

TRANSITION FORM-FACTORS FOR THE INELASTICALLY
SCATTERED LIGHT IONS FROM ^{12}C

SAYED A. E. KHALLAF and HUSSIEN M. ALI AHMED

Department of Physics, Faculty of Science, Asswan, Egypt

Received 20 November 1984

Revised manuscript received 28 January 1985

UDC 539.17

Original scientific paper

The transition form-factors for the inelastic scattering of ^3He , α -particles and ^7Li ions to the first excited state (2^+ , 4.44 MeV) in ^{12}C are calculated. The radial dependence of the transition density for the 2^- excitation of ^{12}C is chosen to be either of standard form for a mass vibration or of Tassie hydrodynamic form. The resulting form-factors are compared with deformed optical potentials where good fits are obtained. The need to calculate the cross sections using the present form-factors is stressed.

1. Introduction

Studies of inelastic scattering to collective target states with proton and light ion projectiles have most often used a transition form-factor $A(r)$ obtained by deforming the optical potential $V(r)$ so that $A(r) \sim \frac{dV(r)}{dr}$. Alternatively, the approach of deforming the target density directly in a folding model has also been employed with considerable success^{1,2}. The later method is an extension of the double-folding model of the optical potential for elastic scattering where the transition form-factor is obtained by folding an effective nucleon-nucleon interaction into the density distribution of the projectile and the transition density of the target nucleus which is obtained from structure calculations. The resulting form-factor then may be used to calculate the inelastic scattering, e. g., in DWBA.

Recently, the hydrodynamic model of Tassie has been widely employed to derive the radial part of the transition densities used in the analysis of the inelastic scattering of α -particles³⁾, ${}^6, {}^7\text{Li}$ ⁴⁾, ${}^{12}\text{C}$ and ${}^{16, 18}\text{O}$ ⁵⁾ from a variety of targets. The Tassie model has been also used in the analysis of the inelastic electron scattering form-factors for ${}^6\text{Li}$, ${}^{60}\text{Ni}$ and ${}^{114}\text{Cd}$ ⁶⁾, ${}^{58}\text{Ni}$, ${}^{60}\text{Ni}$ and ${}^{62}\text{Ni}$ ⁷⁾ and ${}^{206}\text{Pb}$, ${}^{207}\text{Pb}$, ${}^{208}\text{Pb}$ and ${}^{209}\text{Bi}$ ⁸⁾ at low momentum transfers where good fits to the experimental data were obtained.

The main purpose of the present work is to calculate the transition form-factors for the inelastic scattering of ${}^3\text{He}$, α -particles and ${}^7\text{Li}$ ions to the (2^+ , 4.44 MeV) state in ${}^{12}\text{C}$ and to test the applicability of the folding model to transition form-factors in this range of mass numbers. The Tassie or hydrodynamical model is used for the transition density for ${}^{12}\text{C}$ nucleus. The resulting form-factors are compared with deformed Woods-Saxon optical potentials.

2. Theory

For the inelastic transition of multipolarity L , the form-factor for a nucleus-nucleus inelastic scattering may be written as^{1, 9)}

$$A_L(r) = \beta_L R F_L(r) \quad (1)$$

where $\beta_L R$ is the deformation length and $F_L(r)$ is the intrinsic form-factor. In the double-folding model, the intrinsic-form factor is expressed as:

$$F_L(r) = \int \rho_p(r_2) \rho_{tr}(r_1) Y_{L,M}(\hat{r}_1) V_{N-N}(|\vec{r} - \vec{r}_1 + \vec{r}_2|) d\vec{r}_1 d\vec{r}_2. \quad (2)$$

Here $\rho_p(r_2)$ is the density distribution of the projectile (assumed spherical), $\rho_{tr}(r_1)$ is the transition density for the target nucleus and $V_{N-N}(r)$ is an effective nucleon-nucleon interaction. In the single-folding approach for the α -particle inelastic scattering, an alpha-nucleon interaction $V_{\alpha-N}(r)$ is folded into the transition density of the target nucleus, i. e.,

$$F_L(r) = \int \rho_{tr}(r_1) Y_{L,M}(\hat{r}_1) V_{\alpha-N}(|\vec{r} - \vec{r}_1|) d\vec{r}_1. \quad (3)$$

