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ALPHA DECAY WIDTHS AND THEIR SYSTEMATICS IN LEAD AND SUB-LEAD NUCLEI

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The method of non-local barrier theory is applied to the consideration of α-decay widths of lead and sub-lead nuclei. Penetration factor P_L is calculated for a barrier which is taken to be coulombic superposed by non-local α-nucleus potential and semiempirical values of reduced widths (hF_L) calculated from the relationship $F_L = \lambda_L / P_L$. Some new systematics of reduced widths are given and results are discussed.

1. Introduction

Theoretical interest in α-decay in sub-lead region is of recent origin. In the sub-lead region α-activity is found to compete with β-decay and electron capture and this mode of decay becomes discernible for highly neutron-deficient nuclei. Early works in this field were done by Rasmussen, Thompson, Ghiorso and Seaborg[1-2]. With advanced techniques for investigating nuclei away from stability, there has been extensive experimental investigation of α-decay in the transitional region[3-7], but theoretical interpretation of the experimental data proves to be difficult if the barrier is taken to be purely coulombic or that superposed by static α-nucleus potential. For example it has been reported by Hornshoj et al.[3] that the neutron-deficient Pb-isotopes do not follow the usual trend of increasing hF_L with decreasing neutron number N.

Semiempirical evaluation of reduced width (hF_L) from the relationship,

$$\lambda_L = F_L P_L \tag{1}$$

where λ_L is partial decay constant for the mode of decay involving α-angular momentum L, P_L being corresponding penetrability factor and internal transition probability, respectively, is beset with the difficulty that both P_L and F_L are not measurable and one has to make assumption on either of them to elicit information about the other. However, it is reasonable to make assumption about P_L because F_L is much more uncertain as it involves nuclear structure parameters, while the nature of the barrier is inferable from earlier studies[8-10]. The basis of this approach lies in the fact that non-local barrier penetration theory largely reproduces the experimental data in the rare-earth[8], trans-lead[9] and trans-uranium region[10], with F_L/F_O values within reasonable range[8], while on the other hand, it can be seen that coulomb barrier or that superposed by static α-nucleus potential yields values of F_L/F_O incompatibly low[11] compared to the findings of other sources such as studies of α-spectroscopic amplitudes[12]. Further it has been noted by other authors[12-14] that the relative motion between the α-particle and product nucleus obeys strongly non-local equation. Also, in other areas of nuclear physics, such as narrow resonance, it has been noticed that local potential can not accomodate very narrow resonance [15,16] and utility of non-local potential in its study has been demonstrated[17]. However, question may arise that scattering phenomena are usually treated with static potential and prescriptions have been made to obtain equivalent local potential for non-local potential in scattering[18] to avoid the mathematical difficulties involved in integro-differential Schrödinger equation with non-local potential. But it may be mentioned that situation involved in scattering is much different than in α-decay with different boundary conditions, energy ranges and additional complications arising from internal transition probability. It is also to be noted that a static potential is of limited applicability and widely varying parameters are needed to fit scattering data. Hence, there is no a priori reason to believe that non-local effects even if small in scattering should also be so in α-decay.

In view of these and the fact that WKB solution of integro-differential equation of α-decay problem in one-body formalism is found to be good approximation, in this paper the correlation between hF_L and N in neutron deficient lead, mercury, platinum and osmium nucleids has been studied in an alternative approach by using a modified barrier which involves electrostatic potential superimposed by non--local α-nucleus potential which has been found useful as mentioned earlier. P_L is calculated for such potential using equation (2). The calculated values of P_L and F_L are listed in Table 1 in column (7) and (9), respectively. For comparisonal purpose, hF_L with static barrier are also listed in the same table in col. (8). However there is no way to ascertain the relative superiority of reduced widths obtained by these two formalisms, namely static and non-local barrier, until purely theoretical calculations are made for absolute decay rates. Recently some progress have been made in this line[19-20]. However, this problem still remains unresolved quantitatively, and its discussion is avoided for reason of space.

