teachers in writing imaginary mathematical dialogues to strengthen their mathematics teaching knowledge and analyzed how students processed these exercises. Their findings suggested that dialogues incorporating mathematical tasks enhanced pre-service teachers' content knowledge and understanding of student interactions. In a study by Wille (2016), pre-service teachers wrote imaginary mathematical dialogues in the context of algebra learning, revealing instances where mathematical discourse could lead to potential misconceptions or errors. The study noted that pre-service teachers gained awareness of instructional explanations. The unique contribution of this study lies in training pre-service teachers on how to conduct mathematical dialogues from start to finish, culminating in the development of their own dialogues. This comprehensive approach allows a more nuanced understanding of how teachers build and apply mathematical knowledge in practice.

Building on this theoretical foundation, the aim of our research is to explore how pre-service mathematics teachers construct their instructional explanations to address algebraic misconceptions through the use of imaginary dialogues. Imaginary dialogues offer a reflective platform, enabling prospective teachers to gain insights into students' perspectives and refine their instructional methods accordingly (Brodahl & Wathne, 2018; Wille, 2008). Wille (2017) highlighted that pre-service teachers who engage in writing mathematical discourse develop an awareness of their own instructional methods, which can guide students' mathematical learning. By analyzing pre-service teachers' instructional explanations within these dialogues, this study seeks to contribute to effective algebra instruction and inform teacher preparation programs for better management of algebraic misconceptions in classrooms. Our research questions are: i) What types of instructional explanations do pre-service mathematics teachers propose for teaching algebraic concepts? ii) How do pre-service teachers enrolled in the 'Communication in Mathematics Teaching (CMT)' course develop instructional explanations using imaginary dialogues to address algebraic misconceptions effectively?

Methodology

This study employed a qualitative research approach known as a case study to examine the instructional explanations of pre-service teachers in algebra through the use of imaginary dialogues. A case study involves an in-depth analysis of one or more cases (Christensen et al., 2015). In this research, the case being examined was the instructional explanations of pre-service teachers in the domain of algebra, while the topic of the analysis was the pre-service teachers who presented instructional explanations through imaginary dialogues.

Participants

The instructional explanations were investigated through a purposive sampling approach, selecting pre-service teachers based on specific criteria established by the researcher. The selection criteria included successful completion of undergraduate courses in 'CMT' and 'Algebra Teaching', and ensuring participants were equipped to write realistic, classroom-based imaginary dialogues. To further qualify participants, it was essential for them to be familiar with classroom interactions and possess some teaching experience, fulfilled by their participation in 'School Practicum' courses. Moreover, pre-service teachers who had prior experience in drafting imaginary dialogues were specifically chosen. Participants who met these requirements underwent training designed to enhance their ability to create suitable imaginary dialogues. This training spanned four class hours, distributed over two weeks.

The study was conducted during the fall semester of the academic year 2022/2023, involving 28 pre-service middle school teachers enrolled in the 'CMT' course at a state university in Türkiye. Compulsory education in Türkiye extends over 12 years and is segmented into three stages: the first stage comprises a 4-year primary school period (grades 1 to 4), the second stage encompasses a 4-year middle school period (grades 5 to 8), and the third stage consists of a 4-year high school period (grades 9 to 12) (MoNE, 2012). Consequently, the pre-service teachers discussed in this study, once appointed as teachers, would be responsible for teaching students from the 5th to 8th grade levels.

Data collection tools

Semi-structured imaginary dialogues were developed to examine the instructional explanations of pre-service teachers. Misconceptions commonly encountered by middle school students in algebra were identified through a literature review (Bishop et al., 2008; Booth, 1988; Chow, 2011; Foster, 2007; Tirosh et al., 1998). Four scenarios in algebra where students exhibit misconceptions were chosen for creating imaginary dialogues: operations in algebraic expressions, using a balance scale to model and solve equations, writing algebraic-verbal expressions, and explaining the concepts of variables and unknowns.

To reflect real classroom interactions, semi-structured dialogues were designed to simulate actual classroom settings. Prior to developing these semi-structured and unstructured dialogues, authentic classroom interactions during middle school algebra lessons were recorded and transcribed for analysis. The data for these dialogues were obtained from the research compiled in Celik's (2019) doctoral thesis. Before finalizing the data collection tool, 39 class hours of written real classroom dialogues were thoroughly reviewed. Following this, three semi-structured and one unstructured dialogues were created, focusing on misconceptions in algebra learning. In the semi-structured dialogues, the conversation was initiated by either the teacher or the student, with participants filling in the remaining parts from both perspectives. Each of the three semi-structured dialogues included written statements from the teacher or student at the beginning, middle, and end, with participants required to complete the gaps. The purpose of having participants write student statements was to determine if they provided opportunities for inquiry and explanation as instructors. Another aim was to assess whether the teacher responses following the student statements were consistent with the written inputs, thereby evaluating the instructional explanations. During the two-week imaginary dialogue writing training incorporated in the 'CMT' course, the dialogues were initiated and participants were asked to continue them using their imagination. It was observed that in the less-structured dialogues, participants sometimes deviated from the context. Therefore, additional teacher or student statements were included in the middle and at the end of the dialogues in the data collection tool. Furthermore, student expressions such as "*Why is that, teacher?*" and *"I don't understand*" were embedded to assess how pre-service teachers justified their explanations and adapted their instructional methods when facing comprehension difficulties. The only unstructured dialogue focused on the concepts of variables and unknowns. The goal was to observe how pre-service teachers construct their instructional explanations without any guiding teacher or student statements. An example of a semi-structured imaginary dialogue is provided in Appendix 1.

Feedback from field experts specializing in mathematics education was obtained, resulting in revisions to the dialogues to enhance creativity and engagement. An example of a semi-structured imaginary dialogue was provided.

Faruk: "How does it work? Can we add it or not?"

Teacher: "Okay, Faruk, let me ask you a question. What does x+y+z=x+k+z mean to you?"

Faruk: "It cannot be equal because..."

Selen: "Is k a number, teacher? I'm getting more confused."

Teacher: ...

Burcu: ...

Research procedure

To investigate the instructional explanations of pre-service teachers through imaginary dialogues, a training session was conducted as part of the 'Communication in Mathematics Teaching' course led by one of the researchers. This course is included in the pool of elective courses recommended by the Higher Education Council of Türkiye for mathematics teacher education programs. Faculty members in these programs can offer this course as an elective related to teaching methodology if they choose. Within the scope of the study, the 'CMT' course emphasized classroom interactions and the practice of writing imaginary dialogues to enhance the learning environment. Initially, an introduction to high- and low-quality mathematical dialogues that could occur in the classroom was provided.

In this context, discussions were held with pre-service teachers about how a mathematical dialogue might begin, develop, and conclude. During the CMT, pre-service teachers received an eight-week training focusing on how elements within a dialogue, such as student responses, teacher feedback, and the questions and answers between teacher and student, should be structured. Subsequently, pre-service teachers engaged in activities that involved writing imaginary dialogues across different learning domains. This was aimed at helping them develop an understanding of how to appropriately complete a mathematical dialogue. After becoming familiar with the process of writing imaginary mathematical dialogues, semi-structured imaginary dialogue forms were administered to the participants. To ensure that participants relied on their existing knowledge without prior preparation, no information was provided about the specific learning domain of the imaginary dialogues beforehand. The application period for this task was approximately 60 minutes.

Data analysis

The data collected from the completion of semi-structured and unstructured imaginary dialogues by the pre-service teachers were analyzed using the Maxqda 2022 qualitative analysis software. Two researchers individually examined the four questions from the dialogues and took notes of their opinions. They jointly coded the data, aiming to identify themes and codes that represented the integrity of the data. One researcher continued with the coding process, and the second coder independently re-coded 10 % of the data to establish reliability. The researchers discussed and reached a consensus on matching and non-matching codes, finalizing the codes.

During the data analysis, the dialogues were examined within the context of instructional explanations. Consequently, multiple instances of the same type of instructional explanation were identified within a single imaginary dialogue by a pre-service teacher. As a result, the frequency of some emerging codes exceeded the number of participants. For instance, in T27's dialogue on translating algebraic expressions into verbal expressions, it was observed that the participant incorrectly structured the coefficient and variable in three different instances. Maxqda maps summarizing the instructional explanations were created based on the codes for presenting the findings. To ensure reliability, the coding process involved extensive discussions and consensus among the researchers, which took approximately four months. The validity of the data analysis was established through participant confirmation, also known as member checking. An online meeting was conducted with the pre-service teachers after the data analysis, where they were presented with the maps generated from the analysis. Their opinions were sought regarding the findings of each imaginary dialogue. The majority of the participants stated that the findings accurately reflected their pedagogical explanations related to algebra. For example, participant T8 expressed that the findings aligned with their own dialogues and acknowledged struggling to provide appropriate answers during the dialogues. By incorporating participant confirmation and ensuring intercoder reliability, the research aimed to enhance the credibility of the data analysis. The findings were supported by the pre-service teachers' perspectives, strengthening the validity of the study.

Results

In the scope of the research, four different questions were posed to pre-service teachers to explore critical aspects within the domain of algebra learning. The responses to these questions were analyzed and summarized using MAXQDA. The first question focused on instructional explanations related to addition operations in algebraic expressions and equations, structured around a semi-structured dialogue featuring the expressions and . Given that the instructional explanations differ for these two expressions, the dialogue was analyzed as a whole; however, to enhance clarity, the findings are presented separately in the results section. Initially, the explanations provided by pre-service teachers within the hypothetical dialogues for the outcome of the expression are illustrated in Figure 1.

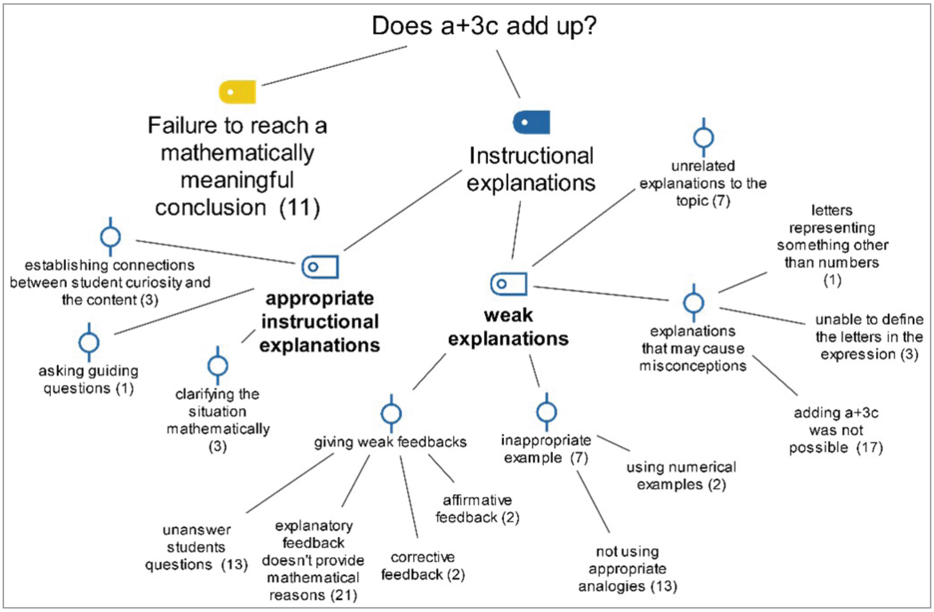

Figure 1. Instructional explanations of pre-service teachers regarding the operation .

The dialogue on performing addition operations in algebraic expressions begins with the teacher asking students what they think about the outcome of the expression . This leads to a series of incorrect student responses, followed by the instructional explanations provided by the pre-service teachers. These responses from the pre-service teachers are evaluated as instructional explanations. As shown in Figure 1, there are relatively few instances of appropriate instructional explanations (f = 7), indicating that most pre-service teachers offered inadequate instructional responses to this algebraic operation. Some pre-service teachers chose to ignore students' questions entirely, leaving them unanswered, while a substantial number (f = 21) provided feedback that, although somewhat explanatory, lacked mathematical justification. Moreover, some teachers provided unrelated explanations (f = 7). For example, one pre-service teacher, T5, responded to a hypothetical student, Gozde, but failed to clarify the mathematical context adequately:

Gozde: "*It doesn't make any sense, teacher. How can we add letters together?*"
T5: "*In mathematics, we sometimes use different symbols to stand for numbers, like x, y, a, b, c.*"

Another pre-service teacher, T7, offered corrective feedback to Gozde's question by saying, "*Your expression is incomplete. How can we add different letters? They represent two different things.*" Some pre-service teachers tried to provide examples to clarify the addition of , often relying on analogies like "*apples and oranges cannot be added*". However, these examples could lead to misconceptions. For example, T1 *("Can we add the letters? Let's say 'a' represents an apple, and 'c' represents an orange. If we say '1 apple + 3 oranges', how many apples do we have in total? Does it make sense to you?")* used the analogy of adding apples and oranges to explain the situation to Gozde. Another pre-service teacher, T26, also tried to explain the situation using a numerical example instead.

The pre-service teachers' weak feedback and examples contained explanations that could lead to conceptual errors. For instance, more than half of the pre-service teachers (f = 17) indicated that the expression could not be evaluated. T17 stated that letters represent something other than numbers, while T11, T14, and T15 could not define the variables in . An example of this can be seen in T14's response: "*Children, the letters here are symbolic. Let's say I have 3 pears and 1 apple. If we call pears 'c' to make it shorter, and call apples 'a,' would it make sense to say 3ac or '3 apple-pears' as Kemal suggested? We would simply add them as 3 pears and 1 apple, right?*" A closer look at Figure 1 reveals that only a small number of pre-service teachers provided appropriate instructional explanations. For instance, T7 addressed the hypothetical student Kemal's error (Kemal suggested that) by using guiding questions to help the student recognize his mistake:

T7: "Kemal, what does the operation there represent?"
Kemal: "It's addition, teacher."
T7: "Then why did you perform multiplication?"

T18 provided an accurate instructional explanation by stating: "*Let me explain this again, everyone. We can only add or subtract when the variables are the same. However, in the case of a + 3c, there are two different variables involved, so we leave it as . That is the result of this operation.*" This clarified the situation mathematically and offering an accurate instructional explanation. Another notable finding is that eleven of the pre-service teachers were unable to reach a mathematical conclusion regarding the outcome of within their dialogues, leaving the expression unresolved. The second mathematical expression analyzed was. A summary of the pre-service teachers' instructional explanations for this equation is presented in Figure 2.

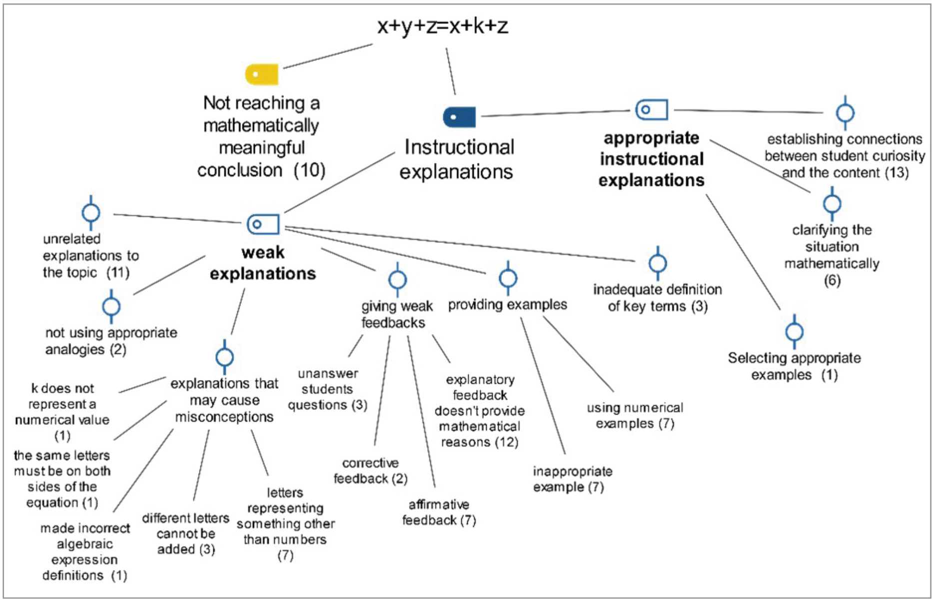

Figure 2. Instructional explanations of pre-service teachers regarding the equation

Although pre-service teachers provided more suitable instructional explanations for the equation compared to the expression , instances of weak instructional explanations were still predominant. Notably, fewer student questions were left unanswered (f = 3) compared to the previous example. However, while some pre-service teachers provided explanatory feedback, it often lacked the necessary mathematical reasoning specific to this equation (f = 12). An illustrative example of this can be seen in the dialogue between T25 and the hypothetical student Selen:

Selen: "Is k a number, teacher? I'm really confused."
T25: "Yes, k is a variable, just like x, y, z."

Some pre-service teachers chose to give short confirmations like "*Yes, that's correct*" instead of more detailed feedback or offered brief corrective feedback without fully addressing the mathematical misconception (e.g., T13 and T7).

