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PHOTON INDUCED INTERNAL BREMSSTRAHLUNG IN ELECTRON CAPTURE

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The cross section for photon induced internal bremsstrahlung in electron capture has been calculated. The possibility of using this phenomenon to measure the mass of the neutrino is analyzed.

1. Introduction

Internal bremsstrahlung in electron capture (IBEC) has been studied for many years and a comprehensive review is available for all but very recent investigations [1]. The resonant nature of p-state contributions was first noted by Glauber and Martin[2] and these have also been analyzed recently by Pisk et al. [3]. De Rújula[4] first suggested that IBEC can be used to investigate the possibility of non-zero neutrino mass. He noted that the sensitivity is considerable enhanced when the EC Q-value is sufficiently low that K capture is forbidden. This is the case for the EC decay of ^{193}Pt and experimental investigations have given an upper limit of 500 eV for the neutrino mass [5].

Recently McGlinn[6] has analyzed photon induced EC in the hope that the threshold behaviour of the phenomenon will be sensitive to the neutrino mass. ^{163}Ho, which normally decays via capture from the 3 s-state was studied and the possibility of photon induced capture from the $2p_{3/2}$ state was investigated. Unfortunately the cross section for this process was found to be prohibitively small ($\sim 10^{-51}$ cm^2) and there is little chance that this could even be used to measure the neutrino mass. We have studied the possibility of photon induced IBEC. Although this is of higher order than the photon induced EC the possibility of resonant enhancements have encouraged us to investigate its suitability for determining the neutrino mass.

2. *Calculation of the cross section*

Feynman diagrams of the process are given in Fig. 1. The corresponding amplitudes are:

$$S_{if} = 2\pi\, e^2 G\, \delta\,(E_{max} + k_1 - k_2 - q)\, B_\mu \int \mathrm{d}\vec{x}\, \mathrm{d}\vec{y}\, [\overline{\psi_\nu}\,(\vec{z})\, \Lambda_\mu$$

$$S\,(\vec{z}, \vec{x})\, \hat{a}_1\,(\vec{x})\, S\,(\vec{x}, \vec{y})\, \hat{a}_2^+\,(\vec{y})\, \psi_0\,(\vec{y}) + (\hat{a}_1 \rightleftarrows \hat{a}_2^+)]. \qquad (1)$$

Here, we have employed the units $\hbar = c = 1$ and $\alpha = e^2/4\pi \approx 1/137$. In Eq. (1) $\Lambda_\mu = \gamma_\mu\,(1 + \gamma_5)$ and G is the vector coupling constant. B_μ is the nuclear matrix element and ψ_ν and ψ_0 are, respectively, the wave functions of the neutrino and the initial state electron. The incident and emitted photons are described by $\hat{a}_1$ and $\hat{a}_2$, respectively, their energies being k_1 and k_2. The EC transition energy is E_{max} and the neutrino energy is q. S represents the electron spinor Green's function.

The complicated structure of the electron Green's function in the external nuclear Coulomb field makes it difficult to evaluate Eq. (1). However, considerable simplifications are possible if large energy incident and emitted photons are involved, and if the incident photon is at a resonance. We consider these two situations and compare their relative magnitudes.

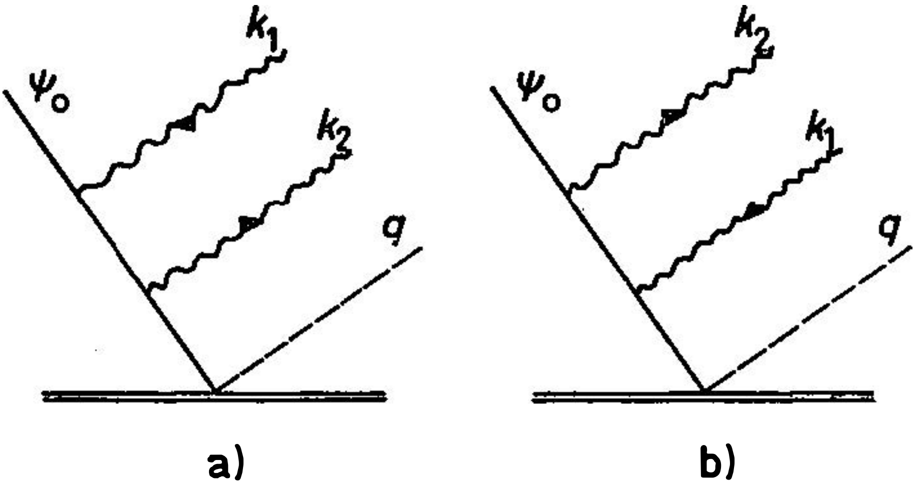

Fig. 1 The Feynman diagrams for the photon-induced IBEC.

