

## IRREDUCIBILITY AND ASYMPTOTIC COMPLETENESS IN RENORMALIZABLE THEORIES: SOME REMARKS

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Remarks, which complement previous works, on irreducibility and asymptotic completeness in renormalizable theories are presented. The analysis may be useful in the derivation of asymptotic completeness in constructive theory, e. g. for the massive Gross-Neveu model in dimension 2 whose existence has been recently established.

### 1. Introduction

This note presents complements to Refs. 1 and 2 on irreducibility and asymptotic completeness in renormalizable theories. Although the analysis can be partly adapted (with complications) to more general models, we mainly consider as in Ref. 1 theories with massive particles in which the only renormalization parts are the 4-point ones (and propagators), and more precisely, for simplicity, a scalar theory with only one type of particle. While the existence of à la  $\Phi_4^4$  theories is doubtful, the analysis can be adapted with minor changes e. g. to the massive Gross-Neveu model in dimension 2 whose existence has been recently established<sup>3,4</sup>. Section 2 presents a different version, of interest from semi-perturbative and probably constructive viewpoints, of the analysis of Ref. 1 of the links between »regularized« and »renormalized« 2-particle irreducible kernels in a  $2 \rightarrow 2$  process and applications: derivation of (2-particle) asymptotic completeness from the irreducibility,

natural in this framework, of renormalized kernels, and (in a way similar to Ref. 1) more explicit justification or simple alternative (algebraic) derivation of renormalized Bethe-Salpeter equations proposed in Refs. 5 and 6. The related analysis of Ref. 1, from an axiomatic-type viewpoint, is presented in Section 3 with some complements. Irreducibility is understood in the analytic sense in momentum space (e. g. 2-particle irreducibility entails analyticity up to the 3-particle threshold, with no singularity at the 2-particle threshold). »Regularized« kernels satisfy regularized, non renormalized, Bethe-Salpeter type equations. »Renormalized« kernels are, from the perturbative viewpoint, (formal) sums of renormalized Feynman amplitudes of irreducible graphs. On the first application mentioned above and its converse in Section 3, the analysis complements the axiomatic quasi-equivalence established in Ref. 7 for regularized kernels and used (as also its extension<sup>8)</sup> to  $3 \rightarrow 3$  processes below the 5-particle threshold in an even theory) in constructive theory<sup>9, 10)</sup> in the derivation of asymptotic completeness for some *superrenormalizable* models; a case in which previous kernels coincide (no regularization needed).

The quasi-equivalence of Refs. 7 and 8 has on the other hand been complemented in Ref. 2 by a semi-heuristic analysis of the way irreducibility properties may yield unitarity, asymptotic completeness and related results in the more general multi-particle case (and more general energy regions), at least in the sense of formal expansions in terms of (regularized) irreducible kernels. This analysis was adapted in Ref. 1 to the renormalized framework in the case of 2-particle irreducibility in a  $2 \rightarrow 2$  process. It is explained in Section 4 how it can be adapted to the more general case, for some renormalizable theories, modulo the same conjectures as in Ref. 2.

## 2. 2-particle irreducibility and asymptotic completeness (semi-perturbative approach)

We start from the formal expansion<sup>6)</sup>  $F = \sum_{n \geq 1} (Go \dots oG)_r$  ( $n$  factors  $G$ ) of the connected  $2 \rightarrow 2$  Green function  $F$  in terms of the renormalized 2-p.  $i.$  kernel  $G$ , which follows from a partial resummation of the perturbative series for the class of theories mentioned in Section 1. Here  $o$  denotes a Feynman-type convolution with 2-point functions on internal lines ( $GoG \equiv \overbrace{G \ G}^{\text{---}}$ , ...) and  $(Go \dots oG)_r$  is (symbolically) the sum of all contributions associated with sets  $U$  (Zimmerman forests) of brackets that include two or more successive factors  $G$  or subbrackets and have no overlap:  $(GoG)_r = GoG - [GoG]$ ,  $(GoGoG)_r = GoGoG - [GoG]oG - Go [GoG] - [GoGoG] + [[GoG]oG] + [Go [GoG]]$  ..., with e. g.  $Go [GoG] \equiv (Go1) [GoG]$ , each bracket  $[ \ ]$  denoting the value of the function at e. g. zero external momenta (renormalization prescription). While individual terms are expected to be infinite, this definition can be made precise by methods of Ref. 6 either by first considering integrands (see Ref. 6), or by first defining  $(Go_\varepsilon \dots o_\varepsilon G)_r$ ,  $\varepsilon > 0$ , where  $o_\varepsilon$  includes a further analytic »regularization« factor on internal lines with sufficient decrease at infinity in euclidean directions to ensure convergence ( $\chi_\varepsilon \rightarrow 1$  when  $\varepsilon \rightarrow 0$ ) and then letting  $\varepsilon \rightarrow 0$ , modulo assumptions<sup>6)</sup> on the decrease of derivatives of  $G$  (related to those of Ref. 5 on the decrease of differences of values of  $G$  at various points) and on 2-point functions.

