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THE APPROACH TO HYPERNUCLEAR STRUCTURE BASED ON BOSON-FERMION DYNAMICAL SYMMETRY AND SUPERSYMMETRY

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We are introducing the idea of boson-fermion symmetry and hypersymmetry for hypernuclei. The case with $j_\pi = j_\nu = j_\Lambda = 1/2$ is investigated.

1. Introduction

The spectroscopy of hypernuclei is a rapidly developing field of nuclear physics[1]. Levels of Λ- and Σ-hypernuclei are excited in the strangeness exchange reactions (K, π) and Ξ-hypernuclei are accessible in double strangeness ecxhange reactions[1-6]. To date, only limited amount of data on light nuclei is available. Theoretically, the interaction of $s_{1/2}$ and $p_{3/2}$ hyperon with the nucleons was studied[2,7]. The excited states of hypernucleus ${}^{A}Z$ were considered as core excited states with the hyperon in s or p orbit and the ${}^{A-1}Z$ nuclear core in one of its excited states.

On the other hand the approach to even-even and odd-even nuclei based on exploiting boson and boson-fermion symmetries, and more generally supersymmetries, has received much attention in recent years[8-13]. In a further step the boson-fermion dynamical symmetry and supersymmetry was extended to odd-odd nuclei[14]. In these considerations Schwinger or Holstein-Primakoff SU(6) boson realization can be used[15,16].

Here we introduce the idea of boson-fermion dynamical symmetry and supersymmetry for hypernuclei.

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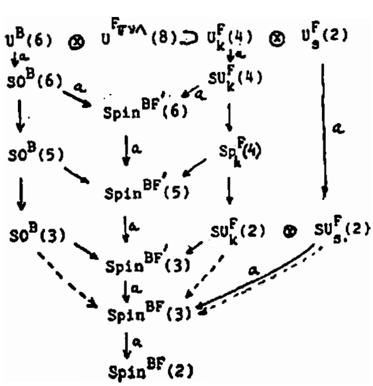
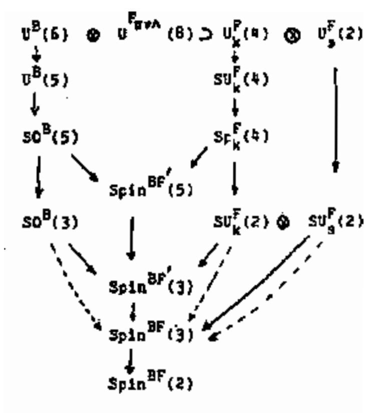
2. *Basic boson-fermion group chains for hypernuclei*

Let us consider a particular case of a hypernucleus ${}^{A}_{\Lambda}Z$ (A odd) with an odd (quasi) proton, odd (quassi) neutron and hyperon in the configurations with the same angular momentum $j = 1/2$, coupled to the even-even boson core ${}^{A-3}Z$ which is characterized by one of dynamical symmetries associated with SU(6) bosons. The system is described by the group $U^{B}(6) \otimes U^{F\pi}(2j+1) \otimes U^{F\nu}(2j+1) \otimes U^{F\Lambda}(2j+1)$ with the SU (3), SU (5) or SO (6) boson group chains.

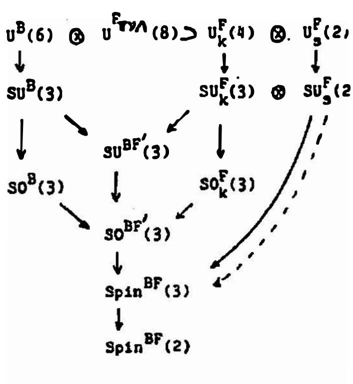
The associated boson-fermion group chains for hypernuclei are



(1)



(2)



(3)

The basis states associated with each group chain appearing in (1—3) are characterised by the quantum numbers which label the irreducible representations (irreps) of the corresponding groups. In forming the boson-fermion chain we have employed a pseudoorbital-pseudospin decomposition, analogous to the one previously used in the boson-fermion treatment of odd-even nuclei[10,13]. The corresponding labels are k and s, respectively. The boson-fermion groups labelled by BF′ and BF are without and with pseudospin part, respectively.