In terms of examples, some pre-service teachers attempted to clarify the equation with examples; however, seven of them used numerical examples exclusively, while seven others provided examples that were not suitable for the specific mathematical context. The dialogue typically began with a hypothetical student, Faruk, questioning the validity of the equation . For instance, in T12's dialogue with another hypothetical student, Burcu, the pre-service teacher tried to clarify the concept using a numerical example but incorrectly assumed the equality of the variables y and k from the start.

Moreover, three pre-service teachers could not accurately define the concepts of equality and variable. T19 asked a student to define the concept of a variable but only included "*Burcu defines variable*" in the dialogue instead of an actual definition.

Similarly, T2 attempted to define equality, and T20 tried to explain the variable concept, yet their definitions fell short of accurately representing these terms.

In some cases, pre-service teachers used inappropriate analogies, while others provided explanations unrelated to the mathematical context (f = 11). For instance, T10, in a dialogue with the hypothetical student Selen, responded to the question "*Is k a number, teacher? I'm really confused,*" with a vague analogy: "*Sometimes we use letters instead of numbers to figure out what they represent. Like when you buy something, don't you ask your friend, 'How much do you think this costs*?'"

Some pre-service teachers' explanations for sometimes led to conceptual errors. These misconceptions included viewing letters as representing non-numerical entities (f = 8), implying that different letters cannot be added together (f = 3), incorrectly defining algebraic expressions (f = 1), insisting that both sides of the equation must contain the same variables (f = 1), and assuming that *k* does not represent a numerical value (f = 1). The following exchange between T3 and the hypothetical student Burcu illustrates the pre-service teacher's implication that different letters cannot be added, as well as the idea that *y* and *k* may not be equal:

> *Burcu: "To say that this equation is equal, don't we need to check if y equals k?"*
> *T3:* "*Yes, Burcu is right. Let's recap: we add terms with the same unknown.*"

Additionally, T1's explanation about the variable *k* suggests that the teacher thought *k* did not represent a numerical value. T13's endorsement of a student's misconception that the variables on both sides of the equation must match could lead to broader misunderstandings in a real classroom setting.

These explanations reveal a notable lack of content knowledge among the pre-service teachers, as evidenced in the examples provided by T1, T3, and T13. Similarly, irrelevant explanations by T11, T21, T22, and T27 stemmed from a lack of curriculum knowledge. For example, T21's response to a question about whether *k* is a number - "*Remember, we used symbols like O and * before. Do you recall that?*" - indicates a lack of understanding of the sequence of instructional topics.

Some pre-service teachers were able to establish a connection between student curiosity and content (f = 13), clarify the equation mathematically (f = 6), and provide contextually appropriate examples (f = 1) in their instructional explanations for the equation . For instance, T14 guided the hypothetical student Burcu to simplify the equation and deduce that y = k. Following this, T14 prompted Burcu to consider whether these different letters could represent the same value. The teacher then responded with an explanation that effectively bridged student curiosity and mathematical content: "*Yes, the letters may appear different, but as we discussed, each letter represents a specific value. If we assign y = 1, then k would also need to be 1, which we can understand from this equality*." This approach allowed T14 to constructively connect the student's inquiry with the underlying mathematical concept.

T8 also clarified Selen's question on whether *k* is a number by providing a mathematical explanation: "*Yes, children, here k represents a number. However, we don't know which* *number it represents. Similarly, y stands for a number, and we don't know its actual value either*." The findings for hypothetical dialogues involving translating algebraic expressions and equations into verbal descriptions are presented in Figure 3.

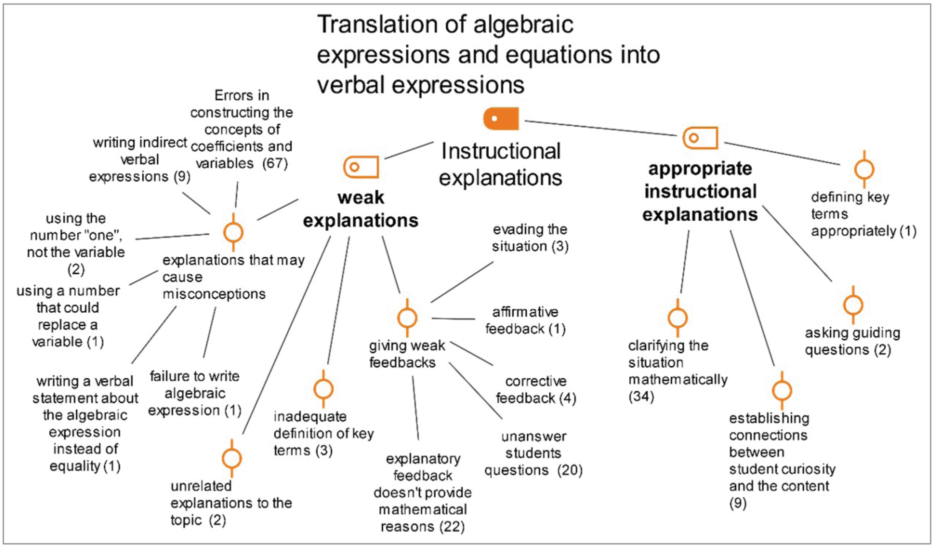

Figure 3. Instructional explanations of pre-service teachers on translating algebraic expressions/equations into verbal statements

In the imaginary dialogues addressing the translation of algebraic expressions into verbal statements, the expressions , and were used. An example of this semi-structured dialogue is presented in Appendix 1. The instructional explanations provided by pre-service teachers regarding the translation of these algebraic expressions and equations into verbal statements often contained elements that could lead to conceptual misunderstandings. Specifically, misinterpretations of coefficient and variable concepts were widespread (f = 67) and appeared in various parts of the dialogues created by nearly all pre-service teachers. A notable gap was the lack of statements guiding students to identify the unknown (f = 43). For example, T9 had the hypothetical student Pınar respond to the equation with the verbal statement: "*If I call my money 'x,' then half of my money plus 6 is 8.*" T9 then approved her response by saying, "*Good*," but failed to prompt Pınar to find the unknown by adding a phrase like "*What is the value of x?" or "How much money do I have?*" Further instructional errors included ignoring that variables should represent equal numbers of items (f = 14), failing to define or misdefining the unknown (f = 8), translating rational coefficients into whole numbers in verbal statements (f = 1), and omitting the multiplicative nature of coefficients (f = 1). These issues suggest a lack of content knowledge. In one dialogue, the teacher asked students to create a verbal statement for the expression , to which the hypothetical student Serap replied, "*I took 5 books out of one of 8 boxes of books*." While the teacher pointed out that Serap had forgotten something, they did not clarify that the variable should represent an equal number of books in each box. Instead, T20 responded with an unrelated explanation, illustrating a lack of understanding that variables often represent identical quantities. Three pre-service teachers failed to define key terms and concepts accurately (e.g., T10). Furthermore, in the semi-structured dialogue, pre-service teachers frequently provided weak feedback to both pre-written and self-composed student responses or left questions unanswered (f = 20). While many attempted to provide explanatory feedback, they often failed to offer mathematical reasoning (f = 24). A few pre-service teachers opted for simple affirmative (f = 4) or corrective (f = 1) feedback. In one scenario, the teacher asked students to create a verbal statement for , leaving space for an incorrect answer by the hypothetical student Aslı. The teacher then prompted Aslı by asking, "*Are you sure, Aslı? Think again*," with the intent of guiding the pre-service teacher to reconsider the verbalization of a rational solution. However, T9 constructed Aslı's response as "*Four less than six times my brother's age is 29*," and later adjusted it to, "*Six times my brother's age plus four is 29*," without addressing the rationality of the result.

Some pre-service teachers, such as T4, T6, and T16, avoided providing mathematical explanations by referencing external sources. For instance, T6 sidestepped Serap's question "*What did I forget, teacher?*" by replying, "*Let's see if you can figure it out in the next problem. I want you to try to solve it yourself*." T4 avoided offering an explanation entirely, stating, "*As written on page 139, there are things to consider when translating algebraic expressions into verbal statements. I've also written these in your notebooks. Does anyone have another idea for this example?*". Despite the prevalence of weak instructional explanations, some pre-service teachers demonstrated appropriate instructional strategies. These included emphasizing that variables represent identical quantities (f = 17), considering the rational nature of results (f = 6), highlighting the importance of identifying the unknown (f = 4), emphasizing the need to consider the value on the right side of the equation (f = 2), and providing accurate guidance on addition/subtraction operations (f = 2). T26's response, "*Would it make sense to have 25/6 beads...?*" illustrates an example of appropriately addressing the rationality of the result.

T5 and T19 employed guiding questions rather than corrective feedback to help their hypothetical students identify and correct their own errors. For instance, T5 asked the hypothetical student Serap, who had incorrectly stated "*I took 5 books out of one of 8 boxes of books*," the guiding question: "*So, how many books are in each box?*" T7 also provided a suitable definition of the unknown, stating that it represents identical items. Finally, the findings regarding pre-service teachers' instructional explanations on solving the equation through the balance will be presented next, summarized in Figure 4.

While examining the general structure of the dialogue on solving equations using a balance model, the exchange begins with a student's desire to solve the question from the book, which asks to represent the equation using a balance. A gap is left for the teacher to guide the solution process. Later, the hypothetical student Ozge asks, "*Why did we solve it this way? Isn't the other way shorter?*" At this point, pre-service teachers are

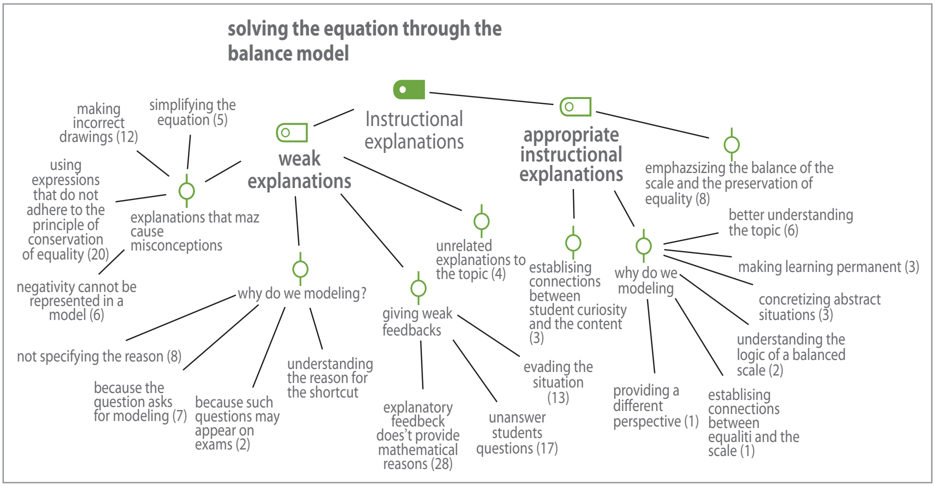

Figure 4. Instructional explanations of pre-service teachers about solving the equation through the balance

expected to provide instructional explanations related to the balance representation. Spaces are also left for additional student questions or comments from Necdet, Class and Munevver, with the dialogue concluding as Munevver gives a correct explanation of the balance process, and the teacher validates her answer.

As seen in Figure 4, similar to other algebraic topics, pre-service teachers' instructional explanations for algebraic expressions/equations using the balance were predominantly weak. Although some pre-service teachers attempted to give explanatory feedback regarding the balance representation, these explanations lacked the connection necessary to bridge the model with mathematical content (f = 28). A significant number of pre-service teachers either made no drawings or balance efforts at all (f = 16) or provided only partial representations (f = 12). Notably, none of the pre-service teachers demonstrated accurate and appropriate use of the balance. For instance, T26 noted in the response "*draws a balance on the board and writes the equation on each side*," yet failed to actually depict or model the solution, solving the equation algebraically instead. Furthermore, nearly half of the pre-service teachers used evasive language instead of engaging in the balance representation (f = 13). Similarly, T28 created a partial drawing but simplified the equation algebraically before attempting any form of balance representation (see Figure 5). This suggests that most pre-service teachers lacked confidence or experience in applying visual or physical representations to solve algebraic equations.

A holistic examination of the hypothetical dialogues created by pre-service teachers regarding the balance representation reveals that in many cases, student questions were left unanswered. Both hypothetical student questions posed by the researchers and questions created by the pre-service teachers themselves for the hypothetical students often remained unaddressed (f = 17). For instance, T9 included a question from the hypothetical student Necdet, asking: "*Why did you subtract 6 from both sides? Can't we directly represent cubes on the balance?*" However, rather than addressing this question, T9 shifted the dialogue to ask a different question to whole class.

Figure 5. Example of T28's representation

In providing instructional explanations about their models, pre-service teachers also made statements that could lead to conceptual misunderstandings (f = 43). As seen in Figure 4, these errors included assertions that negative values cannot be represented in a model (f = 6), simplifying the equation prematurely (f = 5), creating incorrect drawings (f = 12), and using statements that violate the principle of maintaining equality (f = 20). For example, T6 provided instructional explanations that violated the principle of equality by performing operations on only one side of the equation and treating a constant term as positive instead of negative. In another example, T1 made a conceptual error by drawing an insufficient number of objects to represent the model. T1 described a scenario with eight bags of hazelnuts, where six were removed from one bag, finding it equal to 26, thus creating a verbal model related to real life. However, this was not accurately depicted in the drawing. T1 used numbers on one side of the balance and models on the other, failing to include a sufficient number of objects to represent the model, which created an imbalanced and inaccurate visual representation.

Among the pre-service teachers who explained that "*negativity cannot be represented in a model,*" T13 provided the following instructional explanation: "*What are we doing with the two sides of the balance? Look, we need to isolate 8m, but I can't show subtraction on the balance. So, I'm adding 6 kg to both sides.*"

Pre-service teachers' responses to students who were curious about the purpose of using a balance were often superficial and lacked conceptual depth. Their answers ranged from failing to provide any reasoning (f = 8) to stating that using the balance was simply required by the problem (f = 7), that such questions might appear on exams (f = 2), or that using the balance helps in understanding shortcuts for isolating terms (f = 1). For example, T3 responded to the hypothetical student Ozge's question about why a balance is used by saying: "*Ozge, we could solve it differently, but the question requires that we use the balance.*"

Despite these shortcomings, some pre-service teachers provided explanations about the purpose of using a balance, though not explicitly mathematical. These explanations implicitly supported the rationale for using a balance representation, citing reasons such as enhancing understanding (f = 6), making learning permanent (f = 3), making abstract concepts concrete (f = 3), understanding the logic behind the balance (f = 2), offering a different perspective (f = 1), and linking equality with the balance (f = 1). For instance, T20 explained that using the balance reinforces learning but also noted its time-consuming nature: "*Yes, it would be easier to solve this directly as an equation, but we're using the balance to help visualize the concept. We don't solve every problem this way, as it's effective but quite time-consuming*."

T14 also emphasized the value of the balance for better understanding by stating, "*The balance helps you understand the concept of equality better*." Interestingly, none of the pre-service teachers who partially articulated the purpose of using the balance managed to perform a correct representation.

The most effective instructional explanations on representing equations with a balance focused on the principles of balance and maintaining equality (f = 8). Although T14 did not demonstrate the use of balance, they made a verbal connection between balance and equality, explaining: "*The right side of the equation should equal the left side, just like a balance. If we add or subtract something from the right, we need to do the same to the left; otherwise, the equality is disrupted*." Furthermore, T2, T3, and T26 included dialogue segments that established a connection between student curiosity and the content. For example, T3 had the hypothetical student Necdet ask what using the balance is, later allowing Munevver to provide an explanation.

Finally, the pre-service teachers were tasked with creating an unstructured dialogue to explain the relationship between the concepts of variable and unknown. The dialogue began with a hypothetical student, Mehmet, asking: "*Teacher, are unknowns and variables the same thing*?" The remainder of the dialogue was structured entirely by the pre-service teachers. The findings from these dialogues are summarized in Figure 6.

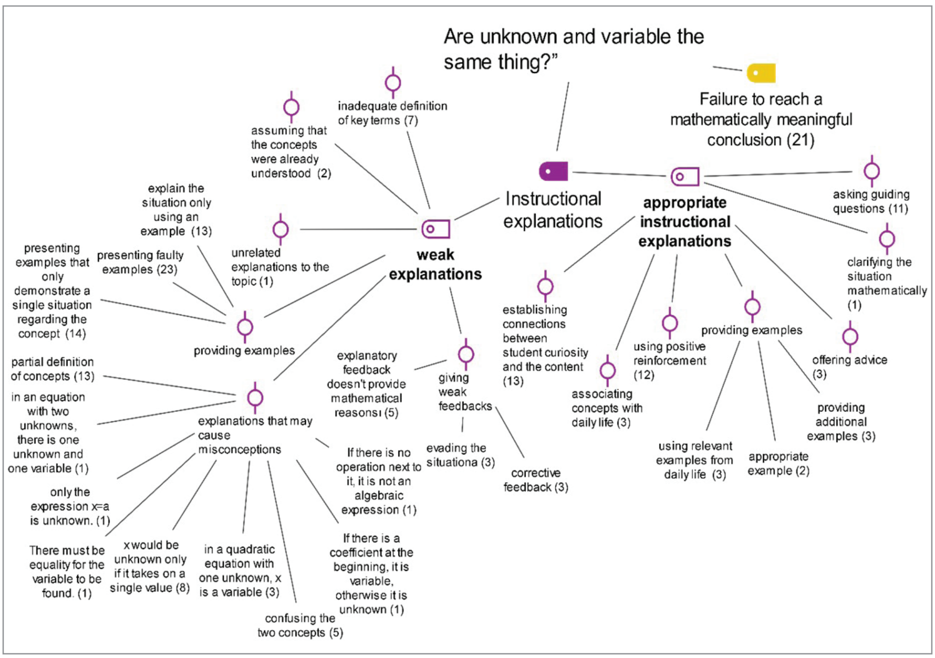

Figure 6. Instructional explanations of pre-service teachers regarding the relationship between the concepts of unknown and variable.