In the present work an effective nucleon-nucleon interaction $V_{N-N}(r)$ and an alpha-nucleon interaction $V_{\alpha-N}(r)$ of the following forms have been used^{10, 11)}:

$$V_{N-N}(r) = -22.332 \exp(-0.46 r^2) \text{ MeV} \quad (4)$$

and

$$V_{\alpha-N}(r) = -37 \exp(-0.25 r^2) \text{ MeV}. \quad (5)$$

The radial dependence of the transition density $\rho_{tr}(r)$ for the 2^+ excitation of ${}^{12}\text{C}$ is chosen to be either of standard form for a mass vibration,

$$\rho'_{tr}(r) = C_s \frac{d\rho(r)}{dr} \quad (5)$$

or of Tassie form

$$\varrho_{tr}^g(r) = C_T r^{L-1} \frac{d\rho(r)}{dr} \tag{6}$$

where $\rho(r)$ is the ground-state density distribution for the target nucleus. The normalizing constants C_S or C_T are determined by assuming that the proton transition density is (Z/A) times the mass transition density and then choosing C_S or C_T to give the measured value of $B(E2)$ for ^{12}C nucleus³⁾, i. e:

$$\int_0^\infty \varrho_{tr}^g(r) r^{L+2} dr = \frac{A}{Ze} [B(EL)]^{1/2} \tag{7}$$

where A and Z are the mass and the charge numbers, of the target nucleus, respectively.

Harmonic oscillator forms have been chosen for the spherical densities of $^{12}\text{C}^{12)}$ and $^7\text{Li}^{13)}$

$$\varrho_{^{12}\text{C}}(r) = \varrho_0 \left(1 + \frac{\nu}{a^2} r^2 \right) \exp(-r^2/a^2) \tag{8}$$

with $a = 1.635$ fm and $\nu = 4/3$ and

$$\varrho_{^7\text{Li}}(r) = (A + Br^2) \exp(-a^2 r^2) \tag{9}$$

with $A = 0.1647$ fm⁻³, $B = 0.03086$ fm⁻¹ and $a = 0.612$ fm⁻¹.

The spherical densities for helions and alpha particles have been obtained using the ground state wave functions of ^3He and an alpha particle of the following forms^{14,15)}:

$$\psi_{^3\text{He}} = N_h \exp\left(-\frac{1}{2} \gamma \sum_{i=1}^3 \varrho_i^2\right) \tag{10}$$

with $\gamma = 0.367$ fm⁻² and

$$\psi_\alpha = N_\alpha \exp\left(-\frac{1}{2} \alpha_0 \sum_{j=1}^4 \varrho_j^2\right) \tag{11}$$

with $\alpha_0 = 0.415$ fm⁻², where the notations of the above parameters are given in Refs. 14 and 15. The derived nuclear matter distributions for helions and alpha particles are in the following forms:

$$\varrho_{^3\text{He}}(r) = 3 \left(\frac{3\gamma}{2\pi}\right)^{3/2} \exp\left(\frac{-3\gamma}{2} r^2\right) \tag{12}$$

and

$$\varrho_\alpha(r) = 4 \left(\frac{4\alpha_0}{3\pi}\right)^{3/2} \exp\left(\frac{-4}{3} \alpha_0 r^2\right). \tag{13}$$

3. Results and discussion

The real parts of the transition form-factors for the inelastic scattering of ^3He , α -particles and ^7Li ions to the 2^+ , 4.44 MeV state in ^{12}C are calculated using the folding model. The intrinsic form-factors are defined by expression (2) where the trivial L dependence of the form-factor due to the different deformation lengths ($\beta_L R$) has been removed¹⁾. The transition density has been deduced from the ground state distribution density for ^{12}C given by expression (8) using the standard form for a mass vibration (expression 5) or the Tassie hydrodynamic model (expression 6). Expression (7) has been used to determine the amplitudes C_S and