It is natural, from a semi-perturbative viewpoint, to introduce approximations  $F_\epsilon$  and  $G_\epsilon$  of  $F$  and  $G$ , which are formal sums of renormalized Feynman amplitudes with a further regularization  $\epsilon$  on all lines. The expansion of  $F$  then becomes:

$$F_\epsilon = \sum_{n \geq 1} \underbrace{(G_\epsilon o_\epsilon \dots o_\epsilon G_\epsilon)_r}_n \tag{1}$$

From its definition,  $\underbrace{(G_\epsilon o_\epsilon \dots o_\epsilon G_\epsilon)_r}_n$  can be written in the form:

$$\sum_{\substack{p \geq 1, n_1, \dots, n_p \\ \sum n_i = n}} \underbrace{[G_\epsilon o_\epsilon \dots o_\epsilon G_\epsilon]}_{n_1} o_\epsilon \dots o_\epsilon \underbrace{[G_\epsilon o_\epsilon \dots o_\epsilon G_\epsilon]}_{n_p} \tag{2}$$

where  $[\ ] \equiv G_\epsilon$  if  $n_i = 1$  and is for  $n_i > 1$  the sum of all contributions to  $(G_\epsilon o_\epsilon \dots o_\epsilon G_\epsilon)_r$  in which an overall bracket  $[ \ ]$  includes all factors  $G$ . (Each term is well defined at  $\epsilon > 0$ ). By formal reorderings, (1) and (2) give:

$$F_\epsilon = \sum_{q \geq 1} \underbrace{\hat{G}_\epsilon o_\epsilon \dots o_\epsilon \hat{G}_\epsilon}_q \tag{3}$$

$$\hat{G}_\epsilon = \sum_{n \geq 1} \underbrace{[G_\epsilon o_\epsilon \dots o_\epsilon G_\epsilon]}_n = G_\epsilon + C_\epsilon \tag{4}$$

where  $C_\epsilon$  is a constant, reexpressed below in terms of  $F_\epsilon$  and  $G_\epsilon$ . The expansion (3) entails the usual (regularized) B. S. equation:

$$F_\epsilon = \hat{G}_\epsilon + F_\epsilon o_\epsilon \hat{G}_\epsilon \tag{5}$$

Since  $C_\epsilon$  is a constant, the 2- $p$ .  $i$ . character of  $G_\epsilon$ , natural from the perturbative, and possibly constructive, viewpoint yields that of  $\hat{G}_\epsilon$ , itself quasi-equivalent by the analysis of Ref. 7 (with  $F$  replaced by  $F_\epsilon$ ) to the 2-particle unitarity or asymptotic completeness relation  $(F_\epsilon)_+ - (F_\epsilon)_- = (F_\epsilon)_+ * (F_\epsilon)_-$ , where  $*$  denotes on-mass-shell convolution  $((o_\epsilon)_+ - (o_\epsilon)_- = *, \forall \epsilon)$ . Although  $G_\epsilon$  and  $C_\epsilon$  are expected to become infinite when  $\epsilon \rightarrow 0$ , they no longer appear in this relation which gives the corresponding result on  $F$  itself in the  $\epsilon \rightarrow 0$  limit. Q. E. D.