The instructional explanations provided by pre-service teachers regarding the relationship between the concepts of "unknown" and "variable" were characterized by a notable variety and abundance of statements that could lead to conceptual misunderstandings. Among those who attempted to define these concepts, only partial definitions were achieved (f = 13). For instance, T20 defined the unknown as follows: "*Mehmet, in an equation, the things we don't know and are asked to find in the algebraic expressions are what we call the unknowns*."

While this explanation partially conveys the meaning, it does not fully capture the mathematical definition of an unknown. Similarly, T23 suggested that *x* qualifies as an unknown only if it has a single value, which could mislead students since an unknown can represent multiple values. T2, attempting to explain "variable," stated that *6x* does not qualify as an algebraic expression because it lacks an operation, which is incorrect and could confuse students.

Struggling to articulate mathematical definitions, pre-service teachers resorted to explaining unknowns and variables through examples. However, many presented flawed examples (f = 23), while others provided examples illustrating only one aspect of the concept (f = 14). T8, like T23, offered an example suggesting that an unknown has only one possible solution.

T11 provided an example, stating, "*In , x is the unknown, whereas in , x is the variable*". This explanation suggests that *x* in a quadratic equation is a variable, which could introduce misconceptions.

Only a few pre-service teachers attempted to define the concepts rather than using examples, but these definitions were largely inaccurate (f = 7). T25's explanation reflects a partial understanding: "*There's a subtle difference, children. We use letters to stand for numbers we don't know, and these letters are called variables. If you see the letter x by itself, it could represent any number, which is why we call it a variable*."

In some cases, pre-service teachers, like T17 and T18, avoided providing instructional explanations altogether, assuming students already understood the concepts. T18 continued the dialogue with the following statement, "*The students and class have expressed correct reasoning, so the teacher understands that they have learned these two concepts*."

Because the dialogue on the relationship between the unknowns and variables was entirely structured by pre-service teachers and involved limited student interactions, hypothetical teachers provided minimal feedback. T10 bypassed the question from the hypothetical student Mehmet, suggesting only that they could find the difference in their notebooks. Similarly, T12 suggested students think about it until the next lesson, while T22 assigned it as homework. A segment of T22's dialogue illustrates this approach: "*Make a note of this in your notebooks, and we'll go over it tomorrow*."

Some pre-service teachers did incorporate corrective feedback in their dialogues. For instance, the exchange between T15 and their hypothetical students went as follows:

Teacher: "No, Mehmet, they are not the same."
Class: "But teacher, we call both the unknown and the variable x."
Teacher: "Yes, that's correct."
Ela: "So, does that mean they're the same?"
Teacher: "No, of course not."

In a few instances, pre-service teachers successfully incorporated student curiosity into the instructional content (f = 13). For example, T3 praised Mehmet for asking his question and encouraged the class to think collectively, promising a treat for correct answers. After listening to several students, T3 concluded the lesson by asking them to note examples of unknowns and variables in their notebooks, though no conclusive definition was reached.

Some pre-service teachers preferred to guide their students toward understanding the concepts with prompting questions, which, while not always reaching a conclusive answer, were valuable in instructional context. T9, for instance, responded to Mehmet's question about the difference between the concepts with the prompt: "*Mehmet, let me ask you something.*" Then the teacher wrote and on the board, asking: "*What do you see here, Mehmet?*"

Though some pre-service teachers struggled to provide appropriate examples, a few offered the correct ones. T21's dialogue, for example, demonstrated a clear understanding of the mathematical relationship between unknowns and variables:

Teacher: "If I can find a specific value for x, then it's an unknown. For instance, in , I found the value of x, so x here is the unknown, right?"
Class: "Yes."
Teacher: "Now let's look at . We haven't found any specific value for x, so it can change. Therefore, we call x here a variable. Understood?"

In addition, some pre-service teachers offered advice to help students understand these concepts. For instance, T4 emphasized understanding over memorization, while T20, within a dialogue that did not fully distinguish the concepts, suggested that reflection and practice are crucial for grasping the difference between unknowns and variables.

Discussion

The study discussed the instructional explanations provided by prospective teachers in the field of algebra using semi-structured imaginary dialogues. The primary aim was to assess the quality of these explanations and to identify any weak pedagogical approaches or misconceptions in algebra. The findings highlighted a significant prevalence of weak explanations compared to appropriate ones, particularly in areas such as operations involving addition in algebraic expressions, verbal translation of equations and algebraic expressions, representation practices, and understanding the concepts of variables and unknowns. Considering that pre-service teachers' algebraic thinking skills are a critical factor guiding the teaching-learning process (Magiera et al., 2013), this study suggests that the weak instructional explanations provided by pre-service teachers may be linked to their insufficient algebraic thinking skills. Consequently, misconceptions, inadequate feedback, errors in providing examples, improper definitions of key ideas and concepts, and explanations unrelated to the topic were commonly observed. These findings underscore the existence of conceptual errors and knowledge gaps among pre-service teachers in the field of algebra.

Conceptual errors regarding coefficients, variables, and verbal expressions for algebraic expressions/equations were found to be common in the instructional explanations provided by pre-service teachers in the study, which aligns with previous research (Adıguzel et al., 2018; Bucher, 2009; Ryan & Williams, 2007). These errors stemmed from misconceptions about performing addition in algebraic expressions and the misinterpretation of letters representing something other than numbers. Similar findings were reported by Wille (2017) in her study on pre-service teachers' imaginary dialogues in algebra, suggesting that conceptual errors in algebra are prevalent among pre-service teachers.

As is well known, algebra is a language (Usiskin, 1997), and the concepts of algebraic expression, equation, variable, and unknown are fundamental components of algebra. An algebraic expression is defined as a mathematical statement that includes both numbers and letters, while an equation is defined as the equality of two algebraic expressions (Baykul, 2009). Furthermore, a variable refers to something that can change or have multiple values, whereas an unknown can be described as something with a fixed value that has not yet been determined (Schoenfeld & Arcavi, 1988). Considering these definitions, incomplete definitions and confusion between the concepts of variables and unknowns were also prominent issues observed in the dialogues. Prospective teachers often failed to provide comprehensive definitions and frequently confused these two concepts. This lack of understanding regarding variables and unknowns has been highlighted in previous studies on the algebraic knowledge of prospective teachers (Boz, 2004; Stephens, 2008; Tanisli & Kose, 2013). Furthermore, the study revealed that instructional explanations by pre-service teachers could potentially lead to misconceptions among students (Jones & Pratt, 2006; Vermeulen & Meyer, 2017). For instance, conceptual errors made by prospective teachers in representing algebraic situations that do not adhere to the principle of equivalence could result in student misunderstandings of the concept or sign of equals.

The study also raised concerns about the inadequate definition of key terms, such as equations, algebraic expressions, and variables, as well as the inclusion of unrelated situations without considering student explanations in the imaginary dialogues written by the participants. These issues point to a lack of subject matter knowledge among the participants, particularly in the field of algebra. Furthermore, when pre-service teachers lacked knowledge in a specific topic, they often resorted to using evasive statements instead of providing clear explanations. Instead of addressing the mathematical situation, they would say things like "*we will cover this later*" or "*the definition is already in the* *textbook*". Similarly, when fictional students in the data collection instrument expected justifications for algebraic situations or concepts, some participants used statements like "*questions like these may appear in exams*" or *"it is done this way because the question requires balance*". These statements reflect the extent to which the lack of content and pedagogical content knowledge is evident in instructional explanations. These findings are consistent with previous studies examining the content and pedagogical knowledge of pre-service teachers in the field of algebra (Asquith et al., 2007; Black, 2007; İdil & Narlı, 2021; Tanisli & Kose, 2013). Moreover, they emphasize the importance of teacher's content and pedagogical knowledge in influencing student achievement, as supported by research (Carpenter et al., 1988; Hill et al., 2005). Therefore, providing significant support to pre-service teachers in developing their content knowledge in algebra is crucial.

Another significant aspect of instructional explanations highlighted in the study is feedback. Pre-service teachers often provided non-explanatory corrective or affirmative feedback such as "*No, that's not right*" or "Y*es, that's correct*", lacking mathematical reasoning and meaningful explanations. However, high-quality interactions between teachers and students necessitate rich and meaningful explanations, as well as interactive and continuous dialogues (Piccolo et al., 2008). At this stage, the way teachers ask questions becomes important. It is known that the way teachers ask questions, such as what, why, and how, plays an important role in student interaction (Li & Huang, 2013) and serves as an important tool to better determine the depth of students' ideas in mathematics instruction (Moyer & Milewicz, 2002). Many studies in the literature on questioning strategies used by teachers have concluded that students' responses can vary depending on the question asked, thus leading to different dialogues in the classroom (Klinzing et al., 1985; Brock, 1986; Singto, 1995; Boaler & Brodie, 2004; Furtak et al., 2018; Paoletti et al., 2018). The study revealed that some participants exhibited the skill to ask probing questions, which stimulated students' mathematical thinking and encouraged them to engage in mathematical questioning. This finding highlights the promising outcome of pre-service teachers' ability to ask probing questions in their imaginary dialogues. Probing questions foster students' mathematical thinking and promote their questioning of algebraic concepts and situations. Previous research supports the notion that such questions stimulate students' mathematical thinking and contribute positively to classroom dialogues (Mason, 2000).

During instructional explanations on algebraic situations, pre-service teachers often relied on the use of examples. Notably, many pre-service teachers used inappropriate analogies, such as "*adding apples and pears cannot be done*," to illustrate the expression . While intended to concretize the concept, this approach, as Booth (1988) noted, not only fosters misconceptions about the meaning of variables but can also be used by students to justify incorrect simplifications. Tirosh et al. (1998) argued that resorting to tangible objects in unclear situations can be more harmful than beneficial, highlighting the need for greater care in the selection of examples. Since examples are essential tools for communication and mathematical discourse in teaching (Leinhardt, 2001), they can be employed to develop concepts, illustrate methods, establish relationships, or demonstrate proofs (Bills et al., 2006). However, when it comes to utilizing examples in instructional explanations, pre-service teachers frequently struggled to provide clear and relevant mathematical examples pertaining to the algebraic situation. Selecting appropriate examples is a complex task influenced by various factors, including instructional objectives and teachers' understanding of students' biases and tendencies (Bills et al., 2006), and it can pose a challenge for educators. Previous studies have also shown that both pre-service teachers and teachers tend to choose weak and incorrect examples (Avcu, 2014; Duran & Kaplan, 2016; Gokbulut & Ubuz, 2013; Rowland et al., 2003). Moreover, Saglam-Kaya (2017) found that pre-service mathematics teachers' perceptions of examples mostly consist of questions or exercises based on procedures, with limited use of non-examples, counter-examples, and examples that require complex cognitive processes. Unfortunately, teachers' limited knowledge of effective example presentation hinders the comprehension of concepts at a more advanced level (Zaslavsky & Peled, 1996).

One notable finding of the study was the inability of many pre-service teachers to arrive at a mathematically meaningful conclusion in their imaginary dialogues. For instance, when discussing expressions such as and whether they can be simplified, many dialogues failed to address this question altogether. This limitation is concerning for classroom dialogues, as the dialogue should progress towards a coherent or alternative outcome (Wells & Arauz, 2006). The inability of pre-service teachers to reach a mathematically meaningful conclusion may stem from their deficiencies in content knowledge and algebraic teaching. Research suggests a strong correlation between prospective teachers' instructional explanations and their domain-specific knowledge (Charalambous et al., 2011; Leinhardt, 2001). Additionally, effective reasoning relies on solid knowledge (Resnick et al., 2018), and a significant number of participants in this study demonstrated inadequate proficiency in providing explanations supported by reasoning when addressing students' questions, responses, and different solution methods. Similar findings were also evident in Hill's (2007) research, which explored the content knowledge of middle school mathematics teachers.

While the study revealed weak instructional explanations provided by pre-service teachers, there were also instances where they were able to provide appropriate explanations. The imaginary mathematical dialogues in the study aimed to explore the ability to use relevant examples from daily life as instructional explanations, considering algebraic thinking as a problem-solving activity in everyday contexts (Kaya & Keşan, 2014). Some pre-service teachers successfully provided appropriate instructional explanations by presenting relevant examples related to the algebraic situation and sometimes incorporating additional real-life examples to enhance students' understanding. However, it is concerning that only a few teachers were able to link algebraic concepts to everyday life, despite the importance of such connections in algebra instruction (Witzel et al., 2001).

When students struggle to grasp a concept, they often seek elaboration, examples, repetition, or demonstrations from their teachers (Darling, 1989). Establishing a connection between students' curiosity and the content is crucial in such cases. Moreover, relating the lesson content to students' familiar experiences and ideas enhances their engagement with the material (Corso et al., 2013). In the study, some pre-service teachers successfully made this connection between student curiosity and the content for all four algebraic situations examined. They were able to provide consistent instructional explanations directly related to the algebraic topic, avoiding irrelevant or incorrect explanations. Furthermore, some pre-service teachers used guiding questions to prompt students' thinking. Questions play a vital role in the teaching-learning process (Pearson & West, 1991), as effective questions encourage students to think and develop their reasoning skills (Shahrill & Clarke, 2014). Research suggests that these types of questions create learning and thinking opportunities (Bozkurt & Polat, 2018) and lead to more accurate student explanations (Franke et al., 2009). The ability to answer students' "why?" questions and ask productive guiding questions reflects specialized content knowledge (Ball et al., 2008), and it is encouraging to see that some pre-service teachers in this study employed guiding questions effectively.

Implications

This study contributes to the field by specifically examining pre-service teachers' pedagogical approaches in algebra through semi-structured imaginary dialogues, a method that offers fresh insights. While prior research has identified general knowledge gaps, our study uniquely explores pre-service teachers' explanations across four distinct algebraic concepts: addition in expressions, verbalization of algebraic forms, solving the equation through the balance, and distinguishing variables from unknowns. Rather than simply confirming existing findings, our results reveal the depth and persistence of specific misconceptions, such as those related to coefficients and variables, providing focused insights for addressing these in teacher education.

Our use of imaginary dialogues offers a replicable way to assess instructional readiness in a controlled environment, suggesting that this approach could benefit teacher education programs by prompting reflection on student misconceptions and difficulties before entering the classroom. Our methodology's contribution lies in using scenario-based tasks to illuminate gaps in instructional explanations, supporting the development of micro-teaching exercises that build pre-service teachers' content knowledge and pedagogical skills in algebra.

Limitations

This study has several limitations that may impact the scope and generalizability of its findings. Firstly, the use of semi-structured imaginary dialogues provides a controlled perspective on pre-service teachers' pedagogical reasoning but does not capture the interactive dynamics of real classroom environments. This limitation may reduce the ecological validity, as authentic classroom interactions could reveal additional instructional challenges and approaches. Additionally, the focus on algebra-specific instructional explanations may limit the applicability of findings to other mathematical domains. Lastly, as the analysis is qualitative, it is based on interpretive insights that could benefit from triangulation with additional data, such as classroom observations or participant interviews. Future research could address these limitations by incorporating micro-teaching sessions or observational studies to examine how pre-service teachers adapt and refine their instructional strategies in real educational contexts.

Conclusion

In conclusion, the study highlighted an alarming state of the instructional explanations provided by prospective teachers in the field of algebra. The prevalence of weak explanations that may lead to conceptual errors and the lack of clarity in examples raised significant concerns about pre-service teachers' subject knowledge, particularly in the context of algebra teaching. To address these issues, teacher education programs should incorporate micro-teaching practices focused on algebra instruction. Such practices would enable pre-service teachers to implement their dialogues in real classroom settings, gaining practical insights and refining their instructional explanations.

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Appendix

Semi-Structured Imaginary Dialogue on Writing Verbal Expressions for Algebraic Expressions/Equations

Teacher: Well done! You've now learned to write algebraic expressions for verbal descriptions. Let's take it a step further and reverse the process. This time let's write verbal descriptions for given algebraic expressions.

The teacher writes "8x-5" on the board, and a murmur arises from the students.

Teacher: But let's set some ground rules here. No more simple phrases like "twice a number" or "half of my apples". I want you to get creative and provide examples from everyday life.

Serap: I took 5 books out of 8 boxes, each filled with books.

Teacher: Great example, Serap! But you're missing one crucial detail.

Serap: ....
....
....

Teacher: ....
....
....

The teacher then writes 12x+6=8 on the board.

Now, let's see who can come up with a description for this one.

Pınar: ....
....
....

Ömer: ....
....
....

Teacher: Good effort. Now, let's try this one: "$6x$+4=29."

Aslı: ....
....
....

