

TRUNCATED STRONG-COUPPLING BASIS STATES FOR THE SU(3) LIMIT OF SU(6) PARTICLE-QUADRUPOLE PHONON MODEL (PTQM)

DENIS K. SUNKO and VLADIMIR PAAR

Prirodoslovno-matematički fakultet, University of Zagreb, Marulićev trg 19, 41000 Zagreb,
Yugoslavia*

Received 25 February 1985

UDC 539.12

Original scientific paper

The truncated strong-coupling (TSC) basis is constructed, using the intrinsic state for odd system (particle j) plus SU(3) coherent quadrupole phonon state) and asymptotic correspondence to Bohr-Mottelson rotational model. In TSC-basis $I \cdot I$ matrix elements satisfy $\Delta K = 0, \pm 1$ rule independently of maximum phonon number N , while for matrix elements of SU(3) particle-vibration interaction this rule is broken. The ensuing conditions for TSC basis to be eigenstates of PTQM are formulated and related to the strong-coupling limit of effective particle-vibration interaction strength.

1. Introduction

The collective quadrupole degree of freedom dominates many features of even- and odd- A nuclei¹⁾. A mathematically transparent but physically rather simplified treatment is provided by SU(6) collective model in quadrupole phonon representation (TQM for even and PTQM for odd nuclei)^{2,5)} and, equivalently, in s, d -boson representation (IBM and IBFM, respectively)^{3,4,6)}.

In the SU(3) limit of PTQM an intrinsic state for odd system can be simply built by coupling a single particle to the coherent SU(3) vibrational state. Ana-

* This project was assisted by the U. S. National Science Foundation under Grant No. YOR 80/001.

logs of Nilsson states and the corresponding bands⁵⁾ are then obtained by the angular momentum projection

$$|K \mathcal{J} M\rangle = P_{MK}^{\mathcal{J}} \{|j K\rangle |C\rangle\}. \quad (1)$$

where $|jK\rangle$ is the single particle state and $|C\rangle$ is the coherent state for SU(3) vibrational core

$$|C\rangle = 3^{-\frac{N}{2}} e^{\sqrt{\frac{N}{2}} b_0^\dagger (N - \hat{N})^{1/2}} |0\rangle, \quad (2)$$

$$\hat{N} = \sum_{\mu} b_{\mu}^{\dagger} b_{\mu}$$

which can be rewritten as

$$|C\rangle = \sum_{I=0}^{2N} \frac{3^{-\frac{N}{2}}}{B_I} |I\rangle_{GSB}, \quad (3.a)$$

$$|I\rangle_{GSB} = \sum_{n\nu} A_{n\nu I}^{norm} |n \nu I\rangle. \quad (3.b)$$

In (2a) b_{μ}^{\dagger} is the creation operator of quadrupole phonon with angular momentum x -projection μ , and N denotes the maximum number of quadrupole phonons included in the configuration space.

In (3b) $|n \nu I\rangle$ denotes n -phonon state of total angular momentum I and with additional quantum numbers ν . The coefficients $A_{n\nu I}^{norm}$ and B_I have been derived in Ref. 5.

The states (1) by its construction contain only core states belonging to $(2N, 0)$ representation.

Using (3a, b) the expression (1) can be brought to the form

$$|K \mathcal{J} M\rangle = \mathcal{N} \sum_I \frac{1}{B_I} \langle j K I 0 | \mathcal{J} K \rangle |j, I_{GSB}; \mathcal{J} M\rangle_{TWC}, \quad (4)$$

with $K = j, j - 1, \dots, \frac{1}{2}$. \mathcal{N} is the normalization factor. The states $|\rangle_{TWC}$ appearing in (3) are defined as

$$|j, I_{GSB}; \mathcal{J} M\rangle_{TWC} = \sum_{m_1 m_2} \langle j m_1 I m_2 | \mathcal{J} M \rangle |j m_1\rangle |I m_2\rangle_{GSB} \quad (5)$$

which resembles the standard weak coupling particle-core states. The states (5) will be referred to as Truncated weak coupling (TWC) basis, because $I_{GSB}(\max) = 2N$.

