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THE HIGHER PION-NUCLEON SCATTERING LENGHTS FROM THE HYPERBOLIC PARTIAL WAVE DISPERSION RELATIONS*

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The higher pion nucleon scattering lenghts are obtained by use of the hyperbolic partial wave relations. It is shown that only contributions from the one nucleon exchange and nearby parts of the t channel cut are needed for a good description of higher scattering lengths. The latest results from the Karlsruhe Helsinki 78(80) partial wave analysis have been used as input.

1. Introduction

The existing data sets for the pion-nucleon differential cross sections and polarisation parameters have appreciable errors and do not cover the whole kinematical region. For these reasons, it is not possible to determine uniquely, from the data alone, all the partial waves which give an appreciable contribution to the measurable quantities.

The aim of this paper is to present results concerning the pion-nucleon scattering lengths for higher orbital momenta.

Several theoretical methods, based on the Mandelstam analyticity of the scattering amplitudes[1], have been used for that purpose.

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The fixed-s dispersion relations (FSDR) for pion-nucleon scattering amplitudes have been used by Koch[2] and Koch and Otto[3] to derive the pion-nucleon scattering lengths for orbital momenta $l < 8$.

The partial wave relations (PWR) and the hyperbolic partial wave relations (HPWR) have been obtained by projections from the fixed t dispersion relations[4] and the hyperbolic dispersion relations[5].

The partial wave dispersion relations (PWDR) have been derived by use of analyticity of the pion-nucleon partial waves[6].

The PWR and HPWR are more direct and natural methods than the FSDR.

As the FSDR approach demands the double fitting procedure, the other three methods allow the direct pion-nucleon scattering lengths calculations. Calculations, by the PWDR method, carried out by Koch and Otto[7], show that evaluations become more and more numericaly complicated as the orbital momentum l increases.

To calculate one of the pion-nucleon partial waves, all other partial waves are needed as input in the PWR and HPWR methods.

Very important t-channel contributions are absent in the PWR method, so that the higher pion-nucleon partial waves, which are not well defined in the present partial wave analysis, are needed. Being derived from the dispersion relations along curves connecting the direct s-and crossed t-channels, HPWR method allows saturation with very few contributions.

As far as the higher pion-nucleon partial waves are concerned, the HPWR method is the most powerful among the mentioned methods.

2. *The hyperbolic partial wave relations*

The dispersion relations along hyperbolas in the Mandelstam plane:

$$(s - a)(u - a) = b \tag{2.1}$$

have been studied by Hite and Steiner[5].

It is possible to chose a subset of hyperbolas (2.1) which fulfills the following conditions:

- it is possible to obtain partial wave projection,
- along the s-and t-channel cuts, respective, partial wave expansions are convergent.

For a fixed value of the parameter a, parameter b is chosen in such a way that hyperbola passes the point s_1, t_1 in the physical region of the Mandelstam plane:

$$(s - a)(u - a) = b(s_1, t_1, a) = (s_1 - a)(2m^2 + 2\mu^2 - s_1 - t_1 - a)$$

(m = nucleon mass; μ = pion mass).

The dispersion relations along these hyperbolas read[5]:

$$\mathrm{Re}\, F^+ (s, t, u) = F_N^+ (s, t, u) + \frac{1}{\pi} \int\limits_{(m+\mu)^2}^{\infty} \mathrm{Im}\, F^+ (s', t', u') \left[\frac{1}{s'-s} + \frac{1}{s'-u} - \frac{1}{s'-a}\right] \mathrm{d}s' + \frac{1}{\pi} \int\limits_{4\mu^2}^{\infty} \mathrm{Im}\, F^+ (t', Z_{t'}) \frac{\mathrm{d}t'}{t'-t},$$

$$\mathrm{Re}\, F^- (s, t, u) = F_N^- (s, t, u) + \frac{1}{\pi} \int\limits_{(m+\mu)^2}^{\infty} \mathrm{Im}\, F^- (s', t', u') \left[\frac{1}{s'-s} - \frac{1}{s'-u}\right] \mathrm{d}s' + \int\limits_{4\mu^2}^{\infty} \mathrm{Im} \left(\frac{\nu}{\nu'} F^- (t', Z_{t'})\right) \frac{\mathrm{d}t'}{t'-t}. \tag{2.3}$$