Teacher: Are you sure, Aslı? Think it over. Yiğit, you've been quiet today—why don't you give it a try?

Yiğit: ....
....
....

Teacher: You're all missing one important point here ....
....
....
....

Unaprjeđenje nastave algebre pomoću imaginarnih dijaloga: analiza načina na koje budući nastavnici objašnjavaju gradivo učenicima

Sažetak

Učinkovita objašnjenja koja učenicima daju nastavnici od ključne su važnosti za proces učenja, a pedagoško znanje nastavnika važan je preduvjet za davanje kvalitetnih objašnjenja. Ovo istraživanje imalo je za cilj ispitati kako budući nastavnici objašnjavaju učenicima *gradivo algebre kako bi im pojasnili pogrešne predodžbe. U istraživanju je kao metodološki alat korišten imaginarni dijalog, a sudjelovalo je 28 budućih nastavnika čija su objašnjenja analizirana pomoću polustrukturiranih imaginarnih matematičkih dijaloga. Rezultati kvalitativne analize pokazali su da su neki budući nastavnici uspješno povezali radoznalost učenika s nastavnim sadržajem, dok ih je mnogo imalo poteškoće pri davanju razumljivih objašnjenja učenicima te nisu uspjeli na adekvatan način razjasniti pogrešne predodžbe učenika i otkloniti poteškoće u učenju. Ovaj rad ističe važnost usklađenosti objašnjenja koja nastavnici daju učenicima sa svakodnevnim iskustvima učenika te naglašava potrebu da budući nastavnici osnaže svoje specifično sadržajno znanje. U radu se također daju preporuke za primjenu aktivnosti mikropoučavanja, poput osmišljavanja i provedbe imaginarnih dijaloga, u sklopu kolegija o nastavi algebre kako bi se unaprijedile nastavničke vještine i poticala samorefleksija.*

Ključne riječi: *nastava algebre; budući nastavnici; objašnjenja nastavnika; imaginarni dijalozi*

Uvod

Veći broj istraživanja o nastavi i izobrazbi nastavnika fokusiran je na profesionalno znanje nastavnika i naglašava ključnu ulogu sadržajnoga znanja i pedagoškoga znanja (Campbell i sur., 2014; Copur-Gencturk i Li, 2023; Depaepe i sur., 2013; Ponte i Chapman, 2016; Şahin i sur., 2016). Sadržajno znanje implicira temeljito razumijevanje sadržaja samoga predmeta koje nastavnici trebaju prenijeti učenicima, a ono je temelj uspješnoga poučavanja i razvoja pedagoškoga znanja nastavnika. Pojam pedagoškoga znanja prvi je spomenuo Shulman (1986), a ono obuhvaća integraciju znanja o samom nastavnom predmetu i općih pedagoških principa. Ovakav jedinstveni spoj jako je važan kako bi se stručno sadržajno znanje nastavnika moglo transformirati u oblike koji su istovremeno i dostupni i poticajni za učenike te im tako omogućavaju temeljito učenje. Shulman (1987) je opisao pedagoško znanje kao „poseban spoj sadržaja i pedagogije koji je jedinstven za nastavnike i predstavlja poseban oblik profesionalnoga razumijevanja" (str. 8). Okvir pedagoškoga znanja obuhvaća tri glavne komponente: znanje o onome što učenici razumiju, znanje o nastavnim strategijama te poznavanje kurikula. Uspješno poučavanje nije samo puko ovladavanje sadržajem, ono zahtijeva sposobnost da se sadržaj učini dostupnim učenicima kroz raznovrsne pedagoške pristupe, uključujući analogije, primjere, objašnjenja, simulacije te obrazovne materijale (Ball i sur., 2008; Blömeke i Delaney, 2012; Mohr-Schroeder i sur., 2017). Ovaj je aspekt iznimno važan u razrednom okružju, pri čemu nastavnici trebaju imati posebne vještine kako bi raspravljali o idejama, odredili u kojoj mjeri učenici razumiju gradivo, jasno objasnili pojmove te stvorili okružje u kojemu će se proces učenja nesmetano odvijati (Ball i Forzani, 2009). Nastavnička su objašnjenja komponenta nastavnih strategija u sklopu pedagoškoga znanja, a jako su važna jer moraju biti usklađena s kognitivnom razinom učenika kako bi im se omogućilo razumijevanje nastavnih sadržaja (Ma, 2020). Unatoč važnosti pedagoškoga znanja, istraživanja pokazuju da tijekom nastavnoga procesa budući nastavnici imaju poteškoće pri davanju jasnih objašnjenja učenicima pomoću kojih bi trebali razjasniti određene matematičke pojmove. Te poteškoće upućuju na potrebu ciljanoga razvoja studijskih programa za izobrazbu nastavnika (Karakus, 2018; Thanheiser, 2009; Toluk-Ucar, 2009).

Algebra, kao neizostavni dio školskoga matematičkog kurikula, naglašava važnost upotrebe matematičkih simbola i njihove primjene. Ona omogućava učenicima stjecanje vještina za upotrebu varijabli i algebarskih izraza kod prikazivanja i analize matematičkih odnosa. Ovakvo temeljno znanje omogućava učenicima rješavanje praktičnih problema kao što je izračun kamata, određivanje najbolje vrijednosti proizvoda koje kupuju, analiza obrazaca rasta te predviđanje na temelju dostupnih podataka. Analiza obrazaca razvija apstraktno mišljenje i algebarske vještine razmišljanja, što je od presudne važnosti za razumijevanje jednadžbi i funkcija u svakodnevnom životu (NCTM, 2000). Unatoč važnosti ovih pojmova, učenici se često muče s njima te nailaze na izazove pri razlikovanju varijabli i konstanti, tumačenju algebarskih izraza te razumijevanju pojma jednakosti. Ti izazovi mogu dovesti do stvaranja pogrešnih predodžbi, a mogu biti i rezultat kognitivnih sposobnosti i nastavnih pristupa (Stewart, 2013). Nastavnici moraju razumjeti procese razmišljanja učenika, razinu njihovoga razumijevanja algebarskih pojmova te česte pogrešne predodžbe kako bi mogli uspješno objasniti gradivo i ukloniti te poteškoće (Guler i Celik, 2021). Nastavnička objašnjenja mogu poslužiti kao mostovi između postojećega znanja učenika i novih algebarskih pojmova, što omogućava strukturirani pristup uklanjanju pogrešnih predodžbi (Charalambous i sur., 2011). Jasna objašnjenja algebarskih pojmova pomažu razjasniti proces razmišljanja, povezati apstraktne pojmove s konkretnim kontekstom te reducirati česte izazove u prepoznavanju i primjeni algebarskih pravila, pojednostavljivanju izraza i rješavanju jednadžbi (Leinhardt, 2001, 2010). Ona često obuhvaćaju vizualne prikaze, primjere koje pokazuju njihovu uporabu u kontekstu te praktičnu demonstraciju kako bi se povećala razina razumijevanja kod učenika (Lachner i sur., 2019).

Sadržajna znanja budućih nastavnika imaju znatan utjecaj na kvalitetu njihovih nastavničkih objašnjenja (Charalambous i sur., 2011; Leinhardt, 2001). Jasnoća i logičnost tih objašnjenja reflektiraju dubinu konceptualnoga znanja nastavnika i njihovu sposobnost da apstraktne sadržaje učine razumljivima učenicima (Leinhardt, 2010). Unatoč temeljnoj ulozi algebre i nastavničkih objašnjenja, istraživanja provedena na uzorku budućih nastavnika o njihovom pedagoškom znanju još su uvijek malobrojna. Zato je potrebno procijeniti kako budući nastavnici konstruiraju svoja objašnjenja kada razjašnjavaju učestale pogrešne predodžbe učenika o algebarskim pojmovima te kakve poteškoće imaju unutar okvira pedagoškoga znanja jer te strategije direktno utječu na kvalitetu nastave matematike te u konačnici i na ostvarenost ishoda učenja. Ovo istraživanje ispituje kako budući nastavnici matematike u višim razredima osnovne škole konstruiraju objašnjenja kada objašnjavaju česte pogrešne predodžbe o algebarskim pojmovima koristeći imaginarne dijaloge kao alat za osmišljavanje objašnjenja unutar okvira pedagoškoga znanja.

Teorijski okvir

Pedagoško znanje

Shulman (1986) razlikuje tri bitne vrste znanja potrebne za uspješno izvođenje nastavnoga procesa: sadržajno znanje, pedagoško znanje i znanje o kurikulu. Sadržajno znanje odnosi se na dubinsko razumijevanje sadržaja, uključujući i sve njegove temeljne principe, strukture te sposobnost prepoznavanja pojmova i činjenica u različitim kontekstima. Sadržajno je znanje od presudne važnosti jer se pomoću njega uspostavlja jaki disciplinarni temelj, što nastavnicima daje dubinu znanja potrebnoga da bi na adekvatan način odgovorili na pitanja učenika i ispravili pogreške u njihovom znanju. Iako se ne odnosi izravno na nastavne metode, sadržajno znanje tvori osnovu na kojoj se grade pedagoške strategije. Pedagoško znanje, kako ga definira Shulman (1986), nadilazi okvire sadržajnoga znanja tako što ga integrira s pedagoškim vještinama posebno osmišljenima za uspješno poučavanje određenoga nastavnog predmeta. Za razliku od općega sadržajnog znanja, pedagoško znanje pomaže nastavnicima da preoblikuju složene matematičke ideje u ostvariva iskustva učenja tako što svoja objašnjenja prilagođavaju kognitivnoj razini učenika i njihovom postojećem znanju (Ball, 1990; Hill i sur., 2008). Pedagoško znanje uključuje tumačenje sadržaja nastavnoga predmeta i odabir odgovarajućih nastavnih pristupa koji potiču aktivnost učenika i razumijevanje sadržaja (Baki, 2020; Grossman, 1990). Pedagoško znanje čine sastavnice kao što su znanje o raznim vrstama prikaza, analogije, primjeri i pojašnjenja, kao i osviještenost nastavnika o tome da učenici često imaju pogrešne predodžbe i da nastavnici moraju razviti strategije kako bi ih razjasnili

(Ball, 1990; Shulman, 1986). Ovdje se naglasak stavlja na razumijevanje načina na koji učenici razmišljaju, uključujući i njihove pogreške i pogrešne predodžbe, što utječe na razvoj učinkovitih nastavnih praksi. I za buduće nastavnike i za one koji već rade u nastavi ključna dimenzija pedagoškoga znanja jest sposobnost davanja jasnih objašnjenja (Karakuş, 2018). Ona pokazuju sposobnost nastavnika da kompleksne matematičke pojmove pretvore u razumljive oblike, da razjasne pogrešne predodžbe koje učenici imaju, odaberu odgovarajuće primjere i prikaze te postupno izgrađuju matematičko razumijevanje. Istraživanja su pokazala da nastavnici s većim pedagoškim znanjem učenicima daju kvalitetnija objašnjenja (Charalambous i sur., 2011; Ma, 2020). Ta objašnjenja služe kao most koji spaja matematičko znanje nastavnika i učinkovite nastavne prakse, što ih čini ključnim pokazateljem kvalitete poučavanja i ishoda učenja koje učenici ostvaruju. Svaka matematička tema (npr. algebra) zahtijeva jedinstven skup pedagoških znanja. Za uspješno poučavanje neophodno je da nastavnici razumiju prirodu predmeta koji poučavaju, česte pogrešne predodžbe učenika te poteškoće povezane s tom temom.

Algebra se često smatra važnom temom u školskoj matematici jer uvodi učenike u pojmove kojima je neophodno ovladati kako bi mogli pratiti matematiku na višoj razini. Među tim pojmovima nalaze se znak jednakosti, varijable, zapisi brojeva i jednadžbe (Usiskin, 1997; Witzel i sur., 2003). Ti glavni elementi osnova su raznih područja u matematici te podržavaju razvoj algebarskoga načina razmišljanja i vještina rješavanja problema (Matthews i Fuchs, 2018). Međutim, mnogi učenici susreću se s izazovima pri razumijevanju strukture algebarskoga sustava i njezinoj primjeni u kontekstu rješavanja problema (Kieran, 1992). Na primjer, česte poteškoće u učenju uključuju tumačenje znaka jednakosti kao relacijskoga simbola, a ne samo kao operatora te točno prebacivanje verbalnih uputa u algebarske izraze. Uvođenje algebre u ranoj dobi predstavlja dodatne izazove, kao što su razvoj algebarskoga načina razmišljanja u toj dobi i razvoj potrebnih vještina i strategija kod nastavnika kako bi mogli uspješno unaprijediti proces učenja učenika. Taj prijelaz s aritmetičkoga na algebarsko razmišljanje – koji se uglavnom može uočiti u višim razredima osnovne škole – varira ovisno o obrazovnom sustavu i strukturi kurikula (Asquith i sur., 2007). Unatoč stalnim naporima, učenici i dalje smatraju da je algebra zahtjevna (Butuner i Guler, 2017), a tome često doprinosi i činjenica da budući nastavnici nemaju adekvatno algebarsko znanje (Guler i Celik, 2021; Tanisli i Kose, 2013). Glavni problemi u poučavanju algebre mogu se svrstati u poteškoće sa specifičnim sadržajem, kognitivne prepreke, nastavni pristup te izazove pri prijelazu s aritmetike na algebru (Wang, 2015). Rješavanje tih problema zahtijeva dobro planiranje nastavnih strategija i poticajno okružje za učenje. Učinkovitost nastavničkih objašnjenja ključan je čimbenik kada se učenicima želi pomoći prevladati prepreke. Međutim, postoji mali broj istraživanja o kvaliteti nastavničkih objašnjenja u nastavi algebre (Guler i Celik, 2016).

Nastavnička objašnjenja u nastavi matematike

Nastavnička objašnjenja u nastavi matematike predstavljaju poseban pedagoški pristup koji karakteriziraju smišljene komunikacijske strategije usmjerene na to da se učenicima omogući lakše razumijevanje i usvajanje matematičkih pojmova, postupaka i razmišljanja (Leinhardt, 2010). Ta objašnjenja tvore jedinstvenu nastavnu metodu koja obuhvaća sustavnu prezentaciju i razradu matematičkih ideja, drugačiju od ostalih metoda. Učinkovitost nastavničkih objašnjenja određuju mnogobrojni međusobno povezani čimbenici: konceptualna jasnoća i dubina, matematička ispravnost i preciznost, prepoznavanje postojećega znanja i pogrešnih predodžbi učenika, upotreba odgovarajućih prikaza i primjera, integracija matematičkih veza i odnosa, kvaliteta diskursa između nastavnika i učenika te usklađenost s ishodima učenja (Stein i Kucan, 2010). Takva objašnjenja, koja se pojavljuju tijekom izravne nastave ili interaktivnih rasprava, promiču aktivno učenje i potiču radoznalost učenika (Inoue, 2009). Kada su usklađena s ciljevima kurikula, odgovarajuća objašnjenja mogu značajno unaprijediti matematičko razumijevanje učenika, kao i njihove metakognitivne sposobnosti (Leinhardt i Schwarz, 1997). Istraživanja pokazuju da se učinkovita i neučinkovita objašnjenja uglavnom razlikuju u pristupu matematičkom razumijevanju. Učinkovita objašnjenja temelje se na postojećem znanju koje su učenici stekli i objašnjavaju učestalih pogrešnih predodžbi. Ona uspostavljaju eksplicitne matematičke veze, koriste precizan matematički jezik te uključuju učenike u smisleni matematički diskurs (Inoue, 2009). Na primjer, kada se obrađuju razlomci, učinkovita nastavnička objašnjenja moraju obuhvatiti tumačenja različitih pojmova (dio-cjelina, omjer, operator, kvocijent, mjera), a ne samo pojednostavljenu primjenu u svakodnevnom životu. Takav pristup zahtijeva pažljiv odabir slijeda bitnih pojmova i eksplicitne veze između različitih prikaza (Grossman i McDonald, 2008). Slično tome, u algebri učinkovita objašnjenja obuhvaćaju simboličke, grafičke i kontekstualne prikaze te istovremeno održavaju visoku kognitivnu razinu, dok proceduralna objašnjenja često rezultiraju površnim učenjem i ograničenim sposobnostima prenošenja znanja (Livers i Elmore, 2018).