2. TSC representation

By applying the orthogonalization procedure to the states (4) we have obtained the supersymmetric basis in which $K = j$ band is uncoupled from other

bands⁸⁾. Here we adopt a different procedure: the basis states (4) are modified in order to establish direct correspondence with Bohr-Mottelson rotational model. The starting point is the fact that K -dependence in (4) is the same as in the Bohr-Mottelson expression for the strong-coupling in terms of weak coupling basis¹⁾. Therefore, we require that in the limit $N \rightarrow \infty$ (3) goes into standard Bohr-Mottelson transformation between weak and strong coupling basis. This appears if we perform the following replacement in (4)

$$\frac{\mathcal{N}}{B_I} \rightarrow \frac{(4I + 2)^{1/2}}{(2\mathcal{J} + 1)}. \tag{6}$$

We extrapolate this relation to the case of finite N and thus we have the new basis

$$|K \mathcal{J} M\rangle_{TSC} = \sum_{I=0}^{2N} \left(\frac{4I + 2}{2\mathcal{J} + 1} \right)^{1/2} \langle j K I O | j K \rangle |j, I_{GSB}; \mathcal{J} M\rangle, \tag{7}$$

to be referred to as Truncated strong coupling (TSC) basis.

We note that the states analogous to (7) have been postulated in IBFM⁶⁾.

It should be pointed out that the unitarity of transformation (7) is broken, as a consequence of truncation of phonon space; the maximum core angular momentum is $I_{max} = 2N$ with $I > 2N$ which satisfy triangular condition in (7) are not available. As a consequence, the TSC basis is not fully orthogonal. The largest \mathcal{J} for which the states (7) are orthogonal, denoted by \mathcal{J}_{max}^o , is

$$2N - j + 1.$$

The maximum angular momentum in the band based on the analog of Nilsson state $|K\rangle$ is

$$\mathcal{J}_{max}(K) = 2N - j + 2K. \tag{8}$$

Thus, we have

$$\mathcal{J}_{max}^o = \mathcal{J}_{max} \left(K = \frac{1}{2} \right), \tag{9}$$

i. e. the states (7) are orthogonal only for the angular momenta contained in the $K = \frac{1}{2}$ band; while in each $K > \frac{1}{2}$ band the $2K - 1$ states, with angular momenta $\mathcal{J} = 2N - j + 2K, 2N - j + 2K - 1, \dots, 2N - j + 2$, are not orthogonal. For these states we apply the Schmidt orthogonalization procedure.

If one single particle (or hole) in the configuration $|j\rangle$ is coupled to SU(3) core, and we consider the states of type (1), the relevant Hamiltonian of the SU(6) particle quadrupole phonon model (PTQM) takes the form^{5, 8)}

$$H'_{PTQM} = \delta I \cdot I + \Gamma (G_2^p G_2^f)_0, \tag{10}$$

with

$$I_\nu = \sqrt{10} (b + \tilde{b})_{1\nu},$$

$$G_{2\mu}^F = b_\mu^+ (N - \hat{N})^{1/2} + (N - \hat{N})^{1/2} \tilde{b}_\mu \pm \frac{\sqrt{7}}{2} (b + \tilde{b})_{2\mu},$$

$$G_{2\mu}^f = (c_j^+ \tilde{c}_j)_{2\mu}.$$

In (10) the SU(3) Casimir operator and the single-particle Hamiltonian are omitted because of the states of type (1) they give only an overall shift.

The matrix elements of (10) in TSC-basis (7) for $\mathcal{J} \leq 2N - j + 1$ are given by

$${}_{TSC} \langle K \mathcal{J} M | H'_{PTQM} | K' \mathcal{J} M \rangle_{TSC} = \delta \cdot {}_{TSC} \langle K \mathcal{J} M | I \cdot I | K' \mathcal{J} M \rangle_{TSC} + \Gamma \cdot {}_{TSC} \langle K \mathcal{J} M | (G_2^B G_2^f)_0 | K \mathcal{J} M \rangle_{TSC}, \quad (11)$$

$${}_{TSC} \langle K \mathcal{J} M | I \cdot I | K' \mathcal{J} M \rangle_{TSC} = \sum_I \frac{4I + 2}{2\mathcal{J} + 1} \langle j K I O | \mathcal{J} K \rangle \langle j K' I O | j K' \rangle \cdot I(I + 1) \quad (12)$$

$${}_{TSC} \langle K \mathcal{J} M | (G_2^B G_2^f)_0 | K' \mathcal{J} M \rangle_{TSC} = \sum_{I'} \frac{1}{2\mathcal{J} + 1} [(4I + 2)(4I' + 2)]^{1/2} \cdot \langle j K I O | \mathcal{J} K \rangle \langle j K' I' O | \mathcal{J} K' \rangle_{TWC} \langle j, I_{GSB}; \mathcal{J} M | (G_2^B G_2^f)_0 | j, I'_{GSB}; \mathcal{J} M \rangle_{TWC}. \quad (13)$$