The $F^\pm$ are crossing even and odd pion nucleon invariant amplitudes. $F_N^\pm$ are the nucleon pole term contributions; $s_{th} = (m + \mu)^2$; $\nu = (s - u)/4m$ and Z_t = cosine of the t-channel scattering angle. Expanding the absorptive parts of the invariant amplitudes in terms of the partial waves, and projecting out the real parts using the projection formulas, the following partial wave relations[5] are obtained:

$$\mathrm{Re}\, F_{l+}^{\pm} (w) = N_{l+}^{\pm} (w) + \frac{1}{\pi} \int\limits_{m+\mu}^{\infty} \mathrm{d}w' \sum_{l'=0}^{\infty} \{K_{ll'}^{\pm} (w', w)\, \mathrm{Im}\, F_{l'+}^{\pm} (w') + K_{ll'}^{\pm} (-w', w)\, \mathrm{Im}\, F_{(l'+1)-}^{\pm} (w')\} + \frac{1}{\pi} \int\limits_{4\mu^2}^{\infty} \mathrm{d}t \sum_{J=0}^{\infty} \{G_{lJ} (w, t)\, \mathrm{Im} f_+^J (t) + H_{lJ} (w, t)\, \mathrm{Im} f_-^J (t)\}, \tag{2.4}$$

$$\mathrm{Re}\, F_{(l+1)-}^{\pm} (w) = N_{(l+1)-}^{\pm} (w) - \frac{1}{\pi} \int\limits_{m+\mu}^{\infty} \mathrm{d}w' \sum_{l'=0}^{\infty} \{K_{ll'}^{\pm} (w' - w)\, \mathrm{Im}\, F_{l'+}^{+} (w') + K_{ll'}^{\pm} (-w' - w)\, \mathrm{Im}\, F_{(l'+1)-}^{\pm} (w')\} + \frac{1}{\pi} \int\limits_{4\mu^2}^{\infty} \mathrm{d}t \sum_{J=0}^{\infty} \{G_{lJ} (-w, t)\, \mathrm{Im} f_+^J (t) + H_{lJ} (-w, t)\, \mathrm{Im} f_-^J (t)\},$$

where w is the total energy in the pion-nucleon centre of mass frame. $F^{\pm}_{l\pm}$ are the s-channel isospin even and odd combinations of the reduced partial waves:

$$F^{+}_{l\pm}(w) = \frac{1}{3}\left(F^{1/2}_{l\pm}(w) + 2\,F^{3/2}_{l\pm}(w)\right);$$

$$F^{-}_{l\pm}(w) = \frac{1}{3}\left(F^{1/2}_{l\pm}(w) - F^{3/2}_{l\pm}(w)\right); \tag{2.5}$$

$$F^{I}_{l\pm}(w) = \frac{e^{2i\delta^{I}_{l+}(w)} - 1}{2\,i\,q^{2l+1}};\; F^{I}_{(l+1)-}(w) = \frac{e^{2i\delta^{I}_{(l+1)-}(w)} - 1}{2\,i\,q^{2l+2}};\; I = 1/2,\ 3/2,$$

where q is the momentum in the pion-nucleon centre of mass frame, and $\delta^{I}_{l\pm}$ corresponding phase shifts.

$f^{J}_{\pm}$ are the t-channel partial waves with parallel and antiparallel nucleon-antinucleon helicities, as defined by Fraser and Fulco[8].

Due to crossing symmetry the summation in (2.4) goes over even (odd) J for even (odd) isospin combinations of the s-channel partial waves.

The kernels $K^{\pm}_{ll'}$ connect the real part of a given partial wave, with orbital momentum l, to the imaginary parts of all s-channel partial waves.

They reproduce the s-channel cut as well as the left u-channel cut.

The kernels G_{lJ} and H_{lJ} connect the real part of a given s-channel partial wave to the imaginary parts of the t channel $\pi\pi \rightarrow N\bar{N}$ partial waves, and reproduce the t-channel cuts-the circle cut and the left hand part of the t-channel cut[5]. $N^{\pm}_{l\pm}$ are projections of the nucleon pole term.