U novijim je istraživanjima utvrđeno nekoliko pristupa pomoću kojih se može povećati učinkovitost nastavničkih objašnjenja. Pokazalo se da samoobjašnjenja, u kojima učenici artikuliraju vlastito razumijevanje nekog matematičkog pojma, produbljuju razumijevanje i povećavaju trajnost znanja (Wittwer i Renkl, 2008). Riješeni primjeri u udžbeniku stukturirani su prikazi postupka rješavanja zadataka te služe kao alati za razumijevanje procesa i temeljnih matematičkih pojmova (Renkl, 2014). Ovakvi se pristupi dopunjavaju rješavanjem problemskih zadataka i eksploratornim razgovorima koji učenicima pomažu otkriti matematičke veze i razviti strategije rješavanja zadataka (Chi i Wylie, 2014). Međutim, istraživanja o nastavničkim objašnjenjima otkrivaju značajne probleme. Pokazalo se da se takva objašnjenja često oslanjaju na proceduralno znanje, a ne na konceptualno razumijevanje (Baki, 2013; Karakuş, 2018; Kinach, 2002), što je možda i posljedica ograničenoga pedagoškog znanja i uvjerenja o poučavanju i učenju matematike (Kul i Celik, 2017; Philipp, 2007; Thompson, 1992; Toluk-Ucar, 2011). Preciznije rečeno, budući nastavnici imaju poteškoća u pronalaženju objašnjenja u kojima koriste adekvatne pojmove te se često fokusiraju na proceduralne korake bez objašnjenja matematičkih veza koje leže u pozadini (Avcu, 2022; Karakuş, 2018; Toluk-

Ucar, 2011). Te poteškoće obuhvaćaju i povezivanje višestrukih prikaza i uspješno objašnjavanje pogrešnih predodžbi koje učenici imaju. Na primjer, Young i Zientek (2011) su došli do saznanja da budući nastavnici imaju poteškoće u objašnjavanju algebarskih pojmova izvan okvira računskih operacija, dok je kod Toluk-Ulcara (2011) uočena njihova ograničena sposobnost da uspješno razjasne pogrešne predodžbe učenika.

Prethodno provedena istraživanja analizirala su nastavnička objašnjenja kroz različite analitičke okvire. Na primjer, Charalambous i sur. (2011) ispitali su nastavnička objašnjenja s obzirom na to kako nastavnici kombiniraju proceduralno i konceptualno znanje, naglašavajući važnost jasnoće, logičkoga povezivanja te kontekstualne osviještenosti tijekom poučavanja. Ovakav okvir potvrđuje da kod nastavnikovih objašnjenja nije važno samo pokazati korake u procesima, nego i pokazati učenicima temeljno znanje koje moraju imati kao podlogu za razumijevanje tih matematičkih pojmova. Slično tome, Thanheiser (2009) je analizirao nastavnička objašnjenja tako što se fokusirao na vezu između sadržajnoga znanja i pedagoških vještina, navodeći da je za davanje jasnih objašnjenja neophodno da nastavnik bude svjestan postojećega znanja i pogrešnih predodžbi koje učenici imaju. Složenost procjene nastavnikova matematičkoga znanja leži u kompleksnoj prirodi samoga predmeta. Krauss i sur. (2008) tvrde da analiza specifičnih aspekata nastavne prakse, kao što su nastavnička objašnjenja, pruža puno smisleniji uvid u situaciju nego opća procjena nastavničkih kompetencija. Ovakav ciljani pristup omogućava detaljnije razumijevanje o tome kako nastavnici razvijaju i primjenjuju vlastito matematičko znanje u praksi. Na temelju ovoga teorijskog okvira, naše istraživanje ispituje kako budući nastavnici matematike strukturiraju objašnjenja kada razjašnjavaju pogrešne predodžbe učenika o algebarskim pojmovima, pružajući uvid u njihovo pedagoško promišljanje i razvoj sadržajnoga znanja.

Imaginarni dijalozi

Nastavnička objašnjenja nastaju tijekom matematičkih dijaloga između nastavnika i učenika. Početak, nastavak i završetak objašnjenja određeni su strukturiranim disciplinarnim dijalogom (Leinhardt, 1993). Imaginarni dijalozi svojevrstan su alat pomoću kojega nastavnik može steći uvid u izazove s kojima se učenici susreću tijekom učenja i u njihova opažanja (Wille, 2008). Imaginarni dijalozi temelje se na pisanim dijalozima ili učeničkim dnevnicima (Bean i Zulich, 1989; Lee, 2004). Clarke i sur. (1993) podijelili su učeničke dnevnike na tri vrste: priče, sažetke i dijaloge, uz napomenu da se dijalozi ističu jer omogućavaju učenicima prepoznavanje izazova i pomažu im artikulirati misaone procese. Wille (2008; 2017) u literaturi koja se bavi nastavom matematike opisuje pisane matematičke dijaloge kao imaginarne dijaloge.

Gössinger i Götz (2023) opisuju imaginarni dijalog kao oblik matematičkoga pisanja, naglašavajući važnost poticanja učenika da prošire pisani i nedovršeni dijalog između dvoje fiktivnih ljudi koji razgovaraju o matematičkom problemu. Štoviše, uloga nastavnika u poticanju takvoga dijaloga u razredu (Webb, 2009; Dobber i van Oers, 2015) od velike je važnosti za buduće nastavnike, koji će tu vještinu trebati u svojoj karijeri. Studije o imaginarnim matematičkim dijalozima provedene na uzorku koji se sastojao od budućih nastavnika i nastavnika koji već rade u struci idu tome u prilog. Na primjer, Brodahl i Wathne (2018) proveli su istraživanje o iskustvima budućih nastavnika s imaginarnim matematičkim dijalozima i njihovom mišljenju o važnosti tih dijaloga za poticanje matematičkoga načina razmišljanja učenika. Spangler i Hallman-Thrasher (2014) uključili su buduće nastavnike u pisanje imaginarnih matematičkih dijaloga kako bi ojačali njihovo znanje o nastavi matematike i analizirali kako su učenici odradili te zadatke. Rezultati njihove studije pokazuju da dijalozi koji obuhvaćaju matematičke zadatke proširuju sadržajno znanje budućih nastavnika i njihovo razumijevanje o interakcijama između učenika. U istraživanju koje je proveo Wille (2016) budući su nastavnici pisali imaginarne matematičke dijaloge u kontekstu učenja algebre, otkrivši pimjere kada je matematički diskurs mogao dovesti do mogućih krivih predodžbi ili pogrešaka. Jedinstven doprinos ovoga istraživanja leži u tome što se buduće nastavnike uči kako provesti matematičke dijaloge od početka do kraja, a vrhunac je izrada vlastitih dijaloga. Ovakav sveobuhvatni pristup omogućava dublje razumijevanje načina na koji nastavnici izgrađuju i primjenjuju matematičko znanje u praksi.

U skladu s ovom teorijom, cilj je našega istraživanja ispitati kako budući nastavnici matematike objašnjavaju pogrešne predodžbe učenicima u području algebre koristeći se imaginarnim dijalozima. Ti su dijalozi oblik refleksije jer pomažu budućim nastavnicima steći uvid u mišljenja učenika i prilagoditi im svoje nastavne metode (Brodahl i Wathne, 2018; Wille, 2008). Wille (2017) je istaknuo da budući nastavnici koji sudjeluju u pisanju matematičkoga diskursa postaju svjesniji vlastitih nastavnih metoda, što im može pomoći bolje voditi učenike tijekom procesa učenja matematičkoga gradiva. Analizirajući objašnjenja budućih nastavnika u tim dijalozima, ovo istraživanje daje svoj doprinos efikasnijoj nastavi algebre te prijedloge za dopunu studijskih programa za izobrazbu nastavnika kako bi oni mogli što bolje objasniti pogrešne algebarske predodžbe u razredu. Pitanja koja postavljamo u istraživanju su: 1) Kakva objašnjenja budući nastavnici matematike predlažu za poučavanje o algebarskih pojmovima?; 2) Kako budući nastavnici koji pohađaju kolegij Komunikacija u nastavi matematike osmišljavaju objašnjenja pomoću imaginarnih dijaloga kako bi učinkovito razjasnili pogrešne algebarske predodžbe?

Metodologija

U ovom istraživanju korišten je kvantitativni pristup koji se još naziva i studijom slučaja, s ciljem analize objašnjenja koja budući nastavnici daju učenicima tijekom poučavanja algebre, pomoću imaginarnih dijaloga. Studija slučaja uključuje detaljnu analizu jednoga ili više slučajeva (Christensen i sur., 2015). Slučaj koji je bio predmet ovoga istraživanja sastojao se od objašnjenja koja budući nastavnici daju učenicima pri obradi algebarskih tema, dok su tema analize bili budući nastavnici koji su prikazali objašnjenja kroz imaginarne dijaloge.

Sudionici

Nastavnička objašnjenja analizirana su metodom namjernoga uzorkovanja, a budući nastavnici odabrani su na temelju posebnih kriterija koje je postavio istraživač. Jedan od kriterija odabira bio je da su sudionici uspješno položili kolegije Komunikacija u nastavi matematike i Nastava algebre na prijediplomskom studiju, što je bio znak da su osposobljeni za pisanje realističnih imaginarnih dijaloga koji se mogu odvijati u razredu. Sljedeći je kriterij bio da sudionici dobro poznaju interakcije u razredu i da imaju nešto iskustva u radu u nastavi, koje su stekli u sklopu kolegija Školski praktikum. Nadalje, odabrani su i budući nastavnici koji su imali prethodno iskustvo u izradi imaginarnih dijaloga. Sudionici koji su zadovoljili sve ove kriterije prošli su edukaciju osmišljenu tako da razvije njihovu sposobnost izrade imaginarnih dijaloga. Ta je obuka trajala 4 školska sata, a odvijala se tijekom dva tjedna.

Istraživanje je provedeno tijekom zimskoga semestra akademske godine 2022./2023., a obuhvatilo je 28 budućih nastavnika u višim razredima osnovne škole koji su pohađali nastavu iz kolegija Komunikacija u nastavi matematike na jednom državnom sveučilištu u Turskoj. Obavezno školovanje u Turskoj traje 12 godina i podijeljeno je na tri razine: prva razina obuhvaća četverogodišnje osnovnoškolsko obrazovanje (od 1. do 4. razreda), druga razina obuhvaća 4 viša razreda osnovne škole (od 5. do 8. razreda), a treća razina obuhvaća četverogodišnje srednjoškolsko obrazovanje (od 9. do 12. razreda) (MoNE, 2012). Budući nastavnici koji su sudjelovali u istraživanju, kada se zaposle, poučavat će učenike od 5. do 8. razreda.

Alati za prikupljanje podataka

Polustrukturirani imaginarni dijalozi izrađeni su s ciljem analize objašnjenja koja budući nastavnici daju učenicima. Pregledom literature utvrđene su pogrešne predodžbe iz područja algebre koje se često javljaju kod učenika u višim razredima osnovne škole (Bishop i sur., 2008; Booth, 1988; Chow, 2011; Foster, 2007; Tirosh i sur., 1998). Za izradu imaginarnih dijaloga odabrana su četiri scenarija iz područja algebre u kojima učenici pokazuju pogrešne predodžbe: operacije u algebarskim izrazima, korištenje modela vage pri rješavanje jednadžbi, pisanje verbalnih algebarskih izraza te objašnjavanje pojmova varijable i nepoznanice.

Kako bi što bolje dočarali interakcije koje se odvijaju u razredu, polustrukturirani dijalozi osmišljeni su tako da simuliraju stvarno razredno okružje. Prije izrade ovih polustrukturiranih i nestrukturiranih dijaloga, za potrebe analize snimljene su i transkribirane autentične interakcije iz razreda tijekom nastave algebre u višim razredima osnovne škole. Podatci potrebni za dijaloge dobiveni su iz istraživanja koje je Celik (2016) naveo u svojoj doktorskoj disertaciji. Prije finaliziranja alata za prikupljanje podataka, detaljno je pregledano 39 školskih sati pisanih autentičnih dijaloga koji su se odvijali u razredu. Nakon toga, izrađena su tri polustrukturirana i jedan nestrukturirani dijalog, a svi su bili fokusirani na pogrešne predodžbe u učenju algebre. U polustrukturiranim dijalozima razgovor je započeo ili nastavnik ili učenik, a sudionici su ih dovršili tako što su pisali dijelove iz obje perspektive, i učenika i nastavnika. Svaki od tri polustrukturirana dijaloga sadržavao je pisane rečenice učenika ili nastavnika na početku, sredini i na kraju, a sudionici su trebali popuniti praznine. Od sudionika se tražilo da napišu rečenice koje bi inače pisali učenici, kako bi se moglo odrediti jesu li kao nastavnici učenicima dali priliku da ih nešto pitaju i kakva su im objašnjenja dali. Drugi je cilj bio procijeniti prate li odgovori nastavnika već napisana pitanja učenika jer su se tako mogla procijeniti nastavnička objašnjenja. Tijekom dvotjedne edukacije za pisanje imaginarnih dijaloga koja se odvijala u sklopu kolegija Komunikacija u nastavi matematike dijalozi su bili započeti, a od sudionika se tražilo da ih nastave koristeći svoju maštu. Uočeno je da su u nešto manje strukturiranim dijalozima sudionici ponekad odstupali od konteksta. Stoga su u alatu za prikupljanje podataka u sredinu ili na kraj dijaloga dodane rečenice napisane iz perspektive učenika ili iz perspektive nastavnika. Nadalje, učenički izrazi poput *„Zašto je to tako, nastavniče?"* i *„Ne razumijem"* dodani su kako bi se procijenilo kako budući nastavnici potkrijepljuju svoja objašnjenja i prilagođavaju nastavne metode kada učenici pokažu teškoće u razumijevanju. Jedini nestrukturirani dijalog bio je fokusiran na pojmove *varijabla* i *nepoznanica*. Cilj je bio promatrati kako budući nastavnici osmišljavaju objašnjenja bez prethodno napisanih rečenica iz perspektive nastavnika ili učenika. Primjer toga je polustrukturirani imaginarni dijalog u Prilogu 1.

Dobivene su povratne informacije od stručnjaka za nastavu matematike, a nakon njih je uslijedila revizija dijaloga kako bi se povećala kreativnost i uključenost učenika. Slijedi primjer polustrukturiranoga imaginarnog dijaloga:

Faruk: „Kako to ide? Možemo li to zbrojiti ili ne?"
Nastavnik: „U redu, Faruk, da te nešto pitam. Što tebi znači $x+y+z=x+k+z$?"
Faruk: „Ne može biti jednako zato što…"
Selen: „Je li k broj, nastavniče? Sve me to zbunjuje."
Nastavnik: …
Burcu: …

Tijek istraživanja

Kako bi se ispitao način na koji budući nastavnici daju objašnjenja učenicima kroz imaginarne dijaloge, u sklopu kolegija Komunikacija u nastavi matematike održana je edukacija koju je vodio jedan od istraživača. Taj je kolegij jedan od izbornih kolegija koje Povjerenstvo za visoko obrazovanje u Turskoj preporuča za uključivanje u studijske programe za izobrazbu nastavnika matematike. Profesori na fakultetima koji izvode nastavu u tim programima mogu, ako žele, taj kolegij ponuditi kao izborni kolegij povezan s metodikom nastave. U sklopu ovoga istraživanja kolegij Komunikacija u nastavi matematike naglašava interakciju u razredu i praksu pisanja imaginarnih dijaloga kako bi se unaprijedilo okružje za učenje. Na početku su studenti prošli kroz uvod u kvalitetne i nekvalitetne matematičke dijaloge koji bi se mogli odvijati u stvarnoj nastavi.

U ovome kontekstu održani su razgovori s budućim nastavnicima o tome kako matematički dijalog može početi, razvijati se i završiti. Tijekom kolegija Komunikacija u nastavi matematike budući nastavnici prošli su kroz osmotjednu edukaciju u kojoj su se fokusirali na način strukturiranja elemenata dijaloga, poput odgovora učenika, povratne informacije nastavnika te pitanja i odgovora između učenika i nastavnika. Nakon toga su budući nastavnici sudjelovali u aktivnostima pisanja imaginarnih dijaloga, uključujući različite domene učenja, što je imalo za cilj pomoći im da bolje razumiju kako na adekvatan način dovršiti matematički dijalog. Nakon što su se sudionici istraživanja upoznali s procesom pisanja imaginarnih matematičkih dijaloga, dobili su polustrukturirane obrasce imaginarnih dijaloga. Kako bi se osiguralo da se sudionici oslanjaju na svoje postojeće znanje, bez prethodne pripreme, nisu im unaprijed dane informacije o specifičnim domenama učenja na koje se imaginarni dijalozi odnose. Za rad na ovome zadatku imali su 60 minuta.

Analiza podataka

Podatci dobiveni nakon što su budući nastavnici završili polustrukturirane i nestrukturirane dijaloge analizirani su pomoću računalnoga programa za kvalitativnu analizu Maxqda 2022. Dva istraživača su zasebno analizirala četiri pitanja iz dijaloga i vodili bilješke o svojim zapažanjima. Zajedno su kodirali podatke, s ciljem prepoznavanja tema i kodova, čime je osiguran integritet podataka. Jedan je istraživač nastavio s procesom kodiranja, a drugi je samostalno ponovno kodirao 10 % podataka kako bi se postigla pouzdanost. Nakon rasprave, istraživači su postigli konsenzus o kodovima koji se podudaraju i ne podudaraju te su tako došli do konačne inačice kodova.

Tijekom analize podataka dijalozi su analizirani u kontekstu nastavničkih objašnjenja. Prepoznati su višestruki primjeri istoga tipa tih objašnjenja u jednom imaginarnom dijalogu budućega nastavnika. Zbog toga je frekvencija nekih kodova premašila broj sudionika. Na primjer, u dijalogu nastavnika br. 27 o pretvaranju algebarskih izraza u verbalne izraze uočeno je da je sudionik netočno sturkturirao koeficijent i varijablu u tri različita primjera.