The TWC-matrix element in (13) is⁸⁾:

$${}_{TWC} \langle j, I_{GSB}; \mathcal{J} M | (G_2^B G_2^f)_0 | j, I'_{GSB}; \mathcal{J} M \rangle_{TWC} = (-)^{j+I-1} \left\{ \begin{matrix} j & I' & \mathcal{J} \\ I & j & 2 \end{matrix} \right\} \cdot \sqrt{\frac{3}{8}} \langle \lambda 0 || Q^{11} || \lambda 0 \rangle \sqrt{2I + 1} \langle (\lambda 0) I', (11) 2 || (\lambda 0) I \rangle, \quad (13.a)$$

where $\lambda = 2N$; $\langle || || \rangle$ and $\langle || \rangle$ are the SU(3) reduced matrix element of $Q^{11} \equiv \{I^B, Q^B \equiv \sqrt{\frac{8}{3}} G_2^B\}$ and the SU(3) Wigner coefficient, respectively⁹⁾.

It turns out that the matrix elements (12) in TSC basis do not vanish for $K' = K, K \pm 1$; the off-diagonal terms satisfy the $\Delta K = 1$ rule which corresponds to Coriolis force in Bohr-Mottelson rotational model.

For $2N + j > \mathcal{J} > 2N - j + 1$ the TSC states (7) are first orthogonalized in the order: $|K = j \mathcal{J} M\rangle, |K = j - 1 \mathcal{J} M\rangle, \dots$, down to the state with lowest K of that \mathcal{J} . With such orthogonalization procedure the $\Delta K = 1$ rule is again satisfied for the matrix elements of $I \cdot I$. In fact, this feature singles out this particular orthogonalization procedure.

On the other hand, the attractivity of TSC basis is spoiled by dynamical interaction $(G_2^B G_2^F)_0$: $\Delta K = 1$ rule is broken for the matrix elements (13), i. e. dynamical interaction in TSC basis is not of the Coriolis type. (However, there exists another basis, supersymmetric, in which dynamical interaction is of Coriolis type⁸⁾). Because in the TSC-basis for finite N all $(G_2^B G_2^F)_0$ matrix elements are different from zero, while the $I \cdot I$ matrix elements are nonvanishing only for $\Delta K = 1$, there is no possibility to cancel off-diagonal matrix elements with $\Delta K > 2$, i. e. no K -band can be uncoupled from the others. The two-parameter Hamiltonian (10) is not diagonal (or block-diagonal) in TSC basis (7), i. e. eigenstates of (10) are superpositions of different TSC states

$$|\mathcal{J} M\rangle_n = \sum_{K=\frac{1}{2}}^j \eta_{Tn}^K |K \mathcal{J} M\rangle_{TSC}, \quad (14)$$

where n labels the n -th state of angular momentum \mathcal{J} .

The exceptions are trivial cases for the states with angular momenta with one- or two-dimensional TSC basis space for diagonalization: $j = \frac{1}{2}$, any \mathcal{J} ; $j = \frac{3}{2}$, any \mathcal{J} ; $j > \frac{3}{2}$, $\mathcal{J} = \frac{1}{2}$; $j > \frac{3}{2}$, $\mathcal{J} = \frac{3}{2}$. In the first and third case the TSC basis space is one-dimensional ($K = \frac{1}{2}$) and in the other two cases two-dimensional ($K = \frac{1}{2}, \frac{3}{2}$); thus the $K \geq 2$ off-diagonal matrix elements are absent. Then the condition

$$\langle K = \frac{3}{2} \mathcal{J} M | H'_{PTQM} | K = \frac{1}{2} \mathcal{J} M \rangle = 0 \quad (15)$$

yields a particular ratio of Γ and δ , to be denoted as $\left(\frac{\Gamma}{\delta}\right)_{TSC}$, for which the two basis states $|K = \frac{1}{2} \mathcal{J} M\rangle, |K = \frac{3}{2} \mathcal{J} M\rangle$ are eigenstates of (10).