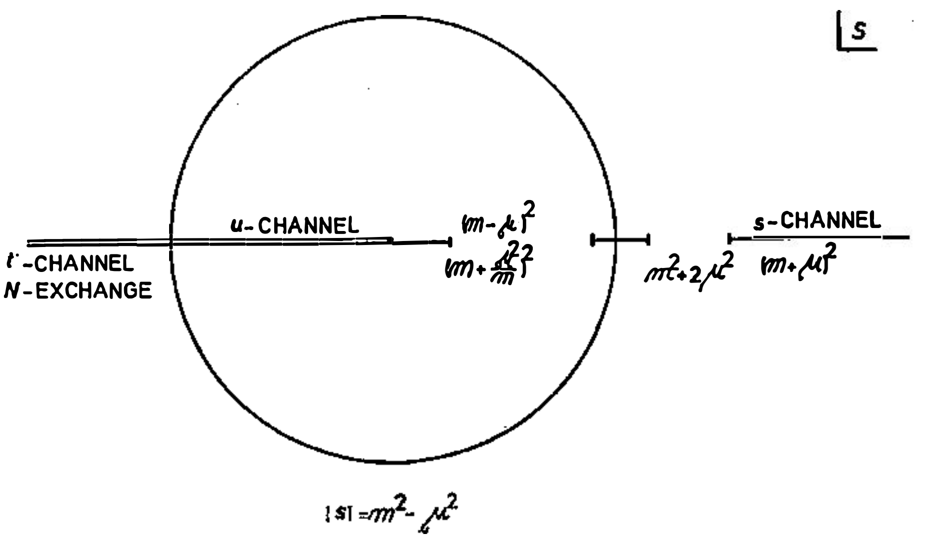

Fig. 1. The cut structure of the pion-nucleon partial waves in the complex s-plane.

The cut structure of the pion nucleon partial waves is shown in Fig. 1. The pion-nucleon scattering lengths are defined as usual,

$$a_{l\pm} = \lim_{q^2 \to 0} F^{\pm}_{l\pm}(w).$$

3. The dominant contributions

For the nonrelativistic potential scattering, it has been shown by Martin[9] that at position s' on the unphysical cut there is no contribution from any component of potential whose range is less than quantity $a(s')$ where $a(s')$ decreases as s' moves along the unphysical cut away from the physical cut.

We cannot apply this result to the pion-nucleon scattering. It is preferable to define a range of interaction associated with position s on any unphysical cut:

$$R = \frac{\hbar}{c\,|m + \mu - w|}. \tag{3.1}$$

Now we have a good description of the pion-nucleon longer range interaction ($R \gtrsim 2.5 \times 10^{-16}$ m).

The crossed cut $-\infty < s < (m - \mu)^2$ and the arc $|\varphi| > 66°$ of the circle cut are associated with range of interaction $R < 2 \times 10^{-16}$m.

For $l > 1$ the centrifugal barrier prevents pions of low and medium energies from penetrating to such short ranges.

The dominant contributions to the higher pion-nucleon partial waves at low energies come from the nearest singularities; the front of the circle cut and the short nucleon exchange cut $\left(m + \frac{\mu^2}{m}\right)^2 < s < m^2 + 2\mu^2$ with associated ranges $R \approx \hbar/\mu c$.

The front of the circle cut is connected with the low energy $\pi\pi \to \pi\pi$ scattering. The expresions given bellow are approximate. More details can be found in Refs. 5 and 10.

The one-nucleon exchange contributions to the pion-nucleon scattering lengths are given by:

$$a^{\pm}_{Nl+} = \pm \frac{m f^2 (-)^l}{w_{th}\, \mu^2 (2m\mu)^l} \frac{\Gamma(l+1)}{\Gamma(l+3/2)},$$

$$a^{\pm}_{N(l+1)-} = \pm \frac{m f^2 (-)^l}{w_{th}\, \mu^2 (2m\mu)^{l+1}} \frac{\Gamma(l+1)}{\Gamma(l+3/2)} \frac{1}{2l+3}. \tag{3.2}$$

Relations above give the ratio:

$$\frac{N^{\pm}_{l+}}{N^{\pm}_{(l+1)-}} = 2\,m\,\mu\,(2\,l+3). \tag{3.3}$$

One-nucleon exchange contribution to the scattering lengths $a^{\pm}_{l+}$ (no spin flip) are for higher orbital momenta greater than those to the scattering lengths $a^{\pm}_{(l+1)-}$ (spin flip). Explanation of this effect, based on the orbital momentum consideration, are given in Ref. 11.

The t-channel cut contributions can be calculated using the t-channel $\pi\pi \to N\tilde{N}$ partial waves known in the region $4\mu^2 < t \lesssim \mathrm{GeV}^2$. Higher, d and f, $\pi\pi \to N\tilde{N}$ partial waves in this region are small and negligible compared to the s and p waves.