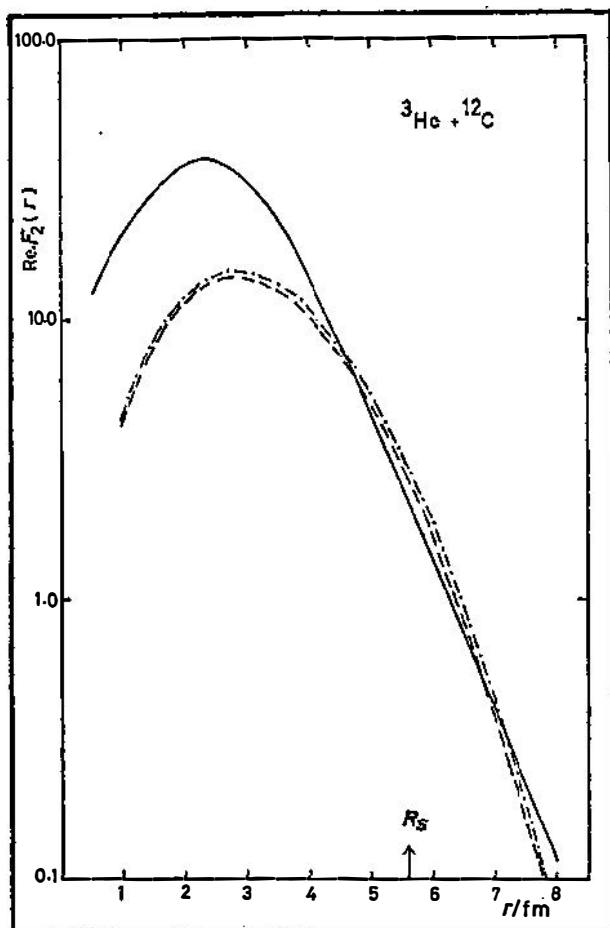


Fig. 1. The intrinsic transition form-factor in $\text{MeV} \cdot \text{fm}^{-1}$ for the inelastically scattered ^3He to the $(2^+, 4.44 \text{ MeV})$ state in ^{12}C . The full curve represents the deformed optical model calculations (dV/dr) with the parameters given in Table 1. The dashed and dashed-dotted curves represent the double-folding model (expression 2) calculations using the Tassie and standard transition models, respectively.

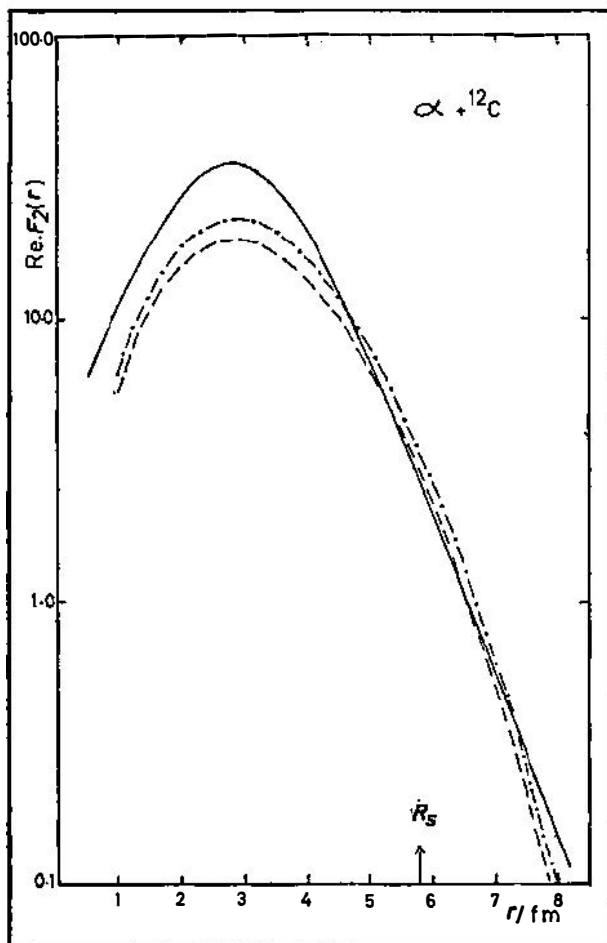


Fig. 2. The same as Fig. 1, but for $\alpha + {}^{12}\text{C}$ reaction.

