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LETTER TO THE EDITOR

FIXED-X EXPECTATION VALUES OF QUARK TRANSVERSE MOMENTUM IN DEEP INELASTIC LEPTON — HADRON PROCESSES*

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We discuss x-dependent k_T distribution obtained in a simple kinematical model. Explicit analytical calculations, using relativistic potential model quark wave functions, are performed.

We consider processes $\gamma + H \to X$, with four-momenta q, P and W for the photon, the hadron and the inclusive collision debris, respectively.

Generally speaking γ could represent a real photon or a virtual electromagnetic signal, space-like or time-like. If, in the spirit of the parton model, we interpret the hadron as a system of organized quarks such that an assault by the intruding photon is first totally absorbed by a single quark, the kinematics requires energy-momentum conservation

$$q + k = k', \tag{1}$$

where k and k' denote the quark four-momenta before and after the photon-quark collision. The act of photon absorption would, generally, turn the recipient quark into a virtual particle with subsequent higher-order decays via intermediary photons or gluons.

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Since our intention, however, is to treat the hadron within confining potential models (i. e. as a system of quarks subject to an averaged quantum-mechanical relativistic potential), photon absorption will not change the quark mass but only its energy in the potential field. This is equivalent to the statement

$$k'^2 = k^2 \tag{2}$$

and requires, by Eq. (1), that the incoming photon is spacelike

$$q^2 < 0. \tag{3}$$

This restricts the applicability of the approach to the lepton-hadron scattering so that the experimental values for q are set by the lepton four-momenta l and l'. As usual, kinematical invariants are expressed either as q^2 and $\nu = q \cdot P/M$, or as the dimensionless pair

$$x = -q^2/(2P \cdot q),\ y = P \cdot q/(P \cdot l). \tag{4}$$

Note that

$$0 \leq x \leq 1 \tag{5}$$

where positiveness of x follows from the lepton vertex and manifests the lepton mass conservation, $l^2 = l'^2$, while the upper limit $x \leq 1$ follows from the hadronic vertex with

$$q + P = W \tag{6}$$

and the requirement

$$W^2 \geq M^2. \tag{7}$$

Strictly speaking Eq. (7) is guaranteed only for nucleonic targets and protects the baryon number conservation against the lightest baryon induced decay. In a general case, with no quantum number restrictions, W would represent a process with a mass increase or a mass defect and Eq. (7) would not be justified.

We went into discussing in some detail the background of the well known restriction (5), in order to make it clear that a confining-potential quark model would not have Eq. (5) inherently built-in but would require a corresponding input in the selection of parameters.

The four-momentum q of the virtual photon, and the initial four-momentum k of the quark before it received the electromagnetic message combine by (1) and (4) into

$$M x = k^0 - (|\vec{q}|/q^0)\ |\vec{k}| \cos \Theta \tag{8}$$

with the angle Θ subtended by $\vec{k}$ and $\vec{q}$. The parton model predictions become significant for large energy transfers, and writing (5) as

$$0 \leq (|\vec{q}|/q^0)^2 - 1 \leq 2M/q^0 \tag{9}$$

shows that the limit of large q^0 implies $|\vec{q}|/q^0 \rightarrow 1$.

So we have

$$x = (k^0 - |\vec{k}| \cos \Theta)/M. \tag{10}$$

Note that although, a priori, x and k represent totally unrelated quantities (one standing for the incoming electromagnetic signal and the other for the four-momentum before collision of one of the constituent quarks) kinematical constraints produce, in the Eq. (10), rather stringent limitations on the allowed absorption processes.

As long as we use the asymptotic freedom property of QCD and treat quarks as free particles inside the hadron Eq. (10) is a numerical-assignement prescription for the values of x. Since, however, we intend to subject quarks to an averaged interquark potential, which hopefully simulates confinement, Eq. (10) should be reinterpreted as a statement connecting non-commuting quantum-mechanical operators. The simplest bag model or potential model implementations assume rotational symmetry and, consequently, the quark wave functions to the corresponding Dirac equation will have simultaneous specification of energy and angular momentum but not of the momentum. With reference to Eq. (10) one should, therefore, determine the momentum probability density $P(\vec{k}) = w(|\vec{k}|)$ and, subsequently, the probability $P(x)$.