The nature of variation of hF_L with N for a series of isotopes is depicted in Fig. 1. Secondly, the correlation between hF_L and ΔE is taken to be the energy difference ($E - E_{min}$) irrespective of proton and neutron number is found and shown in Fig. 2 and discussed in Section 4.

TABLE 1.

Nucleus	Spin and Parity		L of alpha	Partial α-half life/s	Decay energy/MeV	R_t/fm	P_L	$\hbar F_L$/MeV	
	Parent	Product						Static	Non-local
(1)	(2)		(3)	(4)	(5)	(6)	(7)	(8)	(9)
$^{82}_{186}\mathrm{Pb}^{a)}$	0^+	0^+	0	329.16	6.458	8.54	38.75(−27)	0.001	0.002
$^{82}_{188}\mathrm{Pb}^{a)}$	0^+	0^+	0	742.42	6.120	8.55	17.78(−23)	0.010	0.021
$^{82}_{190}\mathrm{Pb}^{a)}$	0^+	0^+	0	8.0(+3)	5.696	8.56	27.73(−25)	0.068	0.129
$^{82}_{192}\mathrm{Pb}^{b)}$	0^+	0^+	0	36.80(+5)	5.220	85.57	85.33(−28)	0.047	0.091
$^{80}_{180}\mathrm{Hg}^{a)}$	0^+	0^+	0	7.25	6.257	8.47	38.68(−22)	0.054	0.102
$^{80}_{182}\mathrm{Hg}^{a)}$	0^+	0^+	0	125.55	5.996	8.48	35.43(−23)	0.034	0.064
$^{80}_{184}\mathrm{Hg}^{f)}$	0^+	0^+	0	2353.84	5.783	8.51	44.46(−24)	0.014	0.027
$^{80}_{186}\mathrm{Hg}^{f)}$	0^+	0^+	0	5.25(+5)	5.320	8.52	28.27(−26)	0.010	0.019
$^{80}_{170}\mathrm{Hg}^{d)}$	0^+	0^+	0	6.0(−3)	6.605	8.36	40.89(−20)	0.633	1.166
$^{78}_{172}\mathrm{Pt}^{d)}$	0^+	0^+	0	0.09	6.459	8.38	13.11(−20)	0.130	0.243
$^{78}_{174}\mathrm{Pt}^{d)}$	0^+	0^+	0	0.90	6.169	8.40	11.14(−21)	0.154	0.286
$^{78}_{178}\mathrm{Pt}^{g)}$	0^+	0^+	0	264.0	5.582	8.43	39.68(−24)	0.148	0.274
$^{78}_{180}\mathrm{Pt}^{f)}$	0^+	0^+	0	173.0(+4)	5.376	8.46	44.59(−25)	0.019	0.037
$^{78}_{182}\mathrm{Pt}^{f)}$	0^+	0^+	0	7.80(+5)	5.034	8.47	81.59(−27)	0.023	0.045
$^{78}_{184}\mathrm{Pt}^{f)}$	0^+	0^+	0	1.03(+8)	4.692	8.49	93.91(−29)	0.015	0.029
$^{78}_{186}\mathrm{Pt}^{f)}$	0^+	0^+	0	5.14(+9)	4.418	8.51	18.36(−30)	0.018	0.030
$^{78}_{188}\mathrm{Pt}^{g)}$	0^+	0^+	0	3.15(+12)	4.00	8.53	19.14(−33)	0.024	0.047
$^{76}_{168}\mathrm{Os}^{d)}$	0^+	0^+	0	4.48	5.814	8.33	25.69(−22)	0.133	0.248
$^{76}_{170}\mathrm{Os}^{d)}$	0^+	0^+	0	59.16	5.535	8.34	16.66(−23)	0.156	0.291

Figures in bracket in column (4) and (7) denote power of 10.

a) Ref. 3 c) Ref. 6 e) Ref. 32 g) Ref. 34
b) Ref. 31 d) Ref. 7 f) Ref. 33

P_L and reduced widths for non-local and static barrier for ground to ground state transitions.