C_T of the transition densities where the proton part of the transition density would yield the measured value of $B(E2)$. The choice of $B(E2) = 42 e^2 \text{ fm}^4$ for ${}^{12}\text{C}$ nucleus⁵⁾ yields $C_S = 0.67301 \text{ fm}$ and $C_T = 0.4969 \text{ fm}^{-1}$.

The substitution of the nucleon-nucleon interaction $V_{N-N}(r)$, the standard or Tassie transition density of ${}^{12}\text{C}$ and the spherical density of ${}^3\text{He}$, α -particle, or ${}^7\text{Li}$ into expression (2) yields the corresponding double-folding intrinsic transition form-factors. The resulting transition form-factors are shown in Figs. 1—3 compared with the deformed optical potentials $\left(\frac{dV(r)}{dr}\right)$, where $V(r)$ is the phenomenological optical potential used to fit the elastic scattering data.

Using expression (3), where the α -nucleon interaction is folded into the transition density of ${}^{12}\text{C}$, the single-folding intrinsic form-factor for the $\alpha + {}^{12}\text{C}$ reaction is obtained. Fig. 4 shows the resulting form-factors using either the Tassie or the standard transition density for ${}^{12}\text{C}$, compared with the deformed optical potential. The optical potentials $V(r)$ used in this work are given in Table 1.

TABLE 1.

Reaction	Energy	V_0	r_0	a_0	Reference
$^3\text{He} + ^{12}\text{C}$	82.1	118.2	1.0	0.8	17
$\alpha + ^{12}\text{C}$	139.0	108.1	1.22	0.76	18
$^7\text{Li} + ^{12}\text{C}$	63.0	58.36	1.488	0.785	19

The optical potential parameters used in the deformed optical potential calculations $\frac{dV(r)}{dr}$ shown in Figs. 1—4. They are parametrized in the form: $V(r) = -V_0 \left[1 - \exp\left(\frac{r-R_0}{a_0}\right) \right]^{-1}$ where $R_0 = r_0 A^{1/3}$. All lengths are in fm and energies in MeV.

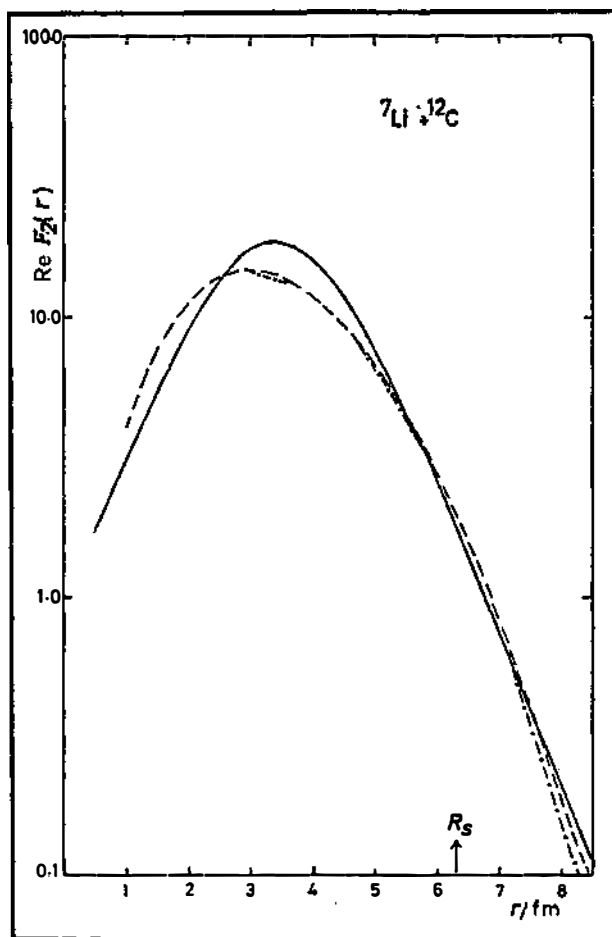


Fig. 3. The same as Fig. 1, but for $^7\text{Li} + ^{12}\text{C}$ reaction.