In the *LAB*-frame the resulting decay momentum $\vec{W}$ coincides with the momentum transfer $\vec{q}$ and the momenta of the individual constituent quarks are viewed with respect to the $\vec{q}$-direction. Of particular interest is the transverse momentum k_T, defined by the assignement

$$k_T = |\vec{k}| \sin \Theta. \tag{11}$$

Eqs. (10) and (11) imply that the eigenvalues of both operators, x and k_T, define classes in the quark momentum space and, consequently, we can determine the conditional probability $P(k_T; x)$, of a k_T-value occurance for a given x, as

$$P(k_T; x) = P(k_T \cap x)/P(x), \tag{12}$$

where $P(k_T \cap x)$ is the *AND* probability for k_T and x. The expectation value of the random variable k_T (for fixed x) is given by

$$\langle k_T \rangle = \langle k_T(x) \rangle = \int_0^\infty k_T\, P(k_T; x)\, \mathrm{d}k_T. \tag{13}$$

The probabilities which enter Eq. (12) are quite easy to determine if we know the $\vec{k}$ distribution function $P(\vec{k}) = w(|\vec{k}|)$: Eqs. (10), (11) define k_T and x as functions of the random variable $\vec{k}$, and therefore[1]

$$P(k_T \cap x) = \int \mathrm{d}^3k\, P(\vec{k})\, \delta(x - (k^0 - |\vec{k}| \cos \Theta)/M)\, \delta(k_T - |\vec{k}| \sin \Theta)$$

$$= 2\pi M k_T w(\{k_T^2 + (k^0 - Mx)^2\}^{1/2}). \tag{14}$$

The x probability density is

$$P(x) = \int_0^\infty P(k_T \cap x)\, \mathrm{d}k_T = 2\pi M \int_{|k^0 - Mx|}^\infty z\, w(z)\, \mathrm{d}z. \tag{15}$$

Finally, $P(k_T; x)$ is given by (12).

Now, a natural choice for $\vec{k}$ distribution function is

$$P(\vec{k}) = |\psi(\vec{k})|^2 \tag{16}$$

where $\psi(\vec{k})$ is an appropriate quark wave function, e. g. the bag model wave function[1], in $\vec{k}$ representation. Of course, for the ground state $P(\vec{k})$ is spherically symmetric, $P(\vec{k}) = w(|\vec{k}|)$.

For $\psi(\vec{k})$ we shall take the relativistic potential model quark wave function[2]

$$\psi(\vec{k}) = N \exp\left(-\frac{\vec{k}^2}{2\xi^2}\right) \begin{pmatrix} U \\ \dfrac{\vec{k} \cdot \vec{\sigma}}{k^0 + m} U \end{pmatrix}. \tag{17}$$

Simple calculation results in

$$P(x) = \frac{M \pi^{-1/2}}{\xi} \frac{(k^0 + m)^2 + \xi^2 + (k^0 - Mx)^2}{(k^0 + m)^2 + 3\xi^2/2} \exp\left\{-\frac{(k^0 - Mx)^2}{\xi^2}\right\}, \tag{18}$$

$$P(k_T; x) = \frac{2 k_T}{\xi^2} \left\{1 + \frac{k_T^2 - \xi^2}{\xi^2 + (k^0 + m)^2 + (k^0 - Mx)^2}\right\} \exp\left\{-\frac{k_T^2}{\xi^2}\right\}, \tag{19}$$

$$\langle k_T(x) \rangle = \frac{\pi^{1/2} \xi}{2} \left\{1 + \frac{\xi^2/2}{\xi^2 + (k^0 + m)^2 + (k^0 - Mx)^2}\right\}, \tag{20}$$

$$\langle k_T^2(x) \rangle = \xi^2 \left\{1 + \frac{\xi^2}{\xi^2 + (k^0 + m)^2 + (k^0 - Mx)^2}\right\}. \tag{21}$$

The function $P(x)$ and the expectation values $\langle k_T(x)\rangle$, $\langle k_T^2(x)\rangle$ are calculated for three sets of parameters:

I $m = 0.272$ GeV, $k^0 = 0.782$ GeV, $\xi = 0.43$ GeV (relativistic potential model[2]),

II $m = 0$, $k^0 = 0.648$ GeV, $\xi = 0.366$ GeV (the same model with zero quark mass),

III $m = k^0 = 0.33$ GeV, $\xi = 0.243$ GeV (harmonic oscillator shell model — HOS[3]).

The corresponding curves are shown in Figs. 1, 2, 3.

The structure function $P(x)$ shown in Fig. 1 has proper behaviour at $x \to 1$ for the set III of parameters (HOS model). In the $x \to 0$ region it is difficult to compare with phenomenologically determined shape[4] because of the dominant quark-antiquark pair production, which is not possible to describe in this simple approach.