For the case $j > \frac{3}{2}$, $\mathcal{J} = \frac{3}{2}$ the condition (15) gives

$$\left(\frac{\Gamma}{\delta}\right)_{TSC} = - \frac{{}_{TSC}\langle K = \frac{3}{2} \mathcal{J} = \frac{3}{2} M | I \cdot I | K = \frac{1}{2} \mathcal{J} = \frac{3}{2} M \rangle_{TSC}}{{}_{TSC}\langle K = \frac{3}{2} \mathcal{J} = \frac{3}{2} M | (G_2^B G_2^F)_0 | K = \frac{1}{2} \mathcal{J} = \frac{3}{2} M \rangle_{TSC}}. \quad (16)$$

For $j = \frac{11}{2}$, $N = 7$ the expression (16) gives

$$\left(\frac{\Gamma}{\delta}\right)_{TSC} = -3465. \quad (17)$$

The states $|K = \frac{3}{2} \mathcal{J} = \frac{3}{2} M\rangle_{TSC}$ and $|K = \frac{1}{2} \mathcal{J} = \frac{3}{2} M\rangle_{TSC}$ are exact eigenstates of (10) with the ratio of parameters given by (17). For reasonable value of moment of inertia \mathcal{J} there is $\delta \equiv \frac{\hbar^2}{2\mathcal{J}} = 0.015$ and (17) gives $\Gamma = -52$, which is an extremely strong particle-vibration coupling strength. (In realistic cases there is $|\Gamma| \approx 1$.)

It should be noted that TSC ratio (16) depends on \mathcal{J} , although rather weakly; thus the states $|K = \frac{3}{2} \mathcal{J} = \frac{3}{2} M\rangle_{TSC}$, $|K = \frac{3}{2} \mathcal{J} = \frac{5}{2} M\rangle_{TSC}$, ... are exact eigenstates of (10) for somewhat different ratios $(\frac{\Gamma}{\delta})_{TSC}$.

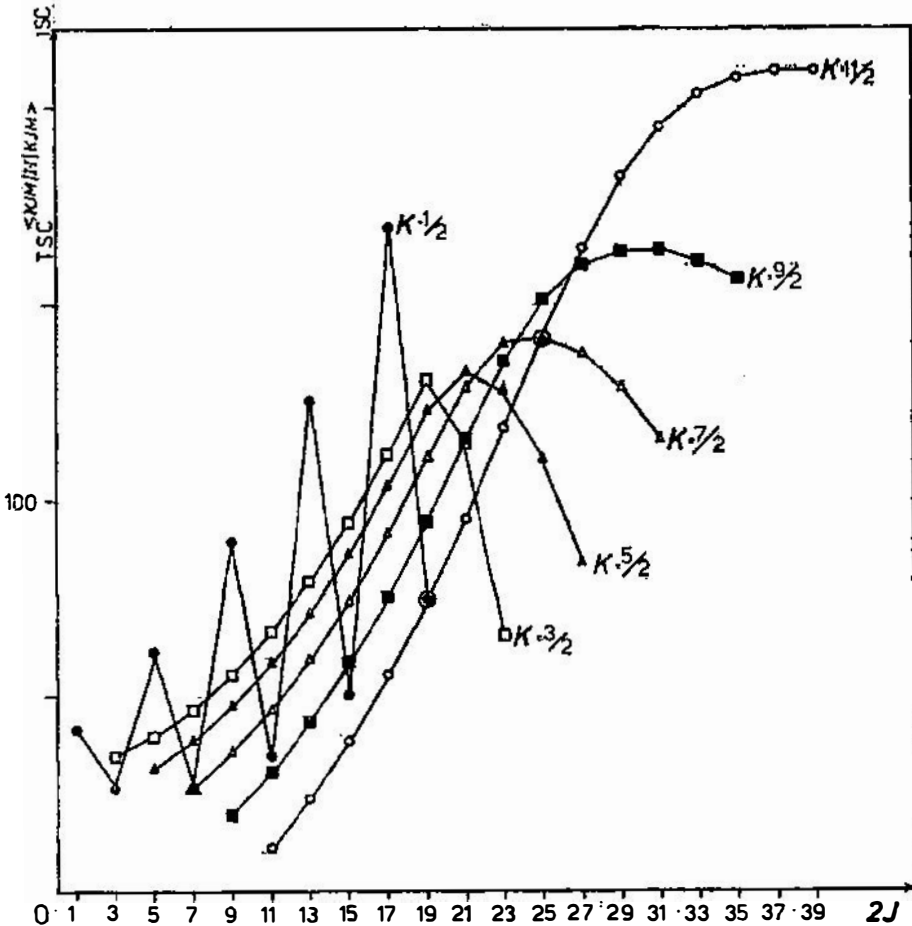


Fig. 1 Diagonal matrix elements ${}_{TSC} \langle KJM || I \cdot I || KJM \rangle_{TSC}$ for $j = 11/2$, $N = 7$.