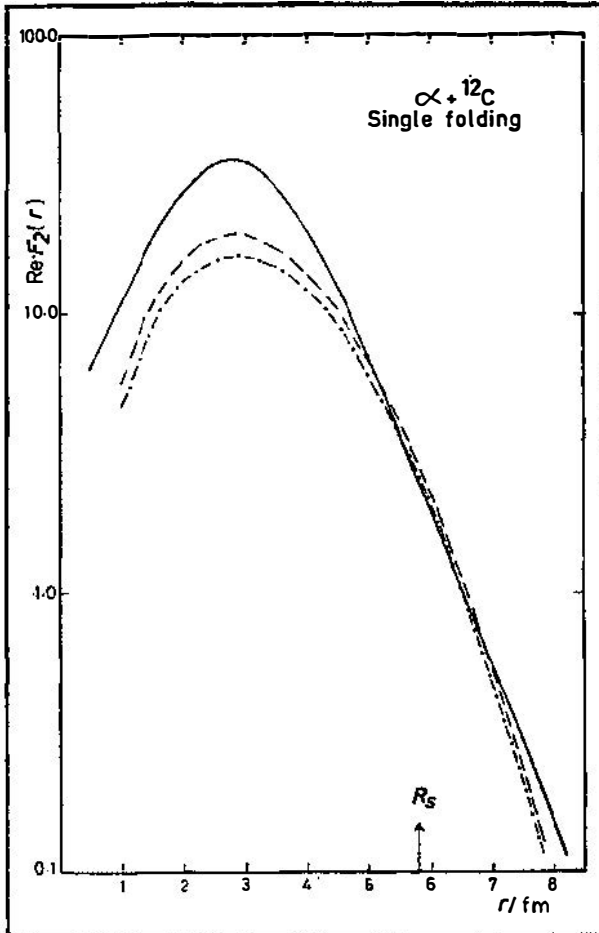


Fig. 4. The same as Fig. 1, but for $\alpha + {}^{12}\text{C}$ reaction using the single-folding model (expression 3).

Figs. 1—4 show that the deformed optical potential model and the folding model give close results in the important region near the strong absorption radius R_s , where R_s can be defined by $R_s = 1.5 (A^{1/3} + A_p^{1/3})$ fm. Here A_p is the projectile mass number. It was shown that elastic and inelastic heavy ion reactions are sensitive to the radial region around R_s ¹⁶⁾, and that the interior region of the form-factors, where the largest differences occur, are not significant to the calculations of the cross sections¹⁾. In the present work one sees that in the vicinity of R_s there are small but not negligible differences in the form-factors mostly in their magnitude. These differences could in principle influence the values of the deformation parameters β_L which are to be extracted from fitting the inelastic scattering data. Thus we expect that both the deformed optical potential and the folding models will give similar cross sections for the reactions under consideration. Using other sets of parameters for the ${}^3\text{He}$, α -particle, ${}^7\text{Li}$ or ${}^{12}\text{C}$ nuclear matter distributions, or using the M3Y interaction^{4,5)} for the nucleon-nucleon interaction may improve the fits shown in Figs. 1—4.

Finally, it seems that the folding model is applicable for the transition form-factors of ^3He , α -particles and ^7Li ions inelastically scattered to the first excited state (2^+ , 4.44 MeV) in ^{12}C . The need to calculate the cross sections using the present form-factors is stressed.

Acknowledgment

Many thanks are due to Professor A. E. Belal for his help and useful discussions.