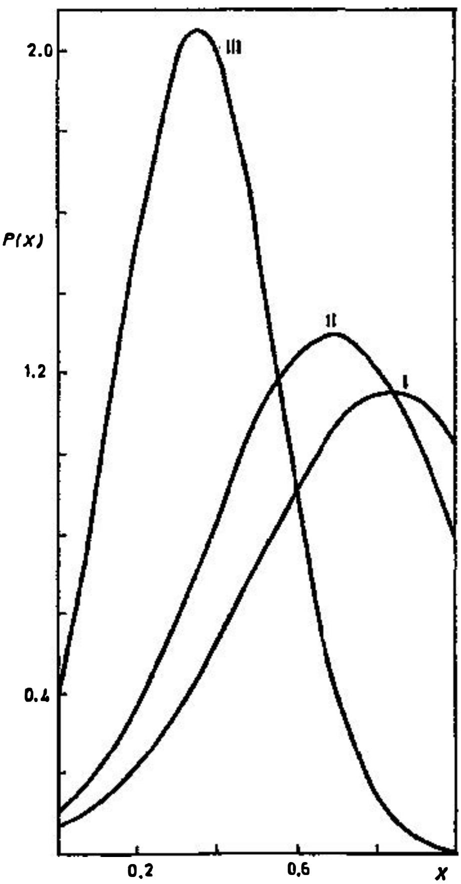

Fig. 1. Plot of the structure function $P(x)$ for the three sets of parameters described in the text.

The I and II curves for $P(x)$ do not behave satisfactorily at $x \to 1$ and they both peak at too large x values ($x = k^0/M$). The agreement of $\langle k_T(x)\rangle$, $\langle k_T^2(x)\rangle$ curves with phenomenology is fairly good in comparison with Refs. 1 and 5. It is interesting that the results obtained in a somewhat different approach[6] qualitatively disagree with the present calculation.

In conclusion it should be stressed that, although the picture of the free valence quarks is completely valid only in the infinite momentum frame, the presented simplified treatment offers an opportunity to compare the working of various models of quark confinement in the partonlike region of energies.

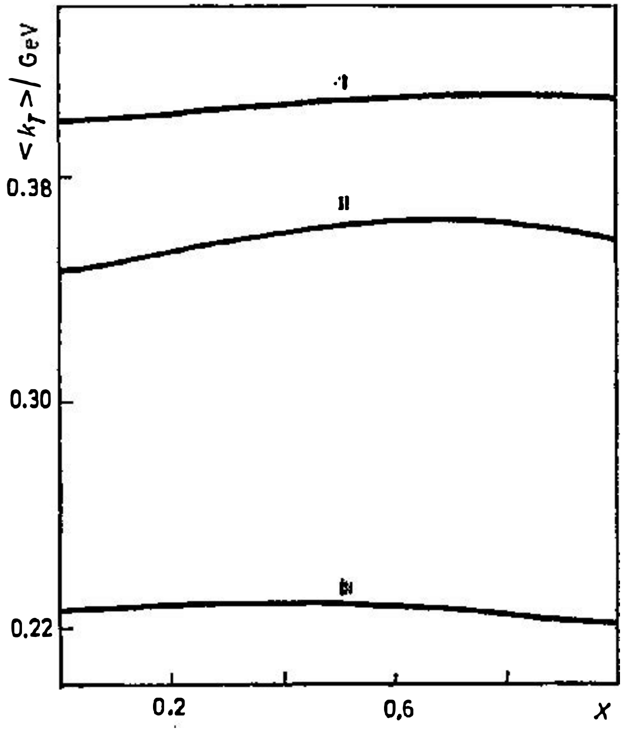

Fig. 2. $\langle k_T\rangle$ as a function of x, for all three cases I, II, III.

The MIT bag model, relativistic potential model and harmonic oscillator shell model were explored in some other partonlike phenomena, like twist-four effects in deep inelastic scattering on nucleonic targets[7,8], for example. We find that the simple HOS model gives the best fit for all calculated functions in the partonlike region of energies.

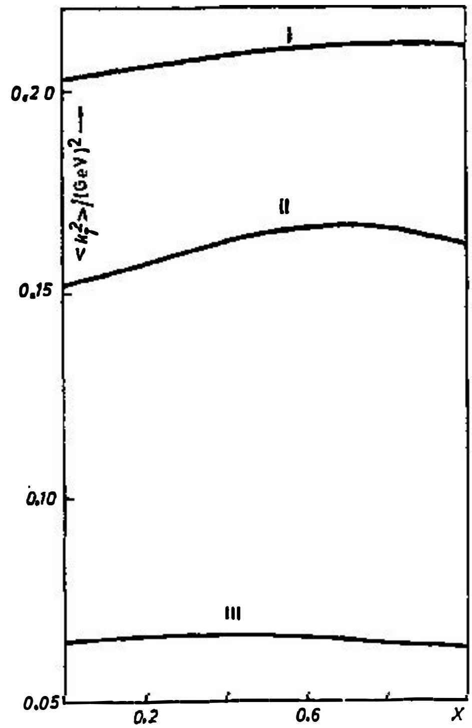

Fig. 3. $\langle k_T^2 \rangle$ as a function of x.

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X-OVISNOST KVARKNOG TRANSVERZALNOG IMPULSA U DUBOKO NEELASTIČNIM PROCESIMA

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Razmotrena je ovisnost o x k_T raspodjele koja je određena u jednostavnom kinematičkom modelu. Proveden je eksplicitni analitički proračun uz upotrebu kvarkovskih valnih funkcija iz raznih relativističkih potencijalnih modela.