Na temelju kodova za prikaz rezultata izrađene su Maxqda mape koje prikazuju sažetak nastavničkih objašnjenja. Kako bi se osigurala pouzdanost, proces kodiranja sastojao se od dugih rasprava i konsenzusa među istraživačima, što je trajalo otprilike četiri mjeseca. Valjanost analize podataka postignuta je kroz potvrdu sudionika, što se još naziva provjerom članova. Nakon analize podataka, s budućim nastavnicima održan je *online* sastanak na kojemu im je prezentirana mapa izrađena na temelju analize. Od njih se tražilo da daju svoje mišljenje o rezultatima svakoga imaginarnog dijaloga. Većina je sudionika izjavila da rezultati točno odražavaju njihova pedagoška objašnjenja u području algebre. Na primjer, sudionik br. 8 izjavio je da su rezultati u skladu s njegovim dijalozima i potvrdio je da je imao poteškoća pri oblikovanju adekvatnih odgovora dok ih je pisao. Pomoću potvrde sudionika i slaganja dvaju istraživača koji su kodirali podatke u istraživanju se želio postići veći kredibilitet analize podataka. Rezultate potvrđuju i perspektive budućih nastavnika, što jača valjanost ovoga istraživanja.

Rezultati

U istraživanju su budućim nastavnicima postavljena 4 pitanja s ciljem ispitivanja ključnih aspekata u području učenja algebre. Njihovi odgovori analizirani su i sažeti pomoću softvera Maxqda. Prvo se pitanje odnosilo na nastavnička objašnjenja u vezi s računskom operacijom zbrajanje u algebarskim izrazima i jednadžbama, a bilo je integrirano u polustrukturirani dijalog u kojemu su se pojavili izrazi i. S obzirom da se nastavnička objašnjenja za ova dva izraza razlikuju, dijalog je analiziran kao cjelina. Međutim, kako bi se postigla veća jasnoća, rezultati su prikazani zasebno u poglavlju Rezultati. Objašnjenja koja su dali budući nastavnici u imaginarnim dijalozima za rezultat izraza ilustrirani su na Slici 1.

Slika 1

Dijalog o izvođenju operacija zbrajanja u algebarskim izrazima počinje tako što nastavnik pita učenike što misle, koji će biti rezultat izraza . To vodi k nizu netočnih odgovora učenika, nakon čega slijede objašnjenja budućih nastavnika. Ta su objašnjenja analizirana kao nastavnička objašnjenja. Kako prikazuje Slika 1, prikupljeno je vrlo malo primjera adekvatnih nastavničkih objašnjenja (f = 7), što upućuje na činjenicu da je većina budućih nastavnika dala neadekvatno objašnjenje za ovu algebarsku operaciju. Neki budući nastavnici odlučili su potpuno ignorirati pitanja učenika te nisu odgovorili na njih, dok je u znatnom broju njihovih odgovora (f = 21) nedostajalo matematičkih pojašnjenja, iako su u određenoj mjeri uspjeli nešto objasniti. Štoviše, neki su budući nastavnici dali nepovezana objašnjenja (f = 7). Na primjer, jedan od njih, budući nastavnik br. 5, odgovorio je na pitanje imaginarnoga učenika Gozde, ali nije adekvatno razjasnio matematički kontekst:

Gozde: „To nema smisla, nastavniče. Kako možemo zbrojiti slova?"
Nastavnik br. 5: „U matematici ponekad umjesto brojeva koristimo različite simbole, kao što su x, y, a, b, c."

Drugi budući nastavnik, nastavnik br. 7, ponudio je korektivnu povratnu informaciju nakon Gozdinoga pitanja, tako što je rekao: *„Izraz kojega si opisao je nepotpun. Kako možemo zbrojiti različita slova? Ona predstavljaju dvije različite stvari."* Neki budući nastavnici pokušali su ponuditi objašnjenja kako bi razjasnili operaciju zbrajanja u izrazu , često se oslanjajući na analogiju poput *„ne možemo zbrajati jabuke i kruške"*. Međutim, takvi primjeri često mogu dovesti do pogrešnih predodžbi. Na primjer, budući nastavnik br. 1 *(„Možemo li zbrajati slova? Recimo da „a" predstavlja jabuku, a „c" krušku. Ako kažemo '1 jabuka + 3 kruške', koliko jabuka ukupno imamo? Ima li vam to smisla?")* iskoristio je analogiju zbrajanja jabuka i krušaka kako bi Gozdi objasnio situaciju. Drugi je budući nastavnik, br. 26, također pokušao objasniti situaciju pomoću numeričkoga primjera. Neadekvatna povratna informacija i primjeri koje su učenicima dali budući nastavnici sadrže objašnjenja koja mogu dovesti do stvaranja pogrešnih predodžbi. Na primjer, više od polovice budućih nastavnika (f = 17) naznačilo je da se izraz ne može procijeniti. Nastavnik br. 17 izjavio je da slova predstavljaju nešto drugo, a ne brojeve, dok nastavnici br. 11, 14 i 15 nisu mogli definirati varijable u izrazu . Primjer toga može se vidjeti u odgovoru nastavnika br. 14: *„Djeco, slova su ovdje simbolična. Recimo da imam 3 kruške i 1 jabuku. Ako kruške zamijenimo slovom „c" da izraz bude kraći, a jabuke zamijenimo slovom „a", bi li imalo smisla reći 3ac ili „3 jabuke-kruške" kako je Kemal predložio? Jednostavno bismo ih zbrojili kao 3 kruške i 1 jabuku, zar ne?"* Detaljnijim promatranjem Slike 1 može se uočiti da je samo mali broj budućih nastavnika dao odgovarajuća objašnjenja učenicima. Na primjer, nastavnik br. 7 ukazao je na pogrešku imaginarnoga učenika Kemala (Kemal je izjavio da je) tako što je pratio pitanja koja učeniku mogu pomoći da prepozna vlastitu pogrešku.

Nastavnik br. 7: „Kemale, što ta operacija predstavlja?"
Kemal: „To je zbrajanje, nastavniče."
Nastavnik br. 7: „Zašto si onda primijenio operaciju množenja?"

Nastavnik br. 18 dao je točno objašnjenje rekavši: *„Da ponovno to objasnim. Možemo zbrajati ili oduzimati samo kada su varijable iste. Međutim, u slučaju izraza a + 3c, postoje dvije različite varijable, tako da izraz ostavljamo takav kakav je: To je rezultat ove računske operacije."* Ovime je u matematičkom smislu razjasnio situaciju i ponudio točno objašnjenje. Drugi važan rezultat istraživanja jest činjenica da jedanaest budućih nastavnika u dijalogu nije moglo doći do matematičkoga zaključka o rezultatu izraza te je on ostao neriješen. Drugi matematički izraz koji je analiziran bio je . Sažetak objašnjenja ove jednakosti koje su dali budući nastavnici prikazan je na Slici 2.

Slika 2

Iako su budući nastavnici dali bolja objašnjenja za izraz nego što je to bio slučaj za izraz , još uvijek su se u velikoj mjeri javljali primjeri neadekvatnih nastavničkih objašnjenja. Ipak, u usporedbi s prethodnim primjerom, važno je primijetiti da je manji broj pitanja koja su učenici postavljali ostao bez odgovora (f = 3). Međutim, dok su neki budući nastavnici dali eksplanatornu povratnu informaciju, ona je često ostala bez potrebnoga matematičkog razmišljanja specifičnoga za ovu jednadžbu (f = 12), što se može ilustrirati dijalogom između budućega nastavnika br. 25 i imaginarne učenice Selen:

Selen: „Je li „k" broj, nastavniče? Stvarno sam zbunjena."
Nastavnik br. 25: „Da, „k" je varijabla, isto kao i x, y, z."

Neki su budući nastavnici odlučili dati kratke potvrdne odgovore poput *„Da, to je točno"* umjesto detaljnije povratne informacije ili su ponudili kratku korektivnu povratnu informaciju a da nisu potpuno razjasnili pogrešnu matematičku predodžbu (npr. nastavnici br. 13 i 7).

Što se tiče primjera, neki su budući nastavnici pokušali razjasniti jednadbžu pomoću primjera. Međutim, sedmero ih je koristilo samo brojčane primjere, dok je drugih sedam dalo primjere koji nisu bili adekvatni za specifični matematički kontekst. Dijalog je, tipično, započeo s imaginarnim učenikom, Farukom, koji postavlja pitanje o valjanosti jednadžbe . Na primjer, u dijalogu nastavnika br. 12 s drugim imaginarnim učenikom, Burcuom, budući je nastavnik pokušao razjasniti pojam koristeći brojčani primjer, no pogrešno je od samoga početka pretpostavio da su varijable *y* i *k* jednake.

Štoviše, troje budućih nastavnika nije moglo točno definirati pojmove *jednakost* i *varijabla*. Nastavnik br. 19 pitao je učenika da definira pojam varijable, ali je u dijalogu zapisao samo „*Burcu definira varijablu*", umjesto da je napisao stvarnu definiciju. Slično tome, nastavnik br. 2 pokušao je definirati jednakost, a nastavnik br. 20 pokušao je objasniti pojam varijable, no njihove definicije nisu uspjele točno prikazati ta dva termina.

U nekim su slučajevima budući nastavnici koristili neadekvatne analogije, dok su drugi davali objašnjenja koja nisu bila povezana s matematičkim sadržajem (f = 11). Na primjer, nastavnik br. 10 je u dijalogu s imaginarnom učenicom Selen odgovorio na pitanje „*Je li* k *broj, nastavniče? Zbunjena sam*" nejasnom analogijom: „*Ponekad koristimo slova umjesto brojeva da bismo shvatili što predstavljaju. Kao kada nešto kupuješ – ne pitaš prijatelja „Što misliš, koliko ovo košta?"*"

Neka od objašnjenja budućih nastavnika za jednadžbu ponekad su dovela do konceptualnih pogrešaka. Te su pogrešne predodžbe uključile: poimanje slova kao prikaza nebrojčanih pojmova (f = 8), implikacije da se različita slova ne mogu zbrajati (f = 3), netočno definiranje algebarskih izraza (f = 1), inzistiranje na tome da obje strane jednadžbe moraju imati iste varijable (f = 1) te pretpostavku da vrijednost *k* ne predstavlja brojčanu vrijednost (f = 1). Sljedeći razgovor između nastavnika br. 3 i imaginarnog učenika Burcua ilustrira da nastavnik implicira da se različita slova ne mogu zbrajati, kao i ideju da *y* i *k* ne mogu biti jednaki:

> *Burcu: „Ako kažemo da je ova jednadžba jednaka, zar ne moramo provjeriti jesu li y i k jednaki?"*
>
> *Nastavnik br. 3: „Da, Burcu je u pravu. Ponovimo: zbrajamo vrijednosti s istom nepoznanicom."*

Osim toga, objašnjenje koje je nastavnik br. 1 dao o varijabli *k* upućuje na to da je mislio da *k* ne predstavlja numeričku vrijednost. Kada je nastavnik br. 13 podržao pogrešnu predodžbu učenika da varijable s obje strane jednadžbe moraju biti jednake, otvorio je put većem broju pogrešnih predodžbi koje se javljaju u razrednom okružju.

Ova objašnjenja otkrivaju značajan nedostatak sadržajnoga znanja budućih nastavnika, kako se može vidjeti u primjerima koje su naveli nastavnici br. 1, 3 i 13. Slično tome, nevažna objašnjenja koja su dali nastavnici br. 11, 21, 22 i 27 nastala su iz njihova nedovoljnoga poznavanja kurikula. Na primjer, odgovor nastavnika br. 21 na pitanje o tome je li *k* broj – „*Prisjeti se, koristili smo simbole poput O i * i ranije. Sjećaš li se toga?*" – upućuje na nedovoljno razumijevanje redoslijeda nastavnih tema.

Neki budući nastavnici mogli su uspostaviti vezu između radoznalosti učenika i nastavnoga sadržaja (f = 13), razjasniti jednadžbu služeći se matematičkim rječnikom (f

= 6) te pružiti primjere koji odgovaraju kontekstu (f = 1) kada su objašnjavali jednadžbu . Na primjer, nastavnik br. 14 vodio je imaginarnoga učenika Burcua k pojednostavljenju jednadžbe i ka zaključku y = k. Nakon toga, nastavnik br. 14 potaknuo je Burcua da razmisli mogu li ta različita slova predstavljati istu vrijednost. Nastavnik je nakon toga odgovorio objašnjenjem koje je uspješno spojilo radoznalost učenika i matematički sadržaj: *„Da, slova mogu izgledati drugačije, ali, kako smo već raspravili, svako slovo predstavlja specifičnu vrijednost. Ako zadamo y = 1, onda k također treba imati vrijednost 1, što možemo shvatiti iz ove vrijednosti."* Takav pristup omogućio je nastavniku br. 14 da na konstruktivan način poveže učenikovu radoznalost s matematičkim pojmom koji leži u pozadini.

Nastavnik br. 8 također je razjasnio Selenino pitanje o tome je li *k* broj tako što je dao matematičko objašnjenje: *„Da, djeco, ovdje k predstavlja broj. Međutim, ne znamo koji broj predstavlja. Slično tome, y također predstavlja broj, no ne znamo ni njegovu stvarnu vrijednost."* Rezultati analize imaginarnih dijaloga koji su uključivali pretvaranje algebarskih izraza i jednadžbi u verbalne opise prikazani su na Slici 3.

Slika 3

U imaginarnim dijalozima koji se bave temom pretvaranja algebarskih izraza u verbalne, korišteni su izrazi, $8x$-5, $\frac{1}{2}x$+6=8 i $6x$+4=29. Primjer ovoga polustrukturiranog dijaloga prikazan je u Prilogu 1. Objašnjenja koja su učenicima dali budući nastavnici o pretvaranju tih algebarskih izraza i jednadžbi u verbalne izraze često sadrže elemente koji mogu dovesti do pogrešnih konceptualnih predodžbi. Preciznije rečeno, pogrešna tumačenja pojmova *varijabla* i *koeficijent* bila su vrlo učestala (f = 67) i javljala su se u raznim dijelovima dijaloga koje su napisali budući nastavnici. Značajan propust bio je nedostatak rečenica kojima se učenike vodi do prepoznavanja nepoznanice (f = 43). Na primjer, nastavniku br. 9 imaginarni učenik Pinar za jednadžbu odgovorio je verbalnim objašnjenjem: *„Ako krenemo od toga da je moj novac 'x', onda je pola mojega novca plus 6 jednako 8."* Nastavnik br. 9 tada je potvrdio odgovor, uz komentar *„Dobro"*, no nije učenika potaknuo da pronađe nepoznanicu dodavši možda *„Koju vrijednost ima 'x'?"* ili *„Koliko onda novca imam?"* Daljnje pogreške u objašnjenjima uključile su zanemarivanje činjenice da varijable trebaju predstavljati isti broj elemenata (f = 14), nedefiniranje ili pogrešno definiranje nepoznanice (f = 8), pretvaranje racionalnih koeficijenata u cijele brojeve u verbalnim izrazima (f = 1) te zanemarivanje multiplikativne prirode koeficijenata (f = 1). Ovi problemi upućuju na nedostatak sadržajnoga znanja. U jednome dijalogu nastavnik je od učenika zatražio da izrade verbalne izraze za izraz , na što je imaginarna učenica Serap odgovorila: *„Uzela sam 5 knjiga iz jedne od 8 kutija s knjigama."* Kada je nastavnik napomenuo da je Serap nešto zaboravila, nije pojasnio da varijabla treba predstavljati jednaki broj knjiga u svakoj kutiji. Umjesto toga, nastavnik br. 20 odgovorio je nevezanim objašnjenjem, pokazujući manjak razumijevanja o tome da varijable često prikazuju identične količine. Troje budućih nastavnika nije ispravno definiralo ključne izraze i pojmove (npr. nastavnik br. 10). Nadalje, u polustrukturiranim dijalozima budući nastavnici često su davali slabu povratnu informaciju o unaprijed napisanim odgovorima učenika i o odgovorima koje su sami napisali, ili jednostavno nisu odgovorili na njihova pitanja (f = 20). Dok ih je puno pokušalo dati eksplanatornu povratnu informaciju, često nisu ponudili matematičko objašnjenje (f = 24). Nekoliko je budućih nastavnika odabralo jednostavnu potvrdnu (f = 4) ili korektivnu (f = 1) povratnu informaciju. U jednome scenariju nastavnik je od učenika zatražio da napišu verbalni izraz za , ostavljajući imaginarnom učeniku Asliju prostor za netočan odgovor. Nastavnik je Aslija tada vodio do odgovora pomoću pitanja: „*Jesi li siguran, Asli? Promisli još jednom*", s namjerom da budući nastavnik razmisli o verbalnom izrazu za racionalno rješenje. Međutim, budući nastavnik br. 9 na sljedeći je način napisao Aslijev odgovor: *„Četiri minus šest puta dob mojega brata je 29."* Kasnije je prepravio odgovor u: *„Šest puta dob mojega brata plus četiri je 29",* a da nije provjerio racionalnost rezultata.