References

- 1) C. B. Dover, P. J. Moffa and J. P. Vary, *Phys. Lett.* **56B** (1975) 4; P. J. Moffa, C. B. Dover and J. P. Vary, *Phys. Rev. C* **16** (1977) 1857;
- 2) G. R. Satchler, J. L. C. Ford Jr., K. S. Toth, D. C. Hensley and E. E. Gross, *Phys. Lett.* **60B** (1975) 43;
- 3) F. E. Bertrand, G. R. Satchler, D. J. Horen, J. R. Wu, A. D. Bacher, G. T. Emery, W. P. Jones, D. W. Miller and A. van der Woude, *Phys. Rev.* **C22** (1980) 1832;
- 4) J. Cook, K. W. Kemper and M. F. Vineyard, *Phys. Rev.* **C26** (1982) 486; J. Cook, N. M. Clarke, J. Coopersmith and R. J. Griffiths, *Nucl. Phys. A* **386** (1982) 346; M. F. Vineyard, J. Cook and K. W. Kemper, *Nucl. Phys. A* **405** (1983) 429;
- 5) G. R. Satchler and W. G. Love, *Phys. Rep.* **55** (1979) 183; R. G. Stokstad, R. M. Wieland, G. R. Satchler, C. B. Fulmer, D. C. Hensley, S. Raman, L. D. Rickertsen, A. H. Snell and P. H. Stelson, *Phys. Rev.* **C20** (1979) 655; E. E. Gross, J. R. Beene, K. A. Erb, M. P. Fewell, D. Shapira, M. J. Rhoades-Brown and G. R. Satchler, *Nucl. Phys. A* **401** (1983) 362;
- 6) R. Yen, L. S. Cardman, D. Kalinsky, J. R. Legg and C. K. Bockelman, *Nucl. Phys. A* **235** (1974) 135;
- 7) M. A. Duguay, C. K. Bockelman, T. H. Curtis and R. A. Eisenstein, *Phys. Rev.* **163** (1967) 1259;
- 8) J. F. Zigler and C. A. Peterson, *Phys. Rev.* **165** (1968) 1337;
- 9) P. J. Moffa, J. P. Vary, C. B. Dover, C. W. Towsley, R. G. Hanus and K. Nagatani, *Phys. Rev. Lett.* **35** (1975) 992;
- 10) W. G. Greenless, G. J. Pyle and Y. C. Tang, *Phys. Rev.* **171** (1968) 1115;
- 11) A. M. Bernstein, *Advances in Nucl. Phys.* **3** (1969) 325;
- 12) H. M. Hussein and O. Zohni, *Nucl. Phys. A* **267** (1976) 303;
- 13) J. Cook and K. W. Kemper, to be published;
- 14) D. R. Thompson, *Nucl. Phys. A* **154** (1970) 442; I. Reichsten and Y. C. Tang, *Nucl. Phys. A* **139** (1969) 144;
- 15) F. S. Chwieroth, Y. C. Tang and D. R. Thompson, *Phys. Rev.* **C9** (1974) 56;
- 16) P. J. Moffa, C. B. Dover and J. P. Vary, *Phys. Rev.* **C13** (1976) 147;
- 17) T. Tanabe, K. Koyana, M. Yasue, H. Yokomizo, K. Sato, J. Kokame, N. Koori and S. Tanaka, *J. Phys. Soc. Japan* **41** (1976) 361;
- 18) S. M. Smith, G. Tibell, A. A. Cowley, D. A. Goldberg, H. G. Pugh, W. Reichart and N. S. Wall, *Nucl. Phys. A* **207** (1973) 273;
- 19) A. F. Zeller, F. W. Lui, R. E. Tribble and D. M. Tanner, *Phys. Rev.* **C22** (1980) 1534.

FORM-FAKTORI PRIJELAZA ZA RASPRŠENJE LAKIH IONA NA ^{12}C

SAYED A. E. KHALLAF i HUSSIEN M. ALI AHMED

Department of Physics, Faculty of Science, Aswan, Egypt

UDK 539.17

Originalni znanstveni rad

Izračunati su form-faktori prijelaza za neelastično raspršenje ^3He , α -čestica i ^7Li iona u prvo pobuđeno stanje (2^+ , 4.44 MeV) u ^{12}C . Radijalna ovisnost prijelazne gustoće izabrana je ili u standardnom obliku vibracije masa ili u Tassievom hidrodinamičkom obliku. Dobiveni form-faktori uspoređeni su s deformiranim optičkim potencijalima gdje je dobiveno dobro slaganje. Naglašena je potreba da se udarni presjeci računaju koristeći tako dobivene form-faktore.