For a given N, the total number of s and d bosons (or, equivalently, the maximum bumber of quadrupole phonons), which labels totally symmetric irrep of $U^B(6)$, $(N, 0, 0, 0, 0, 0)$, we determine the allowed values of other quantum numbers by applying standard group theoretical methods[17,18]. The basis state corresponding to the chain (1a) which presents the highest boson-fermion dynamical symmetry associated with SO (6) boson core are labeled by

$$|N, D_4, S, \Sigma, (\sigma_1, \sigma_2, \sigma_3), (\tau_1, \tau_2), L', \mathcal{J}, M\rangle \tag{4}$$

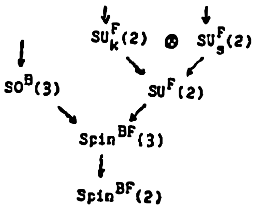
where the quantum numbers label the irreps of the groups $U^B(6)$, $U_k^F(4)$, $U_s^F(2)$, $SO^B(6)$, $Spin^{BF'}(6)$, $Spin^{BF'}(5)$, $Spin^{BF}(3)$, $Spin^{BF}(3)$ and $Spin^{BF}(2)$, respectively. Here, $D_4 = \{\dim\}^F SU(4)$. In Table 1 we present the calculated quantum numbers for $N = 0, 1, 2, 3$.

TABLE 1.

N	(D_4, S)	Σ	$(\sigma_1, \sigma_2, \sigma_3)^{BF'}$	$(\tau_1, \tau_2)^{BF'}$	$2L'$	$2\mathcal{J}$
0	(4, 0)	0	(1/2, 1/2, 1/2)	(1/2, 1/2)	3	3
	$(1, 1/2)^2$	0	$(0, 0, 0)^2$	$(0, 0)^2$	0^2	1^2
1	(4, 0)	1	(3/2, 1/2, 1/2)	(3/2, 1/2)	7, 5, 1	7, 5, 1
				(1/2, 1/2)	3	3
			(3/2, 1/2, −1/2)	(1/2, 1/2)	3	3
	$(1, 1/2)^2$	1	$(1, 0, 0)^2$	$(1, 0)^2$	4^2	$5^2, 3^2$
				$(0, 0)^2$	0^2	1^2
2	(4, 0)	2	(5/2, 1/2, 1/2)	(5/2, 1/2)	11, 9, 7, 5, 3	11, 9, 7, 5, 3
				(3/2, 1/2)	7, 5, 1	7, 5, 1
				(1/2, 1/2)	3	3
			(3/2, 1/2, 1/2)	(3/2, 1/2)	7, 5, 1	7, 5, 1
				(1/2, 1/2)	3	3
		0	(1/2, 1/2, 1/2)	(1/2, 1/2)	3	3
	$(1, 1/2)^2$	2	$(0, 0, 0)^2$	$(2, 0)^2$	$8^2, 4^2$	$9^2, 7^2, 5^2, 3^2$
				$(1, 0)^2$	4^2	$5^2, 3^2$
				$(0, 0)^2$	0^2	1^2
		0	$(0, 0, 0)^2$	$(0, 0)^2$	0^2	1^2
3	(4, 0)	3	(7/2, 1/2, 1/2)	(7/2, 1/2)	15, 13, 11, 9^2, 7, 5, 3	15, 13, 11, 9^2, 7, 5, 3
				(5/2, 1/2)	11, 9, 7, 5, 3	11, 9, 7, 5, 3
				(3/2, 1/2)	7, 5, 1	7, 5, 1
				(1/2, 1/2)	3	3
			(5/2, 1/2, −1/2)	(5/2, 1/2)	11, 9, 7, 5, 3	11, 9, 7, 5, 3
				(3/2, 1/2)	7, 5, 1	7, 5, 1
				(1/2, 1/2)	3	3
		1	(3/2, 1/2, 1/2)	(3/2, 1/2)	7, 5, 1	7, 5, 1
				(1/2, 1/2)	3	3
			(1/2, 1/2, −1/2)	(1/2, 1/2)	3	3
	$(1, 1/2)^2$	3	$(3, 0, 0)^2$	$(3, 0)^2$	$12^2, 8^2, 6^2, 0^2$	$13^2, 11^2; 9^2, 7^2; 7^2, 5^2; 1^2$
				$(2, 0)^2$	$8^2, 4^2$	$9^2, 7^2, 5^2, 3^2$
				$(1, 0)^2$	4^2	$5^2, 3^2$
				$(0, 0)^2$	0^2	1^2
			$(1, 0, 0)^2$	$(1, 0)^2$	4^2	$5^2, 3^2$
				$(0, 0)^2$	0^2	1^2