2.1. *Large photon energies*

In this case the approach is essentially the same as followed in calculations of double internal bremsstrahlung in EC[7]. The nuclear Coulomb field is neglected and the Coulomb electron Green's function is replaced by free electron Green's functions. In addition only the zero momentum component of the initial electron wave function is considered. This approach is analogous to the approach used by Morrison and Schiff[8] and leads to only initial *s*-state electrons contributing.

We calculate the cross section for photon induced IBEC for an initial *ns*-state electron to be:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}k_2\,\mathrm{d}\Omega_2} = \frac{1}{8\,\pi}\frac{\alpha^2}{m^2}\frac{W(n'\,s)}{E_{\max}^2(n's)}\frac{\varphi_{ns}^2(0)}{\varphi_{n's}(0)}\frac{k_2}{k_1}\frac{(E_{\max}(ns)+k_1-k_2)^2}{[m(k_1-k_2)-k_1\,k_2(1-\cos\Theta)]^2}$$
$$\{4(m^2+k_1^2+k_2^2)+4m(k_1-k_2)-4k_1\,k_2(1+\cos\Theta)- \qquad (2)$$
$$-(k_1-k_2)(m+k_1-k_2)(1+\cos\Theta)^2-[2m^2+2m(k_1-k_2)+$$
$$+k_1^2+k_2^2-k_1\,k_2(1+\cos\Theta)]\sin^2\Theta\}$$

where $n' > n$ and $W(n'\,s)$ is an EC probability for the $n's$ initial state and Θ is the angle between the incident and emitted photon. Also, in Eq. (2) m represents the mass of the electron.

The estimate should be reasonable for large photon energies and for low atomic number.

2.2. *Resonant photon energies*

We assume that the incident photon satisfies the energy condition for an electric dipole transition from an initial electron ns state to a $n'p$ state, where $n' > n$. For these resonance conditions only diagram 1a of Fig. 1 is of interest. The corresponding amplitude, the first part of Eq. (1), is simplified by taking into account only the resonant part of the electron Green's function S_{res}.

$$S_{res}(\vec{x},\vec{y}) = -\frac{i}{2}\sum_{n'p}\psi_{n'p}(\vec{x})\,\overline{\psi}_{n'p}(\vec{y}) \qquad (3)$$

where n' is fixed and the summation takes place over the orbital and spin degrees of freedom.

f The finite width of the $ns \to n'p$ line is taken into account by replacing the δ u nction in the first vertex of diagram 1a by the Lorentz-shaped representation

$$\delta(E_{n'p}-E_{ns}-k_1) \approx \frac{1}{\pi}\frac{\Gamma/2}{(E_{n'p}-E_{ns}-k_1)^2+\Gamma^2/4} \qquad (4)$$

where E_{ns} and $E_{n'p}$ are the respective energies of the ns and $n'p$ electron and Γ is the total width of these states. The resonant condition is satisfied if $k_1 = E_{n'p} - E_{ns}$.

With these considerations we can use the Martin-Glauber approach to simplify Eq. (1) to obtain

$$S_{fi} = 16i\,aG\sqrt{\frac{2\pi^3}{a}}\,\frac{1}{\Gamma}\,\delta(E_{\max}(n'p)-q-k_2)\,B_\mu \sum_\alpha \{\bar{v}^\nu(\vec{q})\Lambda_\mu u_\alpha Q_{n'p}(k_2)\,M^{abs}_{\alpha\beta}(ns\to n'p)\} \tag{5}$$

where $a = (aZm)^{-1}$ and α, β are the spin labels of the $n'p$ and ns states, respectively. The function $Q_{n'p}$ is defined in Ref. 4 and could be found with the approach described by De Rújula. The spin functions of the neutrino and electron are v and u, respectively. The momentum of the neutrino is $\vec{q}$ and its spin label is ν. The absorption matrix elements $M^{abs}_{\alpha\beta}$ are calculated in the non-relativistic limit as

$$M^{abs}_{\alpha\beta}(ns\to n'p) = M^{abs}\,\delta_{\alpha\beta} \tag{6}$$

where

$$M^{abs} = -\frac{1}{m}\int \mathrm{d}\vec{r}\,\varphi_{n'p}\,\vec{e}_1\cdot\vec{p}\,\varphi_{ns}. \tag{7}$$