This simplifies our considerations and makes it possible to approximate the t-channel cut contributions with only one term in each summ (2.4).

The final expresions read:

$$a^{+}_{T l+} = \frac{1}{\pi}\int_{4\mu^2}^{\infty} \mathrm{d}t\, G_{l0}(w_{th}, t)\,\mathrm{Im} f^0_+(t),$$

$$a^{+}_{T(l+1)-} = -\frac{1}{\pi}\int_{4\mu^2}^{\infty} \mathrm{d}t\, G_{l1}(-w_{th}, t)\,\mathrm{Im} f^0_+(t), \tag{3.4}$$

$$a^{-}_{T l+} = \frac{1}{\pi}\int_{4\mu^2}^{\infty} \mathrm{d}t\,\{G_{l1}(w_{th}, t)\,\mathrm{Im} f^1_+(t) + H_{l1}(w_{th}, t)\,\mathrm{Im} f^1_-(t)\},$$

$$a^{+}_{T(l+1)-} = -\frac{1}{\pi}\int_{4\mu^2}^{\infty} \mathrm{d}t\,\{G_{l1}(-w_{th}, t)\,\mathrm{Im} f^1_+(t) + H_{l1}(-w_{th}, t)\,\mathrm{Im} f^1_-(t)\}.$$

At the s-channel threshold, the kernels $G_{l1}(\pm w_{th}, t)$, $H_{l1}(\pm w_{th}, t)$, $G_{l0}(\pm w_{th}, t)$ are given by the following expressions[10]:

$$G_{l1}(w_{th}, t) = \frac{3\,m\sqrt{\pi}\,(2\,m\,\mu + t)}{8\,w_{th}^2\,p_t^2}\,\frac{1}{t^{l+1}}\,\frac{\Gamma(l+1)}{\Gamma(l+3/2)},$$

$$H_{l1}(w_{th}, t) = \left[4\mu + 2m\frac{4\,m\,\mu + t}{p_t^2}\right]\frac{1}{4\sqrt{2}\,w_{th}\,t^{l+1}}\,\frac{\Gamma(l+1)}{\Gamma(l+3/2)}, \tag{3.4a}$$

$$G_{l1}(-w_{th}, t) = -\frac{3(4m\mu + t)\sqrt{\pi}}{8w_{th}mt}\left[1 + \frac{m^2}{(2l+3)p_t^2}\right]\frac{1}{t^{l+1}}\frac{\Gamma(l+1)}{\Gamma(l+3/2)},$$

(3.4b)

$$H_{l1}(-w_{th}, t) = \frac{3\sqrt{\pi}}{4\sqrt{2}}\left[\frac{1}{2} - \frac{m^2}{(2l+3)p_t^2}\right]\frac{1}{t^{l+1}}\frac{\Gamma(l+1)}{\Gamma(l+3/2)},$$

$$G_{l0}(w_{th}, t) = -\frac{m\sqrt{\pi}}{2w_{th}p_t^2}\frac{1}{t^{l+1}}\frac{\Gamma(l+1)}{\Gamma(l+3/2)},$$

(3.4c)

$$G_{l0}(-w_{th}, t) = -\frac{\sqrt{\pi}}{2w_{th}t}\left[1 + \frac{m^2}{(2l+3)p_t^2}\right]\frac{1}{t^{l+1}}\frac{\Gamma(l+1)}{\Gamma(l+3/2)},$$

where $p_t^2 = t/4 - m^2$ is the nucleon momentum in the nucleon-antinucleon centre of mass frame, and $w_{th} = (m + \mu)$.

With these approximations, following integrands for the t-channel cut contributions are obtained:

$$G_{l1}(w_{th}, t)\,\mathrm{Im}f_+^1(t) + H_{l1}(w_{th}, t)\,\mathrm{Im}f_-^1(t) \approx$$

$$-\frac{3\sqrt{\pi}}{2w_{th}t^{l+1}}\left[\mathrm{Im}\,\Gamma_2(t)\frac{t}{4m} - \mu\,\mathrm{Im}\,\Gamma_1(t)\right]\frac{\Gamma(l+1)}{\Gamma(l+3/2)},$$

$$G_{l1}(-w_{th}t)\,\mathrm{Im}f_+^1(t) + H_{l1}(-w_{th}, t)\,\mathrm{Im}f_-^1(t) \approx$$

$$-\frac{3\sqrt{\pi}}{4w_{th}t^{l+1}}\left\{\frac{4m\mu + t}{2t}\left[1 + \frac{m^2}{(2l+3)p_t^2}\right]\mathrm{Im}f_+^1(t) + \right.$$