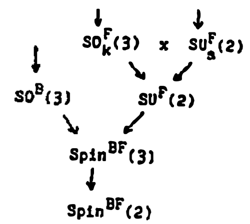
The quantum numbers of the basis states (4) for a hypernucleus.
The label $\mathcal{J}$ denotes the angular momentum of the hypernuclear levels.

The boson-fermion chains (1—3) will be referred to as partially coupled schemes. In addition, there are boson-fermion chains where the pseudoorbital and pseudospin group couple first, i. e. with the chain ends



(5)

associated with the $SO^B(6)$ and $U^B(5)$ core, and with the chain end



(6)

associated with the $SU^B(3)$ core. These chains will be referred to as weakly coupled schemes. The basis states for the scheme (5) associated with $SO^B(6)$ core are

$$|N, \Sigma, v, L, S, J, M\rangle \tag{7}$$

which label the irreps of the groups $U^B(6)$, $SO^B(6)$, $SO^B(5)$, $SO^B(3)$, $SU^F(2)$, $Spin^{BF}(3)$ and $Spin^{BF}(2)$, respectively. In Table 2 we present the calculated quantum numbers (7) for $N = 0, 1, 2, 3$.

3. Boson-fermion dynamical symmetries and supersymmetries for hypernuclei

If the Hamiltonian for the hypernuclear states can be written in terms of the Casimir operators of the groups appearing in any of the chains (1—3, 5, 6) the energy eigenvalues are given by the eigenvalues of the corresponding Casimir operators. In these cases we obtain for each boson-fermion symmetry the corresponding hypernuclear energy formula in terms of the quantum numbers labeling the irreps appearing in the group chain. The parameters appearing in the energy formulas can be determined in several ways: i) by fitting to the experimental hypernuclear levels (when available), ii) by a microscopic derivation starting with a nucleon-nucleon and nucleon-hyperon interaction, iii) by a supersymmetry approach.

TABLE 2.

N	Σ	v	L	$2S$	$2\mathcal{J}$
0	0	0	0	1^2	1^2
				3	3
1	1	1	2	1^2	$3^2, 5^2$
				3	1, 3, 5, 7
		0	0	1^2	1^2
				3	3
2	2	2	4	1^2	$7^2, 9^2$
				3	5, 7, 9, 11
			2	1^2	$3^2, 5^2$
				3	1, 3, 5, 7
		1	2	1^2	$3^2, 5^2$
				3	1, 3, 5, 7
		0	0	1^2	1^2
				3	3
	0	0	0	1^2	1^2
				3	3
3	3	3	6	1^2	$11^2, 13^2$
				3	9, 11, 13, 15
			4	1^2	$7^2, 9^2$
				3	5, 7, 9, 11
			3	1^2	$5^2, 7^2$
				3	3, 5, 7, 9
			0	1^2	1^2
				3	3
		2	4	1^2	$7^2, 9^2$
				3	3, 5, 7, 9
				3	3, 5, 7, 9
			2	1^2	$3^2, 5^2$
				3	1, 3, 5, 7
		1	2	1^2	$3^2, 5^2$
				3	1, 3, 5, 7
		0	0	1^2	1^2
				3	3
	1	1	2	1^2	$3^2, 5^2$
				3	1, 3, 5, 7
		0	0	1^2	1^2
				3	3