In Fig. 1. we present diagonal matrix elements of $I \cdot I$ term in TSC-basis, given by (12) for $K' = K$. Strong signature effect is pronounced for the $K = \frac{1}{2}$ band, while it is small for bands $K \geq \frac{3}{2}$. This clearly resembles the Bohr-Mottelson rotational model. In addition, the effect of truncation of the phonon space at $N = 7$ is clearly seen: the matrix elements for large J decrease, since the core states with large spins, which should contribute, are not available.

It should be pointed out that the expression (11), (12) enable us to diagonalize PTQM Hamiltonian (10) without any reference to the spherical phonon basis. This provides an enormous mathematical simplification: The size of matrices diminishes considerably and instead of numerous coefficients of fractional parentage we need SU(3) Wigner coefficients given by simple analytical formula⁷⁾. In TSC basis the maximum possible dimension of the hamiltonian matrix is $(j + \frac{1}{2}) \cdot (j + \frac{1}{2})$, irrespectively of N .

Of course, the particle-vibration basis $|j, n v I; J M\rangle, n \leq N$ is always applicable to diagonalize PTQM, while the simple relations (11—13) for TSC-basis hold only for the SU(3) limit.

Now we consider matrix elements (10) in the limit $N \rightarrow \infty$.

3. Asymptotic limit

Inserting explicit expressions for $6j$ -coefficients and SU(3) Wigner coefficients into (13, 13a), we have a factor which depends on $\lambda (= 2N)$:

$$\begin{aligned}
 F(\lambda) &\equiv [\lambda(\lambda + 3)]^{1/2} \sum_F \left\{ \begin{matrix} j & I & J \\ I & j & 2 \end{matrix} \right\} \langle (\lambda 0) I'; (11) 2 || (\lambda 0) I \rangle \langle j - K' J K' | I' 0 \rangle = \\
 &= [\lambda(\lambda + 3)]^{1/2} \left\{ \frac{2\lambda + 3}{2[\lambda(\lambda + 2)]^{1/2}} \frac{3X(X - 1) - 4j(j + 1)I(I + 1)}{(2I - 1)(2I + 3)} \langle j - \right. \\
 &- K' j K' | I 0 \rangle - 6 \left[\frac{(\lambda - I + 2)(\lambda + I + 1)}{\lambda(\lambda + 3)} \right]^{1/2} f(I)f(I - 1) \langle j - K' J K' | I - \\
 &- 2 0 \rangle - 6 \left[\frac{(\lambda + I + 3)(\lambda - I)}{\lambda(\lambda + 3)} \right]^{1/2} f(I + 1)f(I + 2) \langle j - K' J K' | I + 2 0 \rangle \}. \\
 &\quad \cdot (-)^{J+I} \left[\frac{(2j - 2)!}{(2I + 1)(2j + 3)!} \right]^{1/2}, \tag{18}
 \end{aligned}$$

with

$$\begin{aligned}
 f(x) &= \frac{1}{2} \left\{ \frac{[j + J + 1]^2 - x^2}{(2x - 1)(2x + 1)} [x^2 - (J - j)^2] \right\}^{1/2}, \\
 X &= j(j + 1) + I(I + 1) - J(J + 1).
 \end{aligned}$$

By using identity

$$[f^2(I+1) + f^2(I) - K^2] \langle j - K \mathcal{J} K | I 0 \rangle = -f(I+1)f(I+2) \langle j - K \mathcal{J} K | I + 2 0 \rangle - f(I)f(I-1) \langle j - K \mathcal{J} K | I - 2 0 \rangle$$

the expression (18) for $\lambda \rightarrow \infty$ becomes

$$F(\lambda \rightarrow \infty) = 2 \lambda j(2j+1) \langle j - K' \mathcal{J} K' | I 0 \rangle. \tag{19}$$

Employing (19) the expression (13) gives

$${}_{TSC} \langle K \mathcal{J} M | \Gamma(G_2^B G_2^F)_0 | K \mathcal{J} M \rangle_{TSC} \stackrel{N \rightarrow \infty}{=} -2 \sqrt{2} \Gamma N \left[\frac{(2j-2)!}{(2j+3)!} \right]^{1/2} [3 K^2 - j(j+1)] \delta_{KK'}. \tag{20}$$

The analogous expression has been given in Ref. 6 for IBFM.