*Renormalized Bethe-Salpeter equations*

The analysis above provides, for the same reasons as in Ref. 1, a more explicit and satisfactory justification of the derivation of integral equations proposed in Ref. 5 between  $F$  and differences of values of  $G$ . We now give, following Ref. 1 in a slightly different form (completed by the proof below of Eq. (8)), the algebraic derivation of a renormalized B.S. equation first proposed in Ref. 6 from a different perturbative analysis. Equations (5) and (4) give:

$$F_\epsilon = G_\epsilon + F_\epsilon o_\epsilon G_\epsilon + C_\epsilon (1 + F_\epsilon o_\epsilon) \tag{6}$$

where the equality  $[F_\varepsilon] = [G_\varepsilon]$  (in view of (1) and of the definition of renormalization) entails (in view of (4), (5), or of (6), at zero external momenta) that  $C_\varepsilon = (1 + [F_\varepsilon o_\varepsilon 1])^{-1} [F_\varepsilon o_\varepsilon G_\varepsilon]$ . Although individual terms  $F_\varepsilon o_\varepsilon G_\varepsilon$ ,  $F_\varepsilon o_\varepsilon 1$  are expected to become infinite when  $\varepsilon \rightarrow 0$ , this suggests that the r. h. s. of (6) has a well defined limit, i. e.:

$$F = G + \lim_{\varepsilon \rightarrow 0} \{F_\varepsilon o_\varepsilon G_\varepsilon - A_\varepsilon [F_\varepsilon o_\varepsilon G_\varepsilon]\} \tag{7}$$

where  $A_\varepsilon = (1 + [F_\varepsilon o_\varepsilon 1])^{-1} (1 + F_\varepsilon o_\varepsilon 1)$  admits the formal expansion:

$$A_\varepsilon = 1 + \sum_{n \geq 1} \underbrace{(G_\varepsilon o_\varepsilon \dots o_\varepsilon G_\varepsilon o_\varepsilon 1)}_n. \tag{8}$$

*Proof.* Equation (8) can be derived at each order either from the expansion (1) of  $F_\varepsilon$  as at the end of Section 3, or from the expansions analogous to (2) (with the last factor  $G_\varepsilon$  replaced by 1) of the terms  $(G_\varepsilon o_\varepsilon \dots o_\varepsilon G_\varepsilon o_\varepsilon 1)_r$ : the latter entail that the r. h. s.  $V_\varepsilon$  of (8) is equal to  $C'_\varepsilon + \sum_{n \geq 1} \hat{G}_\varepsilon o_\varepsilon \dots o_\varepsilon \hat{G}_\varepsilon o_\varepsilon C'_\varepsilon$ ,  $C'_\varepsilon = 1 + \Sigma [G_\varepsilon o_\varepsilon \dots o_\varepsilon \overline{G_\varepsilon o_\varepsilon 1}]$ . Hence, in view of (3),  $V'_\varepsilon = C'_\varepsilon (1 + F_\varepsilon o_\varepsilon 1)$ . Since  $[V_\varepsilon] = 1$ ,  $C'_\varepsilon = (1 + (F_\varepsilon o_\varepsilon 1))^{-1}$  and  $V_\varepsilon = A_\varepsilon$ . Q. E. D.

From assumptions analogous to above on  $G_\varepsilon$ , each term  $(G_\varepsilon o_\varepsilon \dots o_\varepsilon G_\varepsilon o_\varepsilon 1)_r$  admits a finite limit at  $\varepsilon = 0$ , which suggests that  $A_\varepsilon$  has itself a well defined limit  $A$ . Another form of the expression  $\{ \}$  in Eq. (7) in which individual terms should be finite at  $\varepsilon = 0$  can be obtained (see Ref. 6) by simple changes. One of the new terms involves a difference of values of  $G$  and is expected to be finite in view of the assumption on  $G$  previously mentioned. The second one involves a combination of  $F$  and  $A$ . A natural assumption on the decrease of this combination, linked via Fourier transformation to Wilson short-distance expansion in space-time, follows (see Ref. 6).

### 3. 2-particle irreducibility and asymptotic completeness (axiomatic-type approach)

By abuse, we use the same notation  $\hat{G}_\varepsilon$  as in Section 2 now for the axiomatic 2-p. i. kernel<sup>7)</sup> satisfying  $(\forall \varrho > 0)$ :

$$F = \hat{G}_\varrho + F o_\varrho \hat{G}_\varrho. \tag{9}$$

While renormalized kernels are not a priori defined in the axiomatic framework, we now define, following Ref. 1,  $G_\varrho$  (with again the same notation as in Section 2 for a different function) as:

$$G_\varrho = \hat{G}_\varrho + ([F] - [\hat{G}_\varrho]). \tag{10}$$

Eqs. (9) and (10) now yield the renormalized equation:

$$F = G_\rho + F_{o_\rho} G_\rho - A_\rho [F_{o_\rho} G_\rho] \tag{11}$$

where  $A_\rho = (1 + [F_{o_\rho} 1])^{-1} (1 + F_{o_\rho})$ . The fact that  $G_\rho$  (or  $\hat{G}_\rho - [\hat{G}_\rho]$ ) has a well defined limit  $G$  (resp.  $G - [F]$ ) in contrast to  $\hat{G}_\rho$ , as also suitable decrease properties of derivatives or differences of values of  $G_\rho$  and  $G$ , can be viewed in this framework as particular properties of the class of theories considered. On the other hand, asymptotic completeness, if assumed from the axiomatic viewpoint, yields the 2-p. i. character of  $\hat{G}_\rho$ , hence in turn in view of (10) that of  $G_\rho$  and of  $G$  in the  $\rho \rightarrow 0$  limit up to technical assumptions.

A formal expansion of  $F$  analogous to (1), but not given at the outset in the present framework, and a corresponding expansion of  $A_\rho$  hold:

$$F = \sum_{n \geq 1} \underbrace{(G_\rho o_\rho \dots o_\rho G_\rho)}_n, \tag{12}$$

$$A_\rho = 1 + \sum_{n \geq 1} \underbrace{(G_\rho o_\rho \dots o_\rho G_\rho o_\rho 1)}_n. \tag{13}$$

We give below the proof (omitted in Ref. 1) of Eqs. (12) and (13) and relevant remarks.

*Proof.* Equation (12) follows from (11) as checked at each order by induction on  $n$  since (11) is satisfied (formally) when  $F$  is replaced by the r. h. s.  $A_\rho$  of (12); the latter result is obtained by the same algebraic analysis as in Section 2. (Eqs. (2) to (6)). (The regularized kernel that occurs in this analysis does coincide for each  $\rho > 0$  with  $\hat{G}_\rho$  since it finally satisfies the same Fredholm-type equation (9)). Equation (13) follows in turn, also as in Section 2. Q. E. D.

*Remarks.* (i)  $\hat{G}_\rho$  admits, in view of the above analysis, the formal expansion  $\sum [G_\rho o_\rho \dots o_\rho G_\rho]$ . The latter can alternatively be derived more directly by inversion of the expansion  $G_\rho = \hat{G}_\rho + \sum [\hat{G}_\rho o_\rho \dots o_\rho \hat{G}_\rho]$  that follows easily from (10) and (9). It then yields, in view of (9) and of the analogous of (2), an alternative proof of (12).

(ii) It was shown in the proof above that (11) is satisfied by  $A_\rho$ , i. e. that, for each  $n > 1$ ,  $\underbrace{(G_\rho o_\rho \dots o_\rho G_\rho)}_{n+1}$  is more explicitly equal to:

$$\underbrace{(G_\rho o_\rho \dots o_\rho G_\rho)}_n o_\rho G_\rho - \sum (-1)^p \{ \underbrace{(G_\rho o_\rho \dots o_\rho G_\rho)}_1 o_\rho 1 \} \tag{14}$$

$$\left( \prod_{i=1}^p \underbrace{[(G_\rho o_\rho \dots o_\rho G_\rho)]_{n_i}}_{n_i} \right) \underbrace{[(G_\rho o_\rho \dots o_\rho G_\rho)]_m}_{m} o_\rho G_\rho$$

with a sum over  $p \geq 0$ ,  $l \geq 0$ ,  $m \geq 1$ ,  $n_1, \dots, n_p \geq 1$ ,  $l + m + \sum n_i = n + 1$ . (The terms  $l, m, n_1$  arise, respectively, from terms occurring in  $F_{o_\rho} 1$ ,  $[F_{o_\rho} G_\rho]$  and  $(1 + [F_{o_\rho} 1])^{-1} = 1 + \sum (-1)^p [F_{o_\rho} 1]^p$ . Factors  $l = 0$  and  $p = 0$  are equal to 1). The result (14) can be checked alternatively by regrouping contributions to  $(G_\rho o_\rho \dots o_\rho G_\rho)_r$  associated with sets  $U$  in which the same subset  $S$  of brackets  $[ ]$  contain the last factor  $G_\rho$  (and contain, respectively,  $m + 1, m + 1 + n_1, \dots, \dots, m + 1 + n_1 + \dots + n_p = n + 1 - l$  factors  $G_\rho$ . The first term in the r. h. s. of (14) is obtained when  $S$  is empty). This method also yields (13) (and is related in this case to a method of Ref. 6 for  $\Lambda$ ).  $\Lambda_\rho$  is again expected to have a finite limit.

(iii) Expansions of  $\hat{G}_\rho$  in terms of  $G_\varepsilon$  or  $G$  (where  $G_\varepsilon$  is the kernel defined in (10) with  $\rho$  replaced by  $\varepsilon$ ) are given in Ref. 1. They yield corresponding expansions  $G_\rho = G_\varepsilon + \sum_{n \geq 2} (G_\rho)_{n,\varepsilon}$  where  $(G_\rho)_{n,\varepsilon}$  is equal to:

$$\begin{aligned} & \sum_{p \geq 1} (-1)^{p+1} \{ \underbrace{(G_\varepsilon o_\varepsilon G_\varepsilon \dots o_\varepsilon G_\varepsilon)}_{n_1} \}_r o_\rho \dots o_\rho \underbrace{(G_\varepsilon o_\varepsilon \dots o_\varepsilon G_\varepsilon)}_{n_p} - [\text{id.}] = \\ & = \sum_{p \geq 2} \{ \underbrace{[G_\varepsilon o_\varepsilon \dots o_\varepsilon G_\varepsilon]}_{n_1} x_{\varepsilon,\rho} \dots x_{\varepsilon,\rho} \underbrace{[G_\varepsilon o_\varepsilon \dots o_\varepsilon G_\varepsilon]}_{n_p} - [\text{id.}] \} \end{aligned} \quad (15)$$

with in each case a summation over  $n_1, \dots, n_p$ ,  $\sum n_i = n$  and  $x_{\varepsilon,\rho} = o_\varepsilon - o_\rho$ . In contrast to  $o_\varepsilon$  and  $o_\rho$  (whose discontinuity is  $*$ ),  $x_{\varepsilon,\rho}$  is »2-p. i.« ( $x_+ - x_- = 0$ ). The first form of  $(G_\rho)_{n,\varepsilon}$  shows that it is still finite at  $\varepsilon = 0$ . The second one shows how the 2-p. i. character of  $G_\varepsilon$  yields that of  $G_\rho$  at each order.

#### 4. Formal expansions, irreducibility and asymptotic completeness in renormalizable theories

The way the 2-particle irreducibility of the regularized kernel  $\hat{G}_\rho$  yields the 2-particle asymptotic completeness relation  $F_+ - F_- = F_+ * F_-$  (valid in the low region) can be understood as follows<sup>1)</sup> in terms of the formal expansion  $F = \sum \hat{G}_\rho o_\rho \dots o_\rho \hat{G}_\rho$ . A repeated application of the relation  $(o_\rho)_+ - (o_\rho)_- = *$  and the assumed analyticity of  $\hat{G}_\rho$  up to the 3-particle threshold yield, for each  $n$ , the discontinuity formula:

$$\begin{aligned} & \underbrace{(\hat{G}_\rho o_\rho \dots o_\rho \hat{G}_\rho)_+}_n - \underbrace{(\hat{G}_\rho o_\rho \dots o_\rho \hat{G}_\rho)_-}_n = \sum_{1 \leq q < n} \underbrace{(\hat{G}_\rho o_\rho \dots o_\rho \hat{G}_\rho)_+}_q * \\ & \quad * \underbrace{(\hat{G}_\rho o_\rho \dots o_\rho \hat{G}_\rho)_-}_{n-q} \end{aligned} \quad (16)$$

The relation  $F_+ - F_- = F_+ * F_-$  follows in the sense of formal expansions in  $\hat{G}_\rho$ . In Ref. 2 a general formalism of formal series expansions of multiparticle Green functions in terms of regularized Feynman-type integrals with irreducible kernels at each vertex and 2-point functions on internal lines is conjectured. There

is one minimal expansion of this type in each energy region. In theories without renormalization, these expansions correspond to partial regroupings of Feynman integrals (with no regularization). Otherwise, it is assumed that expansions of the same algebraic form hold in terms of regularized integrals, as checked in the  $2 \rightarrow 2$  and  $3 \rightarrow 3$  cases in the low energy region from the axiomatic results of Refs. 7 and 8. Conjectures on discontinuities of these integrals are then proposed and shown to yield corresponding results (again in a formal sense) on the  $S$ -matrix and Green functions: unitarity and asymptotic completeness relations (which involve different boundary values in the general case) as also  $S$ -matrix discontinuity formulae such as those needed in multiparticle dispersion relations (generalized optical theorems).

The previous analysis in the  $2 \rightarrow 2$  case in the low energy region has been adapted in Ref. 1 to the renormalized framework, for the class of theories considered in Sections 2 and 3 on the basis of the formal expansion  $F = \Sigma (Go \dots oG)_r$ , in which case (16) is replaced by:

$$\underbrace{(Go \dots oG)_r^+}_n - \underbrace{(Go \dots oG)_r^-}_n = \sum_{1 \leq q < n} \underbrace{(Go \dots oG)_r^+}_q * \underbrace{(Go \dots oG)_r^-}_{n-q} \quad (17)$$

Equation (17) can be checked<sup>1)</sup> by replacing  $o$  by  $o_\epsilon$  and letting  $\epsilon \rightarrow 0$ . At  $\epsilon > 0$ , it follows from (2) (with  $G_\epsilon$  replaced by  $G$ ): since each factor [ ], which is either equal to  $G$  or is a constant, satisfies the analyticity associated with 2-particle irreducibility, the discontinuity of each term in (14) is as before a sum of terms obtained by replacing one of the operations  $o_\epsilon$  by  $*$ , with plus and minus  $i\epsilon$  boundary values on each side. Equation (17) follows by a simple regrouping of terms, using again equation (2) for  $q$  and  $n-q$  factors  $G$ . (An alternative proof will be found in Ref. 6).

In the more general case, formal series expansions analogous to those of Ref. 2 and generalizing the expansion  $\Sigma (Go \dots oG)_r$  can be conjectured from perturbation theory, with renormalized irreducible kernels at each vertex. The Feynman-type integrals involved are no longer regularized, but renormalized (with a renormalization procedure which again does not apply to individual kernels). This can be checked for the class of theories considered in Section 1, where it amounts to re-

place all sequences 

occurring in the integrals by 

(We also note that, in a theory in which the only vertices from the perturbative viewpoint are 4-point vertices, many irreducible kernels vanish when the degree of irreducibility increases: e. g. there exists no totally  $5-p$ . *i.* graph in a  $3 \rightarrow 3$  channel and correspondingly totally  $r-p$ . *i.* kernels vanish for  $r \geq 5$ . On the other hand, totally  $r-p$ . *i.* kernels in a  $2 \rightarrow 2$  channel reduce to a constant for  $r > 2$ , corresponding to the trivial graph with one vertex). For more general theories, it is no longer possible to »separate« in the same simple way as above renormalization into that

achieved «inside» the irreducible kernels and that of the integrals: this already occurs in the  $2 \rightarrow 2$  case where the expansion  $F = \Sigma (Go \dots oG)$ , is no longer valid if the theory includes from the perturbative viewpoint renormalization parts with more than 4 (external) lines. One may then have to consider from the outset irreducible kernels of higher degrees and renormalized Feynman-type integrals in the expansions will include also renormalization parts with more than 4 lines.

Being given a theory in which expansions of the type described above can be conjectured, the same method as that outlined below equation (17) allows one to obtain discontinuity formulae of the integrals from the conjecture of Ref 2. in the regularized case. These formulae have the same algebraic form as in Ref. 2, except that factors involved on each side of a cut (corresponding to on-mass-shell convolution over a set of internal lines) are themselves renormalized subintegrals. To show this, one may again first introduce a regularization  $\varepsilon$  on internal lines, use decomposition analogous to (2) of renormalized integrals (based on general factorization properties of the renormalization procedure, e. g. in terms of Zimmerman forests), write corresponding discontinuity formulae for individual terms, regroup terms and let  $\varepsilon \rightarrow 0$ . Application to unitarity, asymptotic completeness and  $S$ -matrix discontinuity formulae then follow from the same algebraic analysis as in Ref. 2.

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## IREDUCTIBILNOST I ASIMPTOTSKA POTPUNOST' RENORMALIZABILNIH TEORIJA

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Originalni znanstveni rad

Dane su primjedbe koje upotpunjuju ranije radove o ireducibilnosti i asimptotskoj potpunosti renormalizabilnih teorija.