$$\left. + \frac{m}{\sqrt{2}}\left[\frac{1}{2} - \frac{m^2}{(2l+3)p_t^1}\right]\mathrm{Im}f_-^1(t)\right\}\frac{\Gamma(l+1)}{\Gamma(l+3/2)}, \qquad (3.5)$$

$$G_{l0}(w_{th}, t)\,\mathrm{Im}f_+^0(t) \approx -\frac{\sqrt{\pi}m}{2w_{th}p_t^2}\frac{1}{t^{l+1}}\frac{\Gamma(l+1)}{\Gamma(l+3/2)}\mathrm{Im}f_t^0(t),$$

$$G_{l0}(-w_{th}, t)\,\mathrm{Im}f_+^0(t) \approx -\frac{\sqrt{\pi}}{2w_{th}p_t}\frac{\Gamma(l+1)}{\Gamma(l+3/2)}\frac{\mathrm{Im}f_+^0(t)}{t^{l+1}}.$$

Here $\Gamma_1(t)$ and $\Gamma_0(t)$ are the linear combinations of the t-channel p waves:

$$\Gamma_1(t) = -\frac{m}{p_t^2}\left[f_+^1(t) - \frac{t}{4\sqrt{2}m}f_-^1(t)\right]; \quad \Gamma_2(t) = \frac{m}{p_t^2}\left[f_+^1(t) - \frac{m}{\sqrt{2}}f_-^1(t)\right].$$

The term $t^{-(l+1)}$ causes sharp cut-off for higher orbital momenta. As a consequence, very accurate t-channel low energy partial waves are needed for evaluation of integrals, given in (3.4).

As can be seen from (3.5a), the t-channel contributions to the scattering lenghts a^{-}_{l+} are small owing to cancelation of terms in the bracket; the one nucleon exchange contribution dominates.

For $l \gtrsim 6$ the second term in the bracket becomes small compared to the first, so that the t-channel contributions growe up.

It has been shown that for very high orbital momenta ($l \geq 20$ say) the one nucleon exchange is negligible compared to the t-channel cut contributions[10].

In the corresponding expression for the $a^{-}_{(l+1)-}$, given by (3.5b), there is no cancelations. The t-channel contributions to the s-channel scattering lengths dominate.

Because of the cut-off factor t^{-l+1}, the t-channel cut is well approximated by the interval $4\mu^2 < t < 4\mu^2 + t_1$, where t_1 approaches zero as l increases.

We conclude that the t-channel contributions to higher pion-nucleon scattering lenghts can be explained neither by the ϱ exchange ($J = 1$; $I = 1$) nor by the $(\pi\pi)^0_0$ pair exchange (»ε exchange« $J = 0$; $I = 0$).

The results of our calculations are displayed in Tables 1 and 2. The u-channel contributions, although small, are also listed. As it has been shown in Ref. 10 these contributions are saturated by the p-waves which are dominated by the nucleon isobar Δ (1232) exchange.

TABLE 1.

l	$a^{+}_{l\pm}$	u-chanel	t-channel	n-exchange
3−	1.327×10^{-4}	9.06×10^{-6}	1.661×10^{-4}	-3.755×10^{-5}
3+	4.147×10^{-4}	1.533×10^{-6}	1.733×10^{-4}	2.363×10^{-4}
4−	2.311×10^{-5}	-2.193×10^{-7}	2.131×10^{-5}	1.984×10^{-6}
4+	5.225×10^{-6}	-4.516×10^{-8}	2.211×10^{-5}	-1.688×10^{-5}
5−	3.120×10^{-6}	4.839×10^{-3}	3.23×10^{-6}	-1.143×10^{-7}
5+	4.58×10^{-6}	8.332×10^{-10}	3.331×10^{-6}	1.233×10^{-6}
6−	5.483×10^{-7}	-1.161×10^{-10}	5.414×10^{-7}	6.947×10^{-9}
6+	4.659×10^{-7}	-9.65×10^{-12}	5.575×10^{-7}	-9.533×10^{-8}
7−	9.7×10^{-8}	2.679×10^{-12}	9.745×10^{-8}	-4.367×10^{-10}
7+	1.07×10^{-7}	3.329×10^{-13}	1.001×10^{-7}	6.848×10^{-10}
8−	1.849×10^{-8}	-7.003×10^{-14}	1.846×10^{-8}	2.868×10^{-11}

The isospin even combinations of the pion-nucleon scattering lenghts $a^{+}_{l\pm}$ for $3 \leq l \leq 8$ multiplied by μ^{2l+1}.