Neki budući nastavnici, br. 4, 6 i 16, izbjegli su davanje matematičkih objašnjenja tako što su se referirali na vanjske izvore. Na primjer, nastavnik br. 6 izbjegao je odgovor na Serapino pitanje *„Što sam zaboravila, nastavniče?"* rekavši: *„Da vidimo možeš li to sama shvatiti u sljedećem problemu. Želim da ga probaš sama riješiti."* Nastavnik br. 4 u potpunosti je izbjegao odgovoriti učeniku, rekavši: *„Kako piše na 139. stranici, postoje stvari koje treba uzeti u obzir kada se algebarski izrazi pretvaraju u verbalne. Također sam to zapisao u vaše bilježnice. Ima li itko drugačiju ideju za ovaj primjer?"* Unatoč prevalenciji slabih nastavničkih objašnjenja, neki su budući nastavnici primijenili adekvatne nastavne strategije. One su obuhvatile naglašavanje činjenice da varijable predstavljaju jednake količine (f = 17), provjeravanje racionalnosti rezultata (f =6), naglašavanje važnosti određivanja nepoznanice (f = 4), naglašavanje potrebe da se u obzir uzme vrijednost na desnoj strani jednadžbe (f = 2) te kvalitetan proces vođenja učenika pri operacijama zbrajanja i oduzimanja (f = 2). Odgovor nastavnika br. 26, *„Je li logično da imate 25/6 perlica…?",* ilustracija je primjera adekvatnoga objašnjenja racionalnosti rezultata.

Nastavnici br. 5 i 19 koristili su pitanja, a ne korektivnu povratnu informaciju kako bi pomogli imaginarnim učenicima prepoznati i popraviti vlastite pogreške. Na primjer, nastavnik br. 5 postavio je imaginarnoj učenici Serap, koja je netočno navela *„Uzela sam 5 knjiga iz jedne od 8 kutija s knjigama"*, pitanje: *„Dakle, koliko ima knjiga u svakoj kutiji?"* Nastavnik br. 7 također je ponudio odgovarajuću definiciju nepoznanice, navodeći da ona predstavlja identičnu vrijednost. Rezultati analize objašnjenja koja su dali budući nastavnici učenicima o rješavanju jednadžbe pomoću jednakosti prikazani su na Slici 4.

Slika 4

Tijekom analize opće strukture dijaloga o rješavanju jednadžbi pomoću modela vage razgovor je počeo željom učenika da riješi pitanje iz knjige u kojemu se traži da pomoću modela vage prikaže jednadbžu . Nakon toga je ostavljen prazan prostor u kojemu nastavnik treba opisati kako bi učenika vodio do rješenja zadatka. Kasnije imaginarni učenik Ozge pita: *„Zašto smo zadatak riješili na taj način? Nije li drugi način kraći?"* U ovoj fazi od budućih nastavnika očekuje se da objasne zašto se tu koristi model vage. Također je ostavljen i prostor za dodatna pitanja ili komentare učenika Necdeta, Munevvera i cijeloga razreda. Dijalog završava tako da Munevver daje točno objašnjenje postupka pomoću modela vage, a nastavnik potvrđuje njegov odgovor.

Kako se može vidjeti na Slici 4, slično drugim algebarskim temama, nastavnička objašnjenja budućih nastavnika o algebarskim izrazima/jednadžbama pomoću modela vage bila su uglavnom slaba. Iako su neki budući nastavnici pokušali dati eksplanatornu povratnu informaciju o modelu vage, ta objašnjenja nisu bila dovoljna za povezivanje modela s matematičkim sadržajem (f = 28). Mnogi budući nastavnici ili uopće nisu napravili grafičke prikaze u kojima koriste model vage (f = 16) ili su izradili samo djelomične prikaze (f = 12). Međutim, valja istaknuti da nitko od budućih nastavnika nije pokazao točnu i adekvatnu upotrebu modela vage. Na primjer, nastavnik br. 26 u odgovoru je naveo *„crta vagu na ploči i piše jednadžbu s obje strane"*, no nije točno opisao niti prikazao model rješenja te je jednadžbu riješio na algebarski način. Nadalje, gotovo je polovica budućih nastavnika koristila nejasan jezik umjesto modela vage (f = 13). Slično tome, nastavnik br. 28 izradio je djelomičan prikaz, no jednadžbu je algebarski pojednostavio prije nego je uopće pokušao grafički prikazati vagu (vidi Sliku 5). To upućuje na činjenicu da većina budućih nastavnika ne posjeduje samopouzdanje ili iskustvo u korištenju vizualnih ili fizičkih prikaza pri rješavanju algebarskih izraza.

Slika 5

Holistička analiza imaginarnih dijaloga koje su napisali budući nastavnici o prikazu modela vage pokazala je da u mnogim slučajevima učenici nisu dobili odgovore na svoja pitanja. Oba pitanja koja su istraživači postavili kao pitanja imaginarnih učenika i pitanja koja su napisali sami budući nastavnici iz perspektive imaginarnih učenika ostala su bez odgovora (f = 17). Na primjer, nastavnik br. 9 napisao je pitanje koje je postavio imaginarni učenik Necdet: *„Zašto si oduzeo 6 s obje strane? Ne možemo li izravno prikazati kockica na vagi?"* Međutim, nastavnik br. 9 je, umjesto da odgovori na ovo pitanje, skrenuo dijalog u drugom smjeru tako što je postavio razna pitanja cijelom razredu.

Pri objašnjavanju modela budući nastavnici također su napisali rečenice koje mogu lako dovesti do stvaranja pogrešnih konceptualnih predodžbi (f = 43). Kako se može vidjeti na Slici 4, neki su nastavnici tvrdili da se negativne vrijednosti ne mogu prikazati na modelu (f = 6), neki su prerano pojednostavili jednadžbe (f = 5), neki su nacrtali pogrešne prikaze (f = 12), a neki su koristili tvrdnje koje nisu u skladu s principom održanja jednakosti (f = 20). Na primjer, nastavnik br. 6 dao je učenicima objašnjenje koje se kosi s principom jednakosti tako što je izvodio operacije samo s jedne strane jednakosti i konstantu je tretirao kao pozitivnu, a ne negativnu. U drugom primjeru, nastavnik br. 1 napravio je konceptualnu pogrešku tako što je nacrtao nedovoljan broj predmeta koji su trebali biti prikazani na modelu. Nastavnik br. 1 opisao je scenarij s osam vreća lješnjaka, u kojemu je 6 lješnjaka izvađeno iz jedne vreće te naveo da je to jednako vrijednosti od 26. Na taj je način izradio verbalni model povezan sa stvarnim životom. Međutim, ta situacija nije točno prikazana na crtežu. Nastavnik br. 1 koristio je brojeve s jedne strane jednakosti, a modele s druge, no nije prikazao dovoljan broj predmeta kako bi točno prikazao model. Rezultat toga bio je nejednak i netočan vizualni prikaz.

Među budućim nastavnicima koji su objasnili da „*negativna vrijednost ne može biti prikazana na modelu*", nastavnik br. 13 dao je sljedeće objašnjenje: „*Što radimo s obje strane jednakosti? Pogledajte, trebamo izlučiti 8m, no u jednakosti ne mogu prikazati oduzimanje. Stoga dodajem 6kg na objema stranama.*"

Odgovori budućih nastavnika učenicima koji su imali pitanja o razlogu korištenja modela vage često su bili površni i nisu imali konceptualnu dubinu. Njihovi su odgovori varirali od toga da nisu uspjeli pružiti jasno objašnjenje (f = 8), da se korištenje modela vage jednostavno traži u zadatku (f = 7), da se takva pitanja mogu pojaviti u ispitnim materijalima (f = 2) do toga da korištenje modela vage pomaže pri razumijevanju prečaca za izlučivanje pojmova (f = 1). Na primjer, na pitanje imaginarnoga učenika Ozgea zašto se vaga koristi, nastavnik br. 3 odgovorio je: „*Ozge, mogli bismo ga riješiti i drugačije, ali u pitanju se zahtijeva da koristimo vagu.*"

Unatoč tim nedostatcima, neki su budući nastavnici dobro objasnili zašto se koristi model vage, iako njihova objašnjenja nisu bila eksplicitno matematičke prirode. Ta su objašnjenja implicitno bila u skladu s logikom koja nalaže korištenje prikaza vage jer su navedeni razlozi poput boljega razumijevanja (f = 6), bolje trajnosti znanja (f = 3), pretvaranja apstraktnih pojmova u konkretne (f = 3), razumijevanja logičke pozadine modela vage (f = 2), uvid u drugačiju perspektivu (f = 1) te povezivanja jednakosti s modelom vage (f = 1). Na primjer, nastavnik br. 20 objasnio je da korištenje modela vage olakšava učenje, ali je također napomenuo da ono zahtijeva puno vremena: „*Da, bilo bi jednostavnije ovo riješiti izravno kao jednadžbu, ali koristimo prikaz vage kako bismo vizualizirali pojam. Ne rješavamo svaki matematički zadatak na ovaj način, jer, iako je učinkovit, on oduzima puno vremena.*"

Nastavnik br. 14 naglasio je važnost korištenja modela vage za bolje razumijevanje, rekavši: „*Model vage pomaže vam bolje razumjeti pojam jednakih vrijednosti.*" Zanimljivo je napomenuti da nitko od budućih nastavnika koji su djelomično artikulirali svrhu primjene modela jednakosti nije uspio izraditi točan prikaz.

Najučinkovitije nastavničko objašnjenje o prikazivanju jednadžbi pomoću modela vage temeljeno je na principima jednakosti i održavanju jednakosti (f = 8). Iako nastavnik br. 14 nije demonstrirao primjenu modela vage, verbalno je povezao model vage s pojmom jednakosti, uz objašnjenje: „*Desna strana jednadžbe trebala bi biti jednaka lijevoj strani, kao na vagi. Ako nešto dodamo desnoj strani ili oduzmemo s desne strane, isto trebamo učiniti i na lijevoj strani, inače ćemo poremetiti jednakost.*" Nadalje, nastavnici br. 2, 3 i 26 u dijaloge su uključili segmente u kojima se uspostavlja veza između radoznalosti učenika i sadržaja. Na primjer, nastavnik br. 3 je u ime imaginarnoga učenika Necdeta postavio pitanje o tome zašto se koristi vaga, a na to pitanje Munevver je kasnije odgovorio.

Na kraju, od budućih nastavnika se zahtijevalo da napišu nestrukturirani dijalog kako bi objasnili vezu između pojmova *varijabla* i *nepoznanica*. Dijalog je započeo s pitanjem učenika Mehmeta: „*Nastavniče, jesu li nepoznanice i varijable isto?*" Ostatak dijaloga u potpunosti su strukturirali budući nastavnici. Rezultati analize ovih dijaloga sažeti su na Slici 6.

Slika 6

Objašnjenja koja su učenicima dali budući nastavnici o vezi između pojmova *nepoznanica* i *varijabla* bila su vrlo raznolika i brojna, a mogla bi dovesti do konceptualnih nesporazuma. Oni koji su pokušali definirati ove pojmove dali su samo djelomična objašnjenja (f = 13). Na primjer, nastavnik br. 20 definirao je nepoznanicu na sljedeći način: „*Mehmete, nepoznanica je ono što u jednadžbama ne znamo i što moramo izračunati u algebarskom izrazu.*"

Dok ovo objašnjenje s jedne strane djelomično objašnjava pojam, ono ne obuhvaća u potpunosti matematičku definiciju nepoznanice. Slično tome, nastavnik br. 23 izjavio je da je *x* nepoznanica samo ako ima jednu vrijednost, što bi moglo učenike navesti na krivi zaključak jer nepoznanica može imati višestruke vrijednosti. Nastavnik br. 2, u pokušaju da objasni pojam *varijabla*, izjavio je da *6x* nije algebarski izraz jer ne sadrži matematičku operaciju u sebi, što je netočno i što bi moglo zbuniti učenike.

U pokušaju artikulacije matematičkih definicija, budući su nastavnici pojmove *nepoznanica* i *varijabla* objašnjavali pomoću primjera. Međutim, mnogi od njih koristili su pogrešne primjere (f = 23), dok su drugi dali primjere koji ilustriraju samo jedan aspekt pojma (f = 14). Nastavnik br. 8, kao i nastavnik br. 23, ponudio je primjer, sugerirajući da nepoznanica ima samo jedno moguće rješenje.

Nastavnik br. 11 koristio je primjer: „*U , x je nepoznanica, dok je u izrazu x varijabla.*" Ovo objašnjenje sugerira da je x u kvadratnoj jednadžbi varijabla, što može dovesti do stvaranja pogrešnih predodžbi.

Samo je nekoliko budućih nastavnika pokušalo definirati pojmove bez primjera. No, te su definicije bile uglavnom netočne (f = 7). Objašnjenje nastavnika br. 25 pokazuje djelomično razumijevanje: „*Djeco, postoji mala razlika. Slova koristimo umjesto brojeva koje ne znamo, a ta se slova zovu varijable. Ako vidite da slovo x stoji samo, ono može predstavljati bilo koji broj pa ga zbog toga nazivamo varijablom.*"

U nekim su slučajevima budući nastavnici, npr. nastavnici br. 17 i 18, potpuno izbjegli dati objašnjenje, smatrajući da učenici već razlikuju te pojmove. Nastavnik br. 18 nastavio je dijalog sljedećom rečenicom: „*Učenici i razred kao cjelina pokazali su da razmišljaju točno pa nastavnik smatra da su naučili ova dva pojma.*"

Zbog toga što su dijalog o vezi između nepoznanica i varijabli u potpunosti strukturirali budući nastavnici i zbog toga što je on uključio ograničenu interakciju između učenika, imaginarni nastavnici dali su minimalne povratne informacije. Nastavnik br. 10

zaobišao je pitanje imaginarnoga učenika Mehmeta, uz puku napomenu učenicima da razliku između tih pojmova mogu pronaći u svojim bilježnicama. Slično tome, nastavnik br. 12 predložio je učenicima da dobro razmisle o odgovoru do sljedećega nastavnog sata, dok je nastavnik br. 22 zadatak zadao za domaću zadaću. Dio dijaloga koji je napisao nastavnik br. 22 ilustrira ovakav pristup: *„Zapišite to u bilježnice pa ćemo kroz rješenje proći sutra."*

Neki su budući nastavnici ipak uključili korektivnu povratnu informaciju u svoje dijaloge. Na primjer, razgovor između nastavnika br. 15 i imaginarnih učenika odvijao se ovako:

Nastavnik: „Ne, Mehmete, pojmovi nisu isti."
Razred: „Ali, nastavniče, i nepoznanicu i varijablu označavamo znakom x."
Nastavnik: „Da, to je točno."
Ela: „Dakle, to znači da su iste?"
Nastavnik: „Ne, naravno da ne."

U nekoliko su primjera budući nastavnici uspješno uklopili učeničku znatiželju u nastavni materijal (f = 13). Na primjer, nastavnik br. 3 pohvalio je Mehmeta zbog toga što je postavio pitanje i potaknuo razred na kolektivno razmišljanje te je obećao nagradu za točne odgovore. Nakon što je poslušao nekoliko učenika, nastavnik br. 3 zaključio je nastavni sat tako što je učenicima rekao da u bilježnice zapišu primjere nepoznanica i varijabli, iako nisu došli do konačne definicije.

Neki su budući nastavnici radije vodili učenike do razumijevanja pojmova postavljajući im pitanja, no nisu uvijek došli do konačnoga odgovora. Ipak, ovakav postupak ima veliku vrijednost u nastavnom kontekstu. Nastavnik br. 9 je, na primjer, odgovorio na Mehmetovo pitanje o razlici između ta dva pojma na sljedeći način: *„Mehmete, da te pitam nešto."* Zatim je nastavnik na ploču napisao izraz i te postavio pitanje: *„Što vidiš ovdje, Mehmete?"*

Iako su neki budući nastavnici imali poteškoće u pronalaženju odgovarajućih primjera, nekoliko ih je uspjelo u tome. Na primjer, dijalog nastavnika br. 21 pokazao je jasno razumijevanje matematičke veze između nepoznanica i varijabli:

Nastavnik: „Ako mogu pronaći specifičnu vrijednost za x, tada se radi o nepoznanici. Na primjer, u izrazu pronašao sam vrijednost x. Zato je x ovdje nepoznanica, u redu?"
Razred: „Da."
Nastavnik: „Idemo sada pogledati izraz . Nismo pronašli nikakvu specifičnu vrijednost za x pa ona onda može varirati. Zato je x u ovom izrazu varijabla. Razumijete?"

Osim toga, neki su budući nastavnici pomogli učenicima savjetom o tome kako lakše razumjeti ove pojmove. Na primjer, nastavnik br. 4 naglasio je da je razumijevanje važnije od učenja napamet, dok je nastavnik br. 20, u dijalogu u kojem se nije navelo jasno raspoznavanje razlike između ovih pojmova, naglasio da su refleksija i vježba od ključne važnosti za razumijevanje razlike između nepoznanica i varijabli.

