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INFLUENCE OF THREE-BODY FORCES ON THE $T = 1$ SPECTRUM IN ^{4}He

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A ^{4}He shell-model formalism, including two- and three-body forces, is used to calculate the energy and width of the following $T = 1$ levels of Fiarman and Meyerhof: 2 (2 .4 MeV), 1^- (27.4 MeV), 0^- (29.5 MeV) and 1^- (30.5 MeV). Three-body plus two-body forces lead to significant improvements, in comparison with only two-body forces, in the calculation of energy level estimates for the $T = 1$ states noted above. Improvements in the level width are difficult to ascertain because of the experimental uncertainties in determining the level width.

1. Introduction

Improvements in the calculated 0^+ $T = 0$ ^{4}He spectrum as well as an improved charge form factor and rms radius were recently obtained by introducing a shell-model Hamiltonian which includes two-body plus three-body (TPTB) forces[1]. The success of the TPTB model in describing the $(0^+, 0)$ spectrum suggests that the inclusion of three-body forces may lead to improvements in other ^{4}He levels. A logical choice of ^{4}He levels for investigation is the $T = 1$ spectrum which is fairly well established[2–8]. An investigation of the $T = 0$ spectrum is more uncertain because of the large experimental uncertainties in the position and width of these levels[5,6].

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This paper will apply the TPTB model to the calculation of the level width and energy of the $T = 1$ levels of Fiarman and Meyerhof[5] — i. e. the $(2^-, 1)$ 26.4 MeV, $(1^-, 1)$ 27.4 MeV, $(0^-, 1)$ 29.5 MeV and $(1^-, 1)$ 30.5 MeV levels. The calculations will derive the strengths of the three-body interaction for these states. The reader should note that the TPTB shell-model approach will lead to highly model dependent results[1]. Even though our results will be model dependent, they will provide an indication of the applicability of three-body forces in the ^{4}He $T = 1$ spectrum.

The approach taken in this paper in deriving a three-body interaction is only one of a large number of three-body approaches which have appeared in the literature. The reader is refered to Refs. 5 and 9—33 for a selection of papers which discuss the use of other three-body forces in the $A = 3$ and $A = 4$ systems.

2. *Formalism*

Following the approach used in the three-body study of the ground state and first excited state[1], the three-body force components for $T = 1$ levels will be determined by considering the differences between the measured and calculated (two--body) position of the lowest lying $T = 1$ state $(2^-, 26.4$ MeV$)$[5]. This difference may be minimized by including a three-body term into the standard two-body Hamiltonian

$$H' = H + U \tag{1}$$

where H is the two-body Hamiltonian[6] and U is the three-body term[1]. The choice of the three-body term is motivated by a recent study of the splitting of the ground and first excited state in ^{4}He[1]. The three-body term is defined in terms of a projection operator which selects $A = 3$ triton $(|p_1\rangle)$ or ^{3}He $(|p_2\rangle)$ clusters from the ^{4}He basis state[1,6,34]

$$U = \sum_{j=1}^{2} \sum_{i=1}^{2} |p_i\rangle \Delta_{ji} \langle p_i|. \tag{2}$$

The quantities appearing in Eq. (2) are discussed in detail in Ref. 1. The term Δ_{ji} represents the strength of the three-body interaction. Since the low lying states in the ^{4}He system are composed of a few dominant configurations (at least in terms of the total oscillator content)[1,4,6], the following simplifying assumptions are made for the strengths $\Delta_{ji}(N_{12}, L_{12}, N_B, L_B, i)$

$$\Delta_{ji}(N_{12}, L_{12}, N_B, L_B, i) = \Delta_\varrho \tag{3}$$

where

$$\varrho = 2(N_{12} + N_B) + L_{12} + L_B. \tag{4}$$

In Eqs. (3) and (4), N_i and L_i label the radial and orbital quantum numbers, respectively, in the $\vec{r}_i$ coordinate of the $A = 3$ cluster. The $\vec{r}_{12}$ coordinate labels the $pp\,(nn)$ coordinate and $\vec{r}_B$ labels the coordinate between the center-of-mass of the $pp\,(nn)$ cluster and the remaining nucleon $n\,(p)$ for ^{3}He$(^3$H$)$[34].

The constraint of Eq. (3) implies that the three-body strengths are only dependent on the total oscillator constant (ϱ) of the $A = 3$ cluster state which is defined by Eq. (4). For example, all $A = 3$ clusters with $3\,\hbar\,\omega$ of internal excitation, such as (0s) (0f), (0p) (0d), and (1p) (0s), have the same three-body strength within the framework of our model.

Following the methodology of Ref. 1, Δ_1 and Δ_3 may be obtained by fitting the model $(2^-, 1)$ eigenenergy to the experimental value of 26.4 MeV. This is achieved with the values $\Delta_1 = -0.49$ MeV and $\Delta_3 = -0.65$ MeV. These values are considerably smaller than the $\Delta_0 = +1.86$ MeV and $\Delta_2 = -3.60$ MeV three-body strengths of Ref. 1. However, the Δ_1 and Δ_3 values are reasonable because the differences between two-body model and experimental values in $T = 0$ levels, used to obtain Δ_0 and Δ_2, are considerably larger than the $T = 1$ differences.

The aforementioned Δ_i values complete the specification of the model three-body force. When this force is combined with the model two-body interaction[6,35], the general Hamiltonian of Eq. (1) is completely specified and can be used in an analogous manner to the standard Hamiltonian H in the generalized R-matrix equation[36,37]

$$\sum_{\lambda'} [\langle\lambda\,|H' - E|\,\lambda'\rangle + \sum_{c} \gamma_{\lambda c}\,(b_{\lambda' c} - b_c)\,\gamma_{\lambda' c}]\,A_{\lambda'} = 0. \qquad (5)$$

The quantities appearing in Eq. (5) are defined in detail in Refs. 6 and 37 and will not be discussed further herein. The determination of level energies and widths is achieved from the information appearing in Eq. (5).

3. *Comparison of the TPTB interaction with other three-body interactions*

In this section, we compare the results of the TPTB potential with the historical Fujita-Miyazawa (FM) potential[9] as well as with the more recent Tucson potential[22]. These interactions differ in their level of detail and in the underlying physical assumptions used to establish the respective interactions.

In the work of Fujita and Miyazawa[9], the πN amplitude was assumed to be dominated by the excitation of the Δ resonance. The FM interaction is also based on the assumption that the background amplitude could be properly represented by the isospin symmetric scattering length. As noted by Coelho et al.[33], both assumptions are not universally valid. For example, the Δ dominance of the πN amplitude occurs only when the total center of mass energy is about equal to the mass of the resonance. In addition, contributions from σ and ϱ exchange are often as important as the Δ contribution. These comments show that the Δ based πN amplitude which is the basis of the FM potential does not reproduce the on-shell data, and is not suitable for off-shell extrapolation. These limitations cause the FM potential to be only partially realistic. However, the FM interaction was developed in 1957 and has made a significant contribution to the understanding of multibody forces.

A more modern version of the FM potential is the potential derived by the Tucson group[22] and by Coon and Glöckle[23]. The Tucson potential is based on an off-shell πN amplitude which was constructed by means of chiral symmetry and also reproduces the on-shell data. The basis on chiral symmetry indicates that the Tucson potential is appropriate for off-shell extrapolation. This interaction involves two-pion-exchange three-nucleon forces. The Tuscon potential, which puts the description of the off-shell πN amplitude on the generally accepted phenomenology of current algebra and partial conservation of axial-vector current, is today considered to be one of the best available three-body interactions.

The TPTB interaction proposed in this paper is based on a philosophy which does not attempt to directly describe the πN amplitude. Instead, the TPTB interaction is based on reproducing the ^{4}He level spectrum which is affected by the πN amplitude. In its present form, the TPTB interaction is somewhat simplistic since it is based on projection operators with strengths which are constant in a given oscillator shell. However, future work will recast and generalize the TPTB formalism into a form which is similar to a πN amplitude approach. At that time, a more in depth comparison of the TPTB potential with the Fujita-Miyazawa and Tucson potentials will be possible.

4. *Results and discussion*

Using the Δ_1 and Δ_3 values, calculations for other $T = 1$ levels can be performed. Herein, we restrict consideration to the $T = 1$ levels summarized in the compilation of Fiarman and Meyerhof[5].

Table 1 summarizes the results of two-body (TB) and two- plus three-body (TPTB) calculations for the 2^- (26.4 MeV), 1^- (27.4 MeV), 0^- (29.5 MeV) and 1^- (30.5 MeV) levels. Model calculations using the TPTB force are improved considerably in comparison with TB results for the $T = 1$ states considered herein. The average value of the magnitude of the difference between calculated and experimental level positions is significantly improved with the use of three-body forces. This value decreases from 2.6 MeV for them TB force to a value of 0.25 MeV for the TPTB interaction. The most striking improvement occurs for the 1^- (27.4 MeV) level. The TB force predicts this level occurs at 31.7 MeV and the TPTB interaction predicts a value of 27.9 MeV which is within 0.5 MeV of the experimental position (27.4 MeV).

TABLE 1.

J^π	Excitation energy/MeV		
	TB	TPTB	*Experiment*[a]
2^-	29.0	26.4	26.4
1^-	31.7	27.9	27.4
0^-	30.5	29.1	29.5
1^-	33.0	30.4	30.5

a) Ref. 5.

$T = 1$ level energies with two-body and three-body forces in ^{4}He.

TABLE 2.

J^{π} (E_x MeV)	Level width /MeV		
	TB	TPTB	*Experiment*[a)]
2^-(26.4)	0.7	1.6	≈10.0
1^-(27.4)	2.5	4.2	≈10.0
0^-(29.5)	4.2	3.9	≈10.0
1^-(30.5)	5.7	5.1	≈10.0

a) Calculated from the single particle widths of Ref. 38.

$T = 1$ level widths with two-body and three-body forces in ^{4}He.

The TPTB force also alters the calculated $T = 1$ level widths. Experimental widths for all the $T = 1$ levels considered herein are predicted to be on the order of 10 MeV [35]. The 2^- (26.4 MeV) level width increases from 0.7 MeV (TB) to 1.6 MeV with the TPTB force. The 1^- (27.4 MeV) width also is increased with the use of the TPTB force (2.5 MeV (TB) to 4.2 MeV (TPTB)). However, the calculated TPTB 0^- (29.5 MeV) and 1^- (30.5 MeV) level widths are slightly smaller than the TB widths. In view of the uncertainties in the experimental widths [38], it is not possible to ascertain if TPTB forces significantly improve the calculated level widths.

5. *Conclusions*

The results of this study suggest that three-body forces when combined with standard two-body forces lead to improvements in the energies of the negative parity, $T = 1$ levels of Fiarman and Meyerhof. When these $T = 1$ results are combined with previous $T = 0$ results for the 0^+ spectrum, they provide added evidence for the importance of three-body forces in the ^{4}He system.

References

1) J. J. Bevelacqua, Phys. Rev. **C26** (1982) 1292;
2) P. Kramer and M. Moshinsky, Phys. Lett. **23** (1966) 574;
3) P. P. Szydlik, Phys. Rev. **C1** (1970) 146;
4) P. P. Szydlik, J. R. Borysowicz and R. F. Wagner, Phys. Rev. **C6** (1972) 1902;
5) S. Fiarman and W. E. Meyerhof, Nucl. Phys. **A206** (1973) 1;
6) J. J. Bevelacqua and R. J. Philpott, Nucl. Phys. **A275**(1977) 301;
7) J. J. Bevelacqua, Can. J. Phys. **57** (1979) 1833;
8) J. J. Bevelacqua, Can. J. Phys. **58** (1980) 1555;
9) J. Fujita and H. Miyazawa, Prog. Theor. Phys. **17** (1957) 360;
10) B. A. Loiseau, Y. Nogami and C. U. Ross, Nucl. Phys. **A165** (1971) 601; **A176** (1971) 665 (E);
11) L. M. Delves and M. A. Hennel, Nucl. Phys. **A168** (1971) 347;
12) G. E. Brown, in *Few Particle Problems in the Nuclear Interaction* (Ed. I. Šlaus et al.), North Holland (1972);
13) D. D. Brayshaw, Phys. Rev. **C7** (1974) 1731;
14) J. Žofka and M. Sotona, Czech. J. Phys. **B24** (1974) 1250;
15) M. Sato, Y. Akaishi and H. Tanaka, Suppl. Prog. Theor. Phys. **56** (1974) 76; J. Phys. Soc. Japan **44** (1978) 311;

16) S. A. Coon, M. D. Scadron and B. R. Barrett, Bucl. Phys. **A242** (1975) 467; **A254** (1975) 542 (E);
17) S. Fiarman and S. S. Hanna, Nucl. Phys. **A251** (1975) 1;
18) P. Grangé, M. Martzolff, Y. Nogami, D. W. L. Sprung and C. U. Ross, Phys. Lett. **60B** (1976) 237;
19) P. Grangé and M. Martzolff, Lett. Nuovo Cimento **16** (1976) 156;
20) T. Ueda, T. Sawada and S. Takagi, Nucl. Phys. **A285** (1977) 429;
21) M. Fabre de la Ripelle, Comptes Rendus **288B** (1979) 325;
22) S. A. Coon, M. D. Scadron, P. C. McNamee, B. R. Barrett, D. W. E. Blatt and B. H. J. McKellar, Nucl. Phys. **A317** (1979) 242;
23) S. Coon and W. Glöckle, Phys. Rev. **C23** (1981) 1790;
24) J. Carlson and V. R. Pandharipande, Nucl. Phys. **A371** (1981) 301;
25) J. Torre, J. J. Benayoun and J. Chouvin, Z. Phys. **A300** (1981) 319;
26) Y. Nogami, N. Ohtsuka and L. Consoni, Phys. Rev. **C23** (1981) 1759;
27) Muslim, Y. E. Kim and T. Ueda, Phys. Lett. **115B** (1982) 273;
28) H. T. Coelho, T. F. Das and M. Fabre de la Ripelle, Phys. Lett. **109B** (1982) 255; Lett. Nuovo Cimento **33** (1982) 1;
29) T. K. Das, H. T. Coelho and M. Fabre de la Ripelle, Phys. Rev. **C26** (1982) 2288;
30) R. A. Brandenburg and W. Glöckle, Nucl. Phys. **A377** (1982) 379;
31) W. Glöckle, Nucl. Phys. **A381** (1982) 343;
32) A. Bömelburg, Phys. Rev. **C28** (1983) 403;
33) H. T. Coelho, T. K. Das and M. R. Robilotta, Phys. Rev. **C28** (1983) 1812;
34) J. J. Bevelacqua, Can. J. Phys. **57** (1979) 404;
35) J. P. Elliott, A. D. Jackson, H. Mavromatis, E. A. Sanderson and B. Singh, Nucl. Phys. **A121** (1968) 241;
36) A. M. Lane and D. Robson, Phys. Rev. **151** (1966) 774; **178** (1969) 1715; **185** (1969) 1403;
37) R. J. Philpott and J. George, Nucl. Phys. **A233** (1974) 164;
38) C. Werntz and W. E. Meyerhof, Nucl. Phys. **A121** (1969) 8.

UTJECAJ TROČESTIČNIH SILA NA $T = 1$ SPEKTAR U He^4

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Energije i širine $T = 1$ nivoa u ^{4}He: 2^- (26.4 MeV), 1^- (27.4 MeV), 0^- (29.5 MeV) i 1^- (30.5 MeV) izračunati su koristeći formalizam modela ljuske, te uključivši sile dva tijela i tri tijela. Proračun izvršen kada su pored sile dvaju tijela uvedene i sile triju tijela daje bitno poboljšanje u proračunu energija. Teško je procijeniti da li ima poboljšanja u izračunatoj širini stanja uslijed eksperimentalnih neodređenosti.