We note that in $N \rightarrow \infty$ limit the matrix element (12) can be also derived directly in much simpler way by using the intrinsic state for odd system, leaving out the angular momentum projection; indeed, there is no \mathcal{J} -dependence in (20). Using (1) it is easy to show

$$\begin{aligned} & \lim_{\substack{N \gg J \\ N \rightarrow \infty}} {}_{TSC} \langle K \mathcal{J} M | (G_2^B G_2^F)_0 | K' \mathcal{J} M \rangle_{TSC} = \\ & = \sum_{\mu} \frac{(-)^{\mu}}{\sqrt{5}} \langle C | G_{2\mu}^B | C \rangle \langle j K | G_{2-\mu}^F | j K \rangle. \end{aligned} \tag{21}$$

Here $|j K\rangle |C\rangle$ is the intrinsic state appearing in (1). Inserting straightforward phonon and single-particle matrix elements into (21) we obtain again the result (20).

For $N \rightarrow \infty$ it follows from (12) and (20) that

$$\frac{{}_{TSC} \langle \Gamma(G_2^B G_2^F)_0 \rangle_{TSC}}{{}_{TSC} \langle \delta I \cdot I \rangle_{TSC}} \approx \frac{\Gamma N}{\delta}. \tag{22}$$

This means that for large N the interaction term dominates. Since for $N \rightarrow \infty$ the TSC basis states become the eigenstates of the Hamiltonian by construction, one would be tempted to conclude that they are eigenstates of the interaction term alone for any N . This is not true, because the absence of large angular momenta ($> 2N$) for finite N always introduces off-diagonal elements in the interaction matrix. However, they are generally rather small, and if we set $\delta = 0$ the TSC states reasonably approximate the exact solutions even for $N < 10$.

If N is not large, in the previously discussed exceptional cases ($j = 3/2$ for any \mathcal{J} and $\mathcal{J} = 3/2$ for any j) we can use the core term to get rid of the unwanted off-diagonal matrix element. This fine-tuning requires a very large ratio Γ (see (17)), and states $\mathcal{J} \neq 3/2$ are still well approximated by the TSC solutions. (The

states $\mathcal{J} = 3/2$ become, of course, identical to the TSC states.) Thus we can stipulate that the condition

$$\frac{\Gamma N}{\delta} \equiv \frac{\Gamma_{eff}}{\delta} \gg 1$$

has to be satisfied in order for the TSC states to be good approximation to the exact solution. In other words, the term *strong coupling* refers to Γ_{eff} , and not necessarily to Γ itself.

In realistic situation of unique parity states with particle-vibration interaction strength Γ only one or two orders of magnitude larger than δ , we decompose the wave functions in the TSC basis. It turns out that each is dominated by a particular K -component, though for some states with only 50% or even less, i. e. K is an approximate quantum number⁸⁾.

Concluding, we have investigated the TSC limit of PTQM/IBFM and its asymptotic limit (geometrical case). In combination with the new supersymmetric limit which we have constructed recently^{8,9)} this opens a puzzling questions of two rotation-like limits in the quantal case, while only one limit is known in the Bohr-Mottelson geometrical model. On the other hand, we have extended the concept of dynamical symmetry and supersymmetry to odd-odd nuclei¹³⁻¹⁵⁾ and to hypernuclei^{16,17)}.

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ODREZANA BAZA JAKOG VEZANJA ZA SU (3) GRANICU SU (6) MODELA
ČESTICE I KVADRUPOLNOG FONONA

DENIS K. SUNKO i VLADIMIR PAAR

Prirodoslovno-matematički fakultet, Sveučilište u Zagrebu, Marulićev trg 19, Zagreb

UDK 539.12

Originalni znanstveni rad

Odrezana baza jakog vezanja (TSC) konstruirana je koristeći unutrašnje stanje neparnog sistema (čestica j plus SU (3) koherentno kvadrupolno fononsko stanje) i asimptotsku korespondenciju s Bohr-Mottelsonovim modelom. U TSC bazi matični elementi operatora $I \cdot I$ zadovoljavaju $\Delta K = 0, \pm 1$ izbornu pravilo neovisno o maksimalnom broju fonona N , a za matične elemente SU (3) interakcije između čestice i vibracija to pravilo je narušeno. Dobiveni su uvjeti da TSC stanja budu svojstvena stanja za PTQM i povezani su s efektivnom jačinom interakcije u granici jakog vezanja.