2. *Penetrability factor*

Expression for penetrability factor for non-local barrier has already been derived[8]. However, since all the transitions considered here involve even L, expression for P_L reduces to:

$$P_L = \exp\left(-2(2M)^{1/2}/\hbar \int\limits_{R_t}^{2Ze^2/E_\alpha} \left[2\,(Z-2)\,e^2/r - V_0 f(r)\left\{\frac{1}{2}(1+\mathrm{erf}(x))\right\} - \right.\right.$$

$$\left.\left. - E_\alpha + \hbar^2/2M\,(L(L+1)/r^2)\,(1+\eta(x) f(r))\right]^{1/2} \mathrm{d}r\right), \qquad (2)$$

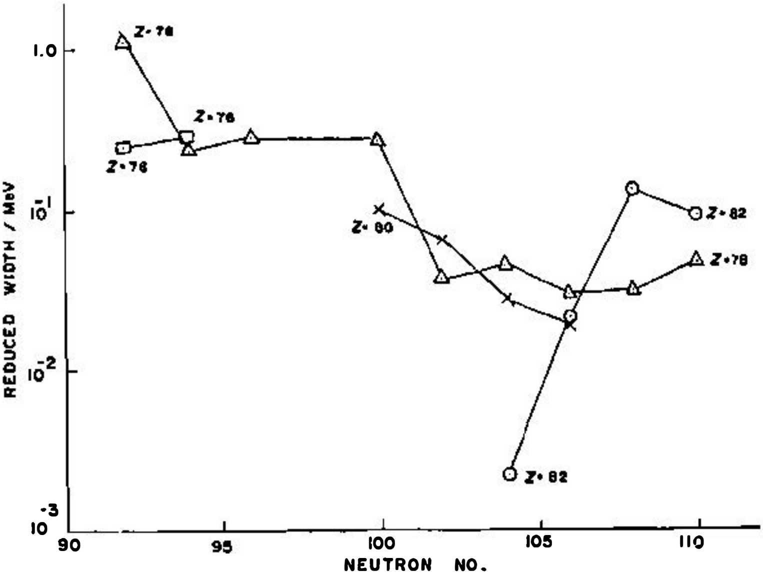

Fig. 1. Systematics of hF_L as a function of neutron number N of parent. Points for Pb, Hg, Pt and Os isotopes have been represented by O, X, Δ and □, respectively.

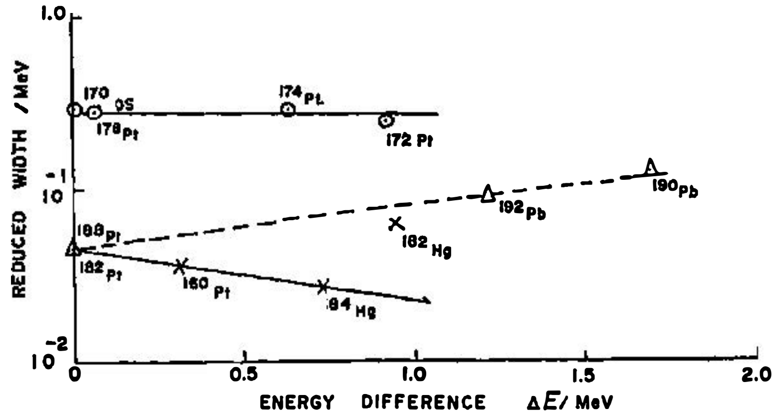

Fig. 2. Systematics of hF_L in terms of shell model configurations. Neutron orbitals $f_{7/2}$, $f_{5/2}$ and $p_{3/2}$ have been represented by O, Δ and X, respectively. Reduced widths of ^{188}Pt and ^{282}Pt are very close in value and hence the points overlap in the figure.

where M is the reduced mass of the α-particle, R_i is the inner turning point, $(Z - 2)$ is the charge number of the product, E_α is α-particle energy corrected for recoil of the product and electron screening, $V_0 F(r)$ is the static part of α-nucleus potential[21],

$$\eta(x) = (M b^2/2\hbar^2)\, V_0\, [1/2\,(1 + \mathrm{erf}(x))]$$

$$x = (r - R)/b,$$

b being the range of non-locality.