The quantum numbers of the basis states (7) for a hypernucleus. The label $\mathcal{J}$ denotes angular momentum of the hypernuclear levels.

The supersymmetry approach will be based on the assumption that hypernuclear levels belong to the supersymmetry multiplets together with nuclear levels. Previously, the supersymmetry approach was used for even-even and odd-even nuclei[2] and was extended to encompass also the odd-odd nuclei[14]. Now we suggest the extension of the supersymmetry to incorporate also the hypernuclei levels. For $j_\pi = j_\nu = j_\Lambda = j$ the corresponding graded Lie algebra would be $U(6/(2j+1)_\pi \otimes (2j+1)_\nu \otimes (2j+1)_\Lambda)$. Owing to the fact that Λ differs from π and ν, this group should be regarded as broken to its subgroup $U(6/(2j+1)^2_{\pi\nu} \otimes (2j+1)_\Lambda)$ where the fermion sector is split into the direct product $U^{\pi\nu}[(2j+1)^2] \otimes U^\Lambda(2j+1)$, of which $U^\pi(2j+1) \otimes U^\nu(2j+1) \otimes U^\Lambda(2j+1)$ is a subgroup.

Keeping this in mind the totally supersymmetric irreps $[N_0\}$ would be labelled by a new quantum number $\mathcal{N}_0 = N + M_\pi + M_\nu + M_\Lambda$.

It is interesting to note that there is a simple selection rule for Λ-decays $\Lambda \to \to p + K^-$, $\Lambda \to n + K^0$ in hypernucleus, transforming the hypernucleus into a nucleus. Namely because of $j = 1/2$ there is no totally antisymmetric irrep of SU(2) and therefore only the basis states (4) with $D_2 = 4$ and the basis states (7) with $S = 3/2$ can undergo the hypernucleus $\to$ nucleus transition.

In a similar way as in the $j = 1/2$ case, we can consider the hypernuclear states with odd proton, odd neutron and hyperon in $j = 3/2$ configurations, coupled to the boson core characterized by a dynamical symmetry[19,20]. As an illustration of nuclear/hypernuclear supersymmetry in that case, the nuclei ^{64}Zn, ^{63}Cu, ^{62}Cu and the hypernucleus ^{61}Cu* would belong to the $[\mathcal{N}_0 = 4\}$ supermultiplet, with $(N = 4, M_\pi = M_\nu = M_\Lambda = 0)$, $(N = 3, M_\pi = 1, M_\nu = M_\Lambda = 0)$, $(N = 2, M_\pi = = 1, M_\nu = 1, M_\Lambda = 0)$ and $(N = 1, M_\pi = 1, M_\nu = 1, M_\Lambda = 1)$, respectively. Therefore, in such a case we obtain the $\pi 3/2 \times \nu 3/2 \times \Lambda 3/2$ spectrum of ^{61}Cu* by inserting into the hypernuclear energy formula the nonvanishing parameters from the $SU^{BF}(5)$ parametrization for ^{63}Cu[19].

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PRISTUP HIPERNUKLEARNOJ STRUKTURI ZASNOVAN NA BOZONSKO-FERMIONSKOJ DINAMIČKOJ SIMETRIJI I SUPERSIMETRIJI

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Uvedena je ideja bozonsko-fermionske simetrije i supersimetrije za hiperjezgre. Istraživan je slučaj s $j_\pi = j_\nu = j_\Lambda = 1/2$.