TABLE 2.

l	$a^{+}_{l\pm}$	u-channel	t-channel	n-exchange
3−	9.32×10^{-5}	4.78×10^{-6}	5.24×10^{-5}	3.76×10^{-5}
3+	-2.47×10^{-4}	8.93×10^{-7}	-9.74×10^{-6}	-2.36×10^{-4}
4−	1.86×10^{-6}	-1.07×10^{-7}	2.96×10^{-6}	-1.98×10^{-6}
4+	1.65×10^{-5}	-2.15×10^{-8}	-3.38×10^{-7}	1.69×10^{-5}
5−	5.56×10^{-7}	2.36×10^{-9}	4.42×10^{-7}	1.14×10^{-7}
5+	-1.23×10^{-6}	1.292×10^{-9}	2.05×10^{-9}	-1.23×10^{-6}
6−	5.54×10^{-8}	-4.84×10^{-11}	6.23×10^{-8}	-6.95×10^{-9}
6+	9.53×10^{-8}	-1.83×10^{-11}	4.16×10^{-9}	9.15×10^{-8}
7−	1.05×10^{-8}	2.23×10^{-13}	1.0×10^{-8}	4.4×10^{-10}
7+	-5.75×10^{-9}	4.15×10^{-13}	1.1×10^{-9}	-6.85×10^{-9}
8−	1.72×10^{-9}	-3.01×10^{-14}	1.74×10^{-9}	-2.87×10^{-11}

The isospin odd combinations of the pion-nucleon scattering lenghts a^- for $3 \leq l \leq 8$ multiplied by μ^{2l+1}.

3. *Numerical evalution and discussion*

To calculate the pion-nucleon scattering lengths using the HPWR, the s-channel as well the t-channel partial waves are needed. In our calculation, the s-channel partial wave input consists of the latest Karlsruhe-Helsinki 78 (80) partial wave analysis[12]. The analysis uses the fixed-t dispersion relations as a theoretical constraint to insure a unique solution. The results are available in the region:

$$20 \text{ MeV}/ck \lesssim 10 \text{ GeV}/c,$$

where k is the pion momentum in the pion-nucleon laboratory frame.

The t-channel $\pi\pi \to N\tilde{N}$ partial waves have been taken from the Karlsruhe--Helsinki 78 t-channel partial wave analysis[12,13]. The $\pi\pi \to N\tilde{N}$ partial waves up to $J = 3$ (s, p, d and f) are derived in the region $4\,\mu^2 < t \lesssim 1\text{ GeV}^2$.

The $\pi\pi \to \pi\pi$ partial waves have been used as an additional experimental input. This insures a precise determination of the $\pi\pi \to N\tilde{N}$ partial waves near to the t-channel threshold $t = 4\mu^2$, as required for the determination of the pion--nucleon higher scattering lenghts.

Although we applied the HPWR method to calculate the scattering lengths for higher orbital momenta, some comments on applications to the lower scatering lenghts should be made. To apply the HPWR method to low momenta scattering lenghts the t-channel input for $t > 1\text{ GeV}^2$ is needed.

Some information on the two pion resonances ϱ', ϱ'', g, f, h which lies in this region are awailable[14].

A new $p\tilde{p} \to \pi\pi$ scattering and polarisations data[15] have been the basis for a partial wave analysis recently carried out by Martin and Morgan[16], but up till now authors did not find a unique solution. To awoid all these problems the best way to calculate the pion-nucleon scattering lengths for low momenta is to apply the HPWR derived from once suptracted hyperbolic dispersion relations[17].

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VIŠE PION-NUKLEONSKE DULJINE RASPRŠENJA IZ HIPERBOLIČKIH DISPERZIJSKIH RELACIJA ZA PARCIJALNE VALOVE

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Više pion-nukleonske duljine raspršenja računate su pomoću hiperboličkih disperzijskih relacija za parcijalne valove. Pokazano je da je za opisivanje viših duljina raspršenja dovoljno uzeti u obzir jednonukleonsku izmjenu i najbliže dijelove reza u t-kanalu. Računi su provedeni na temelju posljednjih podataka o parcijalnim valovima što su ih dobile grupe u Helsinkiju i Karlsruheu 1978 (1980).