Rasprava

U istraživanju se raspravlja o objašnjenjima u području algebre koja budući nastavnici daju učenicima u obliku polustrukturiranih imaginarnih dijaloga. Glavni je cilj bio procijeniti kvalitetu njihovih objašnjenja i prepoznati slabosti u pedagoškim pristupima ili pogrešne predodžbe u području algebre. Rezultati su pokazali znatno više slabih objašnjenja nego onih odgovarajućih, posebno u matematičkim operacijama zbrajanja u algebarskim izrazima, pretvaranju jednadžbi i algebarskih izraza u verbalni oblik, načinima izrade vizualnih prikaza te razumijevanju pojmova *varijabla* i *nepoznanica*. Uzevši u obzir činjenicu da su vještine algebarskoga načina razmišljanja kod budućih nastavnika od ključne važnosti za uspješno vođenje procesa učenja i poučavanja (Magiera i sur., 2013), u ovom istraživanju došlo se do zaključka da neadekvatna objašnjenja budućih nastavnika mogu biti povezana s njihovim nedostatnim vještinama algebarskoga razmišljanja. Kao rezultat toga, uočene su česte pogrešne predodžbe, neodgovarajuća povratna informacija, pogreške u odabiru primjera, neodgovarajuće definicije ključnih ideja i pojmova te objašnjenja koja nisu povezana s temom. Ovakvi rezultati naglašavaju da postoje konceptualne pogreške i praznine u znanju kod budućih nastavnika u algebarskoj domeni.

Konceptualne pogreške u koeficijentima, varijablama i verbalnim izrazima za algebarske izraze/jednadžbe bile su vrlo česte u objašnjenjima budućih nastavnika koji su sudjelovali u istraživanju, što je u skladu s ranije provedenim istraživanjima (Adıguzel i sur., 2018; Bucher, 2009; Ryan i Williams, 2007). Te su pogreške nastale zbog pogrešnih predodžbi o izvođenju računske operacije zbrajanje u algebarskim izrazima te pogrešnoga tumačenja slova koja predstavljaju nešto drugo osim brojeva. Do sličnih je rezultata došla Wille (2017) u svojem istraživanju o imaginarnim dijalozima budućih nastavnika u području algebre. Njezini rezultati upućuju na činjenicu da su konceptualne pogreške u algebri jako prisutne kod budućih nastavnika.

Kao što je već poznato, algebra je jezik (Usiskin, 1997), a pojmovi poput algebarskih izraza, jednadžbe, varijable i nepoznanice njezine su temeljne sastavnice. Algebarski izraz definira se kao matematički izraz koji uključuje i brojeve i slova, dok se jednadžba definira kao jednakost dvaju algebarskih izraza (Baykul, 2009). Nadalje, varijabla se odnosi na nešto što može imati ili ima višestruke vrijednosti, dok se nepoznanica definira kao nešto što ima stalnu vrijednost koja još nije određena (Schoenfeld i Arcavi, 1988). Vezano uz te definicije, nepotpune definicije i nerazlikovanje pojmova *varijabla* i *nepoznanica* također su bili učestali problemi uočeni u dijalozima. Budući nastavnici često nisu uspjeli precizno definirati te pojmove i često ih ni sami nisu razlikovali. To nerazumijevanje varijabli i nepoznanica istaknuto je i u ranijim istraživanjima o algebarskom znanju budućih nastavnika (Boz, 2004; Stephens, 2008; Tanisli i Kose, 2013). Nadalje, istraživanje je pokazalo da bi objašnjenja budućih nastavnika potencijalno mogla dovesti do stvaranja pogrešnih predodžbi kod učenika (Jones i Pratt, 2006; Vermeulen i Meyer, 2017). Na primjer, konceptualne pogreške koje su napravili budući nastavnici u prikazivanju algebarskih situacija, a koje nisu u skladu s principom jednakosti, mogle bi rezultirati pogrešnim shvaćanjem pojma ili znaka jednakosti.

Istraživanje je također izazvalo zabrinutost zbog toga što su se u imaginarnim dijalozima koje su napisali sudionici pojavile neodgovarajuće definicije ključnih termina, kao što su jednadžbe, algebarski izrazi i varijable, kao i uvođenje nepovezanih situacija bez razmišljanja o tome kako su učenici objasnili neke pojmove. Ovi problemi ukazuju na nedostatno sadržajno znanje sudionika, pogotovo u području algebre. Nadalje, kada budući nastavnici nisu imali dovoljno znanja o određenoj temi i nisu mogli dati jasno objašnjenje, često su koristili rečenice kojima su izbjegavali dati odgovor. Umjesto da se fokusiraju na matematičku situaciju, rekli bi nešto poput *„o tome ćemo reći nešto više kasnije"* ili *„ta se definicija već nalazi u udžbeniku"*. Isto tako, kada su alatu za prikupljanje podataka imaginarni učenici očekivali objašnjenja za algebarske izraze ili pojmove, neki su sudionici izjavili *„takva se pitanja mogu pojaviti na ispitu"* ili *„rješava se na ovaj način zato što se u pitanju zahtijeva primjena modela vage"*. Ovakve izjave pokazuju u kojoj je mjeri nedostatak sadržajnoga i pedagoškoga znanja prisutan u nastavničkim objašnjenjima. Rezultati su u skladu s rezultatima ranijih istraživanja u kojima se ispitivalo sadržajno i pedagoško znanje budućih nastavnika u području algebre (Asquith i sur., 2007; Black, 2007; İdil i Narlı, 2021; Tanisli i Kose, 2013). Štoviše, rezultati naglašavaju važnost sadržajnoag i pedagoškoga znanja za učenička postignuća, što je u skladu s ostalim istraživanjima (Carpenter i sur., 1988; Hill i sur., 2005). Stoga je iznimno važno pružiti odgovarajuću podršku budućim nastavnicima u razvoju sadržajnoga algebarskog znanja.

Drugi važan aspekt nastavničkih objašnjenja koji je istaknut u istraživanju jest povratna informacija. Budući nastavnici često su davali korektivne ili afirmativne povratne informacije koje nisu bile eksplanatorne prirode (npr. „*Ne, to nije točno*" ili „*Da, to je točno*"), tj. u kojima nije bila objašnjena matematička podloga i koje nisu sadržavale smislena objašnjenja. Međutim, kvalitetna interakcija između nastavnika i učenika zahtijeva detaljna i smislena objašnjenja, kao i interaktivni i stalni dijalog (Piccolo i sur., 2008). U ovoj fazi, način na koji nastavnici postavljaju pitanja postaje jako važan. Poznato je da nastavnici postavljaju pitanja koja počinju riječima *što*, *tko* i *zašto*, jer ona igraju važnu ulogu u interakciji s učenicima (Li i Huang, 2013) i važan su alat za određivanje dubine učeničkoga razumijevanja u nastavi matematike (Moyer i Milewicz, 2002). Rezultati mnogih studija koje se mogu pronaći u literaturi o nastavničkim strategijama postavljanja pitanja pokazuju da učenički odgovori mogu varirati ovisno o tipu postavljenoga pitanja, što vodi različitim dijalozima u razredu (Klinzing i sur., 1985; Brock, 1986; Singto, 1995; Boaler i Brodie, 2004; Furtak i sur., 2018; Paoletti i sur., 2018). Istraživanje je pokazalo da su neki sudionici demonstrirali vještinu postavljanja stimulativnih pitanja, što je kod učenika potaknulo matematički način razmišljanja i dalo im poticaj da se i sami uključe u postavljanje pitanja. Taj rezultat analize imaginarnih dijaloga naglašava bitnu sposobnost budućih nastavnika da postavljaju stimulativna pitanja, što kod učenika jača matematički način razmišljanja i promiče njihovo propitivanje algebarskih pojmova i situacija. Ranija istraživanja idu u prilog mišljenju da takva pitanja stimuliraju matematički način razmišljanja i pozitivno utječu na dijaloge tijekom nastavnoga procesa (Mason, 2000).

Dok su objašnjavali algebarske situacije, budući su se nastavnici često oslanjali na primjere. Neki su koristili neodgovarajuće analogije, poput *„ne možemo zbrajati jabuke i kruške"*, kako bi bolje ilustrirali izraz . Iako su pokušavali konkretno objasniti pojam, ovakav pristup, kako smatra Booth (1988), ne samo da kod učenika stvara pogrešne predodžbe o značenju varijabli, nego ga učenici mogu koristiti i kao objašnjenje za netočno pojednostavljene pojmove. Tirosh i sur. (1998) tvrde da korištenje konkretnih predmeta u nejasnim situacijama može donijeti više štete nego koristi te ističu potrebu pažljivoga odabira primjera. Kako su primjeri glavni alat za komunikaciju i matematički diskurs u procesu poučavanja (Leinhardt, 2001), može ih se koristiti pri objašnjavanju pojmova, ilustriranje metoda, uspostavljanje veza ili demonstraciju dokaza (Bills i sur., 2006). Međutim, kada se radi o korištenju primjera u nastavničkim objašnjenjima, budući nastavnici često su imali problema u pronalaženju jasnih i relevantnih matematičkih primjera vezano uz algebarsku situaciju. Odabir odgovarajućih primjera kompleksan je zadatak na koji utječu različiti faktori, uključujući i nastavne ciljeve i nastavnikovo razumijevanje učeničkih predodžbi i tendencija (Bills i sur., 2006), a može predstavljati nastavnicima veliki izazov. Ranija su istraživanja također pokazala da i nastavnici i budući nastavnici često biraju slabe i netočne primjere (Avcu, 2014; Duran i Kaplan, 2016; Gokbulut i Ubuz, 2013; Rowland i sur., 2003). Štoviše, Saglam-Kaya (2017) je došao do saznanja da budući nastavnici matematike imaju percepcije o primjerima koji se uglavnom sastoje od pitanja i zadataka temeljenih na postupcima, a jako malo koriste suprotne primjere, primjere koji su u suprotnosti s teorijom ili primjere koji zahtijevaju kompleksne kognitivne procese. Nažalost, ograničeno znanje nastavnika o primjeni učinkovitih primjera učenicima onemogućava razumijevanje pojmova na naprednijoj razini (Zaslavsky i Peled, 1996).

Bitan rezultat ovoga istraživanja je nesposobnost mnogih budućih nastavnika da dođu do matematički smislenoga zaključka u imaginarnim dijalozima. Na primjer, kada se radi o izrazima kao što je i o tome mogu li oni biti pojednostavljeni, u mnogim dijalozima taj problem uopće nije spomenut. Ta je situacija zabrinjavajuća kada se radi o dijalozima u razrednom kontekstu jer bi dijalog trebao ići prema koherentnom ili alternativnom ishodu (Wells i Arauz, 2006). Nesposobnost budućih nastavnika da dođu do matematički smislenoga rješenja proizlazi možda iz njihovoga nedostatnog sadržajnog znanja i nedovoljnoga iskustva u poučavanju algebre. Istraživanja pokazuju da postoji jaka veza između objašnjenja koja učenicima daju budući nastavnici i njihovoga poznavanja određenoga područja (Charalambous i sur., 2011; Leinhardt, 2001). K tome, za uspješno objašnjavanje potrebno je čvrsto znaje (Resnick i sur., 2018), a znatan broj sudionika u ovom istraživanju pokazao je nedovoljno razvijenu vještinu navođenja primjera uz smisleno objašnjenje u odgovaranju na učenička pitanja, komentiranju njihovih odgovora i različitih metoda rješavanja zadataka. Slični rezultati mogu se vidjeti u Hillovom (2007) istraživanju, u kojemu je analizirano sadržajno znanje nastavnika matematike u višim razredima osnovne škole.

Iako je istraživanje pokazalo neadekvatna objašnjenja koja su učenicima dali budući nastavnici, također su uočeni i primjeri odgovarajućih objašnjenja. Imaginarni matematički dijalozi korišteni u istraživanju imali su za cilj ispitati sposobnost budućih nastavnika da koriste relevantne primjere iz svakodnevnoga života u svojim objašnjenjima, uzimajući algebarski način razmišljanja kao mogućnost rješavanja problema u svakodnevnom životu (Kaya i Keşan, 2014). Neki su budući nastavnici dali efikasna objašnjenja tako što su prikazali relevantne primjere povezane s algebarskom situacijom i ponekad uključili i dodatne primjere iz stvarnoga života kako bi kod učenika razvili dublje razumijevanje. Međutim, zabrinjava činjenica da je samo nekoliko nastavnika moglo povezati algebarske pojmove sa svakodnevnim životom, unatoč važnosti takvih poveznica u nastavi algebre (Witzel i sur., 2001).

Kada učenici imaju poteškoća u razumijevanju nekoga pojma, često od nastavnika traže dodatna pojašnjenja, primjere, ponavljanje ili demonstraciju (Darling, 1989). Uspostavljanje veze između učeničke znatiželje i nastavnoga sadržaja u tim je slučajevima od ključne važnosti. Štoviše, povezivanje nastavnoga sadržaja s iskustvima i idejama učenika povećava njihovu interakciju sa sadržajem (Corso i sur., 2013). U istraživanju su neki budući nastavnici uspješno povezali učeničku radoznalost i sadržaj svih četiriju analiziranih situacija. Također su mogli dati dosljedna objašnjenja koja su bila izravno povezana s algebarskom temom, ne koristeći nerelevantna ili netočna objašnjenja. Nadalje, neki su budući nastavnici koristili pitanja pomoću kojih su učenike navodili na razmišljanje. Pitanja imaju važnu ulogu u procesu učenja i poučavanja (Pearson i West, 1991) jer učinkovita pitanja potiču učenike na razmišljanje i razvijaju njihovo kritičko mišljenje (Shahrill i Clarke, 2014). Istraživanja pokazuju da ovakve vrste pitanja stvaraju prilike za učenje i razmišljanje (Bozkurt i Polat, 2018) te vode točnijim učeničkim objašnjenjima (Franke i sur., 2009). Sposobnost da se na učenička pitanja koja počinju sa „Zašto?" odgovori adekvatno i da se postave pitanja koja učenike vode odgovoru odraz je specifičnoga sadržajnog znanja (Ball i sur., 2008) te je ohrabrujuće vidjeti da su neki budući nastavnici koji su sudjelovali u ovom istraživanju uspješno koristili takva pitanja.

Implikacije

Vrijednost ovoga istraživanja jest u tome što ono specifično analizira pedagoške pristupe koje budući nastavnici koriste u nastavi algebre kroz polustrukturirane imaginarne dijaloge, metodu koja može pružiti uvid u situaciju na nov, svjež način. Dok su u prethodnim istraživanjima utvrđene praznine u općem znanju, naše istraživanje je jedinstveno po tome što se u njemu analiziraju objašnjenja koja daju budući nastavnici kroz četiri različita algebarska pojma: zbrajanje u algebarskim izrazima, verbalizaciju algebarskih oblika, rješavanje jednadžbi primjenom modela vage te razlikovanje varijabli i nepoznanica. Naše istraživanje ne daje puku potvrdu rezultata prethodno provedenih istraživanja, nego otkriva dubinu i trajnost specifičnih predrasuda, poput onih koje se odnose na koeficijente i varijable, pružajući uvid u ono što treba popraviti u studijskim programima za izobrazbu nastavnika.

Korištenje imaginarnih dijaloga lako je primjenjiv način za procjenu sposobnosti budućih nastavnika za vođenje nastavnoga procesa u kontroliranim uvjetima. Takav bi pristup mogao biti od velike koristi u studijskim programima za izobrazbu nastavnika jer od njih traži refleksiju o poteškoćama i pogrešnim predodžbama učenika prije nego što uistinu počnu raditi u nastavi. Metodički doprinos našega istraživanja jest u korištenju zadataka koji se temelje na scenarijima pomoću kojih se mogu rasvijetliti pogreške u nastavničkim objašnjenjima. To ide u prilog zadatcima u mikropoučavanju jer se pomoću njih gradi sadržajno i pedagoško znanje budućih nastavnika u području algebre.

Ograničenja

Ovo istraživanje ima nekoliko ograničenja koja mogu utjecati na opseg i generalizaciju rezultata. Kao prvo, upotreba polustrukturiranih imaginarnih dijaloga daje kontroliranu perspektivu o pedagoškim vještinama budućih nastavnika, ali ne bavi se interaktivnom dinamikom razrednoga okružja. Ovo bi ograničenje moglo umanjiti valjanost okružja jer bi autentične razredne interakcije mogle otkriti dodatne nastavne izazove i pristupe. Uz to, fokusiranost na nastavnička objašnjenja usko vezana uz područje algebre mogla bi ograničiti primjenjivost rezultata na druge matematičke domene. Na kraju, budući da je analiza kvalitativna, ona se temelji na interpretaciji rezultata koja bi se mogla triangulirati s dodatnim podatcima, kao što su opažanja u razredu ili intervjui sa sudionicima. U budućim bi se istraživanjima ta ograničenja mogla analizirati uvođenjem mikropoučavanja ili opservacijskih studija kako bi se ispitao način na koji budući nastavnici svoje nastavne strategije prilagođavaju i usavršavaju u stvarnom obrazovnom kontekstu.

Zaključak

Kao zaključak, ovo je istraživanje naglasilo zabrinjavajuće stanje vezano uz objašnjenja koja budući nastavnici daju učenicima u području algebre. Veliki broj neadekvatnih objašnjenja koja mogu voditi konceptualnim pogreškama i nejasni primjeri izazvali su zabrinutost o sadržajnom znanju budućih nastavnika, pogotovu u području algebre. Kako bi se poradilo na rješavanju tih problema, studijski programi za izobrazbu nastavnika trebali bi primjenjivati dijaloge u stvarnom razrednom okružju, pomoću čega nastavnici mogu steći praktični uvid i usavršiti način na koji daju objašnjenja.