In Eq. (7) $\vec{e}_1$ represents the incident photon polarization vector and the Schrödinger-Coulomb wave function $\varphi' s$ are used, the $\varphi_{n'p}$ states being defined in the frame defined by the emitted photon polarization vector $\vec{e}_2$.

The differential cross section for the induced IB is calculated to be

$$\frac{\mathrm{d}\sigma}{\mathrm{d}k_2\,\mathrm{d}\Omega_2} = \frac{2}{\pi}\,a^2 a^2\,\frac{W(n''s)}{E_{n'p}-E_{n's}}\,\frac{1}{E^2_{\max}(n''s)\,\Gamma}\,\frac{\varphi^2_{ns}(0)}{\varphi^2_{n''s}(0)} \sum_{\mathrm{pol}} |M^{abs}|^2\,k_2\,(E_{\max}(n'p)-k_2)^2\,Q^2_{n'p}(k_2) \tag{8}$$

where $n'' > n$ and $W(n''s)$ is the EC probability for the $n''s$ electron state. The summation in Eq. (8) is over the incident and emitted photon polarizations.

3. *Discussion*

We can compare the relative importance of the two mechanisms by estimating the ratio R of the resonance cross section (Eq. 8) to the high energy cross section (Eq. 2). Assuming an initial $1s$ electron state and $1s \to 2p$ resonance absorption we find for $\Theta = 90^\circ$

a) For $k_1 \sim k_2 \ll m$

$$R \sim \left(\frac{k_1}{\Gamma}\right)^2 \left(\frac{k_1}{m}\right)^2 Q^2_{2p}(k_2) \tag{9}$$

b) For $k_1 \ll k_2 \sim m$

$$R \sim \left(\frac{m}{\Gamma}\right)^2 \frac{k_1}{m}\,Q^2_{2p}(k_2) \tag{10}$$

As the level width Γ is much less than k_1 and the electron mass m, we find the resonance mechanism to be generally dominant and more favourable for a neutrino mass determination.

As in ordinary IBEC the neutrino mass can in principle be determined from the shape of the IB near the end point. The number of events in the region of interest can be determined by integration of Eq. (8). For a neutrino mass m_ν we obtain

$$\sigma_g = \int_{4\pi} \mathrm{d}\Omega \int_{E_{max}(n'p)-m_\nu}^{E_{max}(n'p)} \mathrm{d}k \left[\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega\, \mathrm{d}k}\right]. \tag{11}$$

For $m_\nu = 30$ eV we obtain $\sigma_g \sim 4 \times 10^{-58}$ cm^2 for photon induced IBEC with ^{193}Pt, assuming resonance absorption via the $1s \to 2p_{3/2}$ electron states. In the case of photon induced IBEC with ^{163}Ho, assuming resonance absorption via the $2s \to 3p_{3/2}$ electron states, we obtain $\sigma_g \sim 10^{-56}$ cm^2.

Alternatively one may conceive of an experiment involved with the threshold sensitivity to neutrino mass. But the cross sections for the complete IB energy range are only 1.1×10^{-50} cm^2 for ^{193}Pt, and 3.4×10^{-53} cm^2 for ^{163}Ho. Such small cross sections preclude not only any realistic determination of the neutrino mass but also the possibility of detecting photon-induced IB process.

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INDUCIRANO UNUTRAŠNJE ZAKOČNO ZRAČENJE PRI UHVATU ELEKTRONA

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Izračunat je udarni presjek za proces unutrašnjeg zakočnog zračenja pri uhvatu elektrona, koji je induciran upadnim fotonom. Razmatrana je i analizirana mogućnost da se ovaj proces upotrijebi za određivanje mase neutrina.