3. *Results*

The integral (2) has been machine computed on a DEC-20 Computer using the method described in Ref. 8. Sources of decay data are given in appropriate places.

For systematics of hF_L, the nuclei with same shell model state have been placed in a group. The relevant data for ground to ground state transitions are listed in Table 2.

TABLE 2.

Nucleus	Neutron number	Neutron orbital	E/MeV	ΔE/MeV	hF_L/MeV
^{170}Os	94		5.535	0.0	0.291
^{178}Pt	100	$f_{7/2}$	5.582	0.047	0.274
^{174}Pt	96		6.169	0.634	0.286
^{172}Pt	94		6.459	0.924	0.243
^{188}Pt	110		4.00	0.0	0.047
^{192}Pb	110	$f_{5/2}$	5.220	1.220	0.091
^{190}Pb	188		5.696	1.696	0.129
^{182}Pt	104		5.054	0.0	0.045
^{190}Pt	102	$p_{3/2}$	5.376	0.322	0.037
^{184}Hg	184		5.783	0.729	0.027
^{182}Hg	102		5.996	0.942	0.064

Variation of reduced widths with energy difference $\Delta E = (E - E_{min})$ for nuclei with the same shell model configurations.

4. *Discussion*

Using the results presented in column (9) of Table 1 the nature of variation of hF_L with N for ground to ground state transitions is represented in Fig. 1.

It may be seen from Fig. 1 that almost all the nuclei considered here in general follow the general trend of decreasing hF_L with increasing N. It is expected that hF_L for Pb isotopes should be low corresponding to a major closed shell at $Z = 82$ and the values on Table 1 are compatible to this expectation. Also, it is seen that hF_L systematically increases as one goes down from $Z = 82$ to $Z = 76$, i. e. away from the closed shell at $Z = 82$. Individual cases are discussed below:

Lead ($Z = 82$) isotopes: It is worthwhile to note that for the isotopes ^{192}Pb and ^{190}Pb, the trend that hF_L should decrease with increasing neutron number is reproduced while the trend was just the reverse in the study of Hornshoj at al.[3]. However, this trend is broken for ^{186}Pb and ^{188}Pb. It may be noted that hF_L for these nuclei have been calculated by using data which were obtained by estimation from the cross sections of their productions from heavy ion induced reactions rather than direct experiment[3]. Toth et al.[5] have shown that for ^{188}Pb this anomalous behaviour is probably due to incorrect data.

There has been speculation that the anomalous behavior of Pb-isotopes is caused by change in nuclear structure either in Pb or Hg isotopes. It has been pointed out that even-A Hg isotopes are not strongly deformed[20,35]. It seems unlikely that shape forbiddeness may be one plausible factor for the anomalous behaviour of Pb-isotopes. It is known that even deformed radium isotopes undergo decay to nearly spherical radon isotopes almost unhidered[3]. Further, equilibrium deformations of Hg and Pb in ground states are the same[24]. For the nuclei which have either of their nucleon number near a closed shell, there is a large scale competition between rotational and vibrational states and with configuration mixing between them in general. Configuration mixing in interacting boson model has been applied for the isotopes of Hg with $A = 180 - 184$ by Duval and Barrel[25]. Tonozuka and Harada[26] made calculations with strongly isospin dependent pairing interaction but that turned out to be inconclusive. Regarding explanation of anomalous behaviour of ^{186}Pb, it may be said that no theory yet could reconcile this.

Mercury ($Z = 80$) isotopes: for the nuclei considered here, reduced widths systematically decreases with increasing N. Reduced widths of ^{186}Hg is somewhat smaller and this is perhaps due to the fact that α-decay in it connects two different neutron levels ($f_{5/2} \rightarrow p_{3/2}$).

Platinum ($Z = 78$) isotopes: The isotopes considered here follow the general trend of decreasing reduced width with increasing N. ^{184}Pt has somewhat smaller reduced width and this is perhaps, here too, because α-decay connects two different neutron levels.

Osmium ($Z = 76$) isotopes: The two isotopes ^{168,170}Os considered here have almost same reduced widths.

Systematics of reduced widths: Usually systematics of reduced widths are given in terms of reduced widths — neutron number for a series of isotopes. Here a systematics is attempted from the correlation among the nuclei and it is found that if the nuclei having same shell model configurations for the corresponding nuclei with same Z and N are grouped together, a trend is obtained. However, question may arise regarding the applicability of shell model states in this region where most of the nuclei are not spherical. Regarding this it may be stated that these assignements do not affect the computed reduced widths because it is known that even-even nuclei have states 0^+, 2^+, 4^+, etc, and less number of rotational states than odd nuclei. Besides, even-even nuclei in the neighbourhood of magic numbers are more likely to be spherical. Further, it has been shown by Lemmer[7] that the introduction of a short range non-local potential accommodates to a large extent the effect of nuclear deformation. Also, Mang[28] has calculated reduced widths of polonium and astatine isotopes from the overlap of shell model wave functions. Based on these considerations, a systematics of reduced widths has been proposed for the considered nuclei although shell model states may not be proper description for them and Nillson components should be considered. Nevertheless, it is found that the reduced widths of the nuclei can be correlated by considering shell model configuration. For this the values of hF_L given in Table 1 are plotted against ΔE and shown in Fig. 2. Since the shell model states of protons of all the nuclei considered here are same, the points are denoted by neutron levels only. ^{186}Pb and ^{188}Pb have not been included because they show anomalously low values of hF_L as already discussed.

Now returning to Fig. 2 it may be seen that nuclei ^{170}Os, ^{172,174,178}Pt having $94 \leq N \leq 100$ and same neutron orbital $f_{3/2}$[20] have almost same reduced widths though they differ largely in decay energy and lies on a distinct line. Similarly the nuclei ^{180,182}Pt and ^{184}Hg having $102 \leq N \leq 104$ and neutron orbital $p_{3/2}$[30] have comparable reduced widths and can be accommodated in another distinct line. However ^{182}Hg having the same neutron state $p_{3/2}$ shows considerable scatter. Likewise ^{188}Pt, ^{190}Pb and ^{192}Pb having $108 \leq N \leq 110$ and neutron orbital $f_{5/2}$ can be represented on yet another line. Reduced widths of ^{184}Pt and ^{186}Hg are somewhat smaller which have already been discussed and not shown in the figure.

It may also be seen from the same Fig. 2 that the lines representing nuclei with neutron orbital $p_{3/2}$ and $f_{5/2}$ show slight slant but the line representing the nuclei with higher spin state of neutron ($f_{7/2}$) is almost horizontal reflecting L--dependence of non-locality.

5. *Conclusion*

i) It is found that the reduced widths with non-local barrier are greater than those with static barrier by a factor of about two.

ii) Reduced widths are comparable for nuclei with same shell model configurations irrespective of charge number and over wide range of decay energy.

iii) Expect for ^{186}Pb the trend of decreasing reduced width with increasing neutron number is reproduced.

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ŠIRINE ALFA RASPADA I NJIHOVA SISTEMATIKA U PODRUČJU OLOVA

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Metoda prolaska nelokalne barijere primijenjena je na proučavanje širina alfa raspada u jezgrama oko olova. Izračunata je penetrabilnost za barijeru sastavljenu od Coulombskog dijela superponiranog na nelokalni potencijal alfa čestica-jezgra, kao i semiempirička vrijednost reducirane širine izračunata iz relacije $F_L = \lambda_L / P_L$. Dana je nova sistematika reduciranih širina i